Performance Evaluation of Stabilized F-bar Aided Edge-based Smoothed Finite Element Method with Four-node Tetrahedral Elements (SymF-barES-FEM-T4) for Contact Problems

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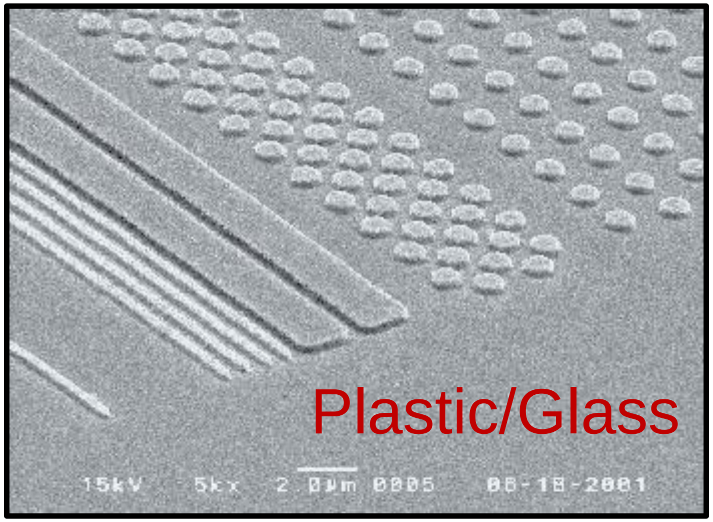
Motivation

What we want to do:

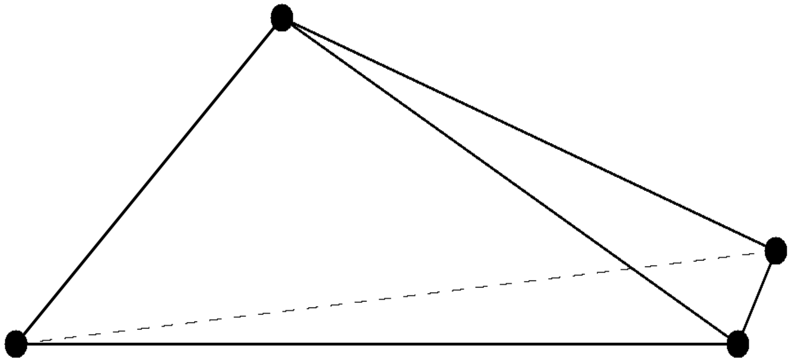
- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



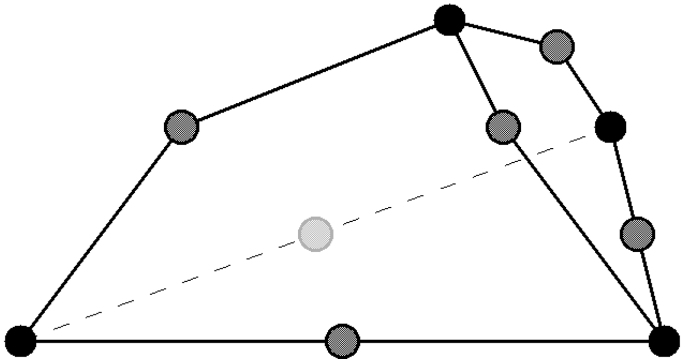
Issue in Ordinary T10 Elements

Issue

When we use 10-node tetrahedral (T10) mesh,
the number of nodes gets much larger
to express complex shapes without element kink.



✓ Skewed T4



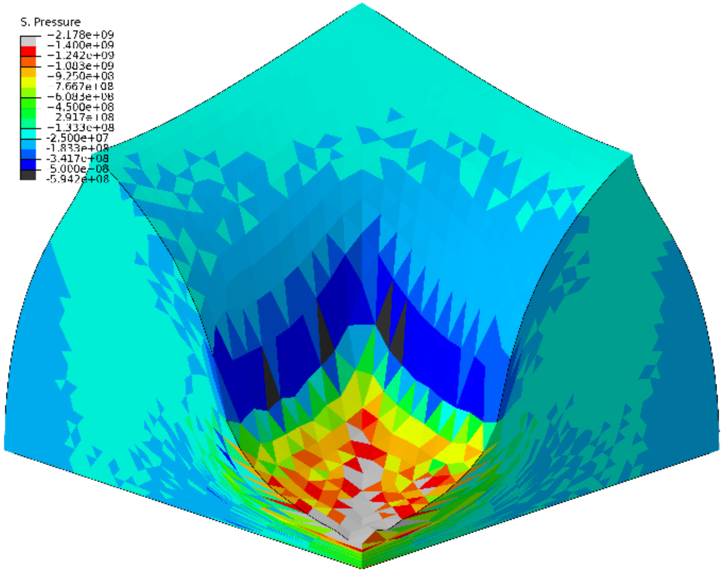
✗ Kinked T10

Possible Solutions

1. Robust T10 element (method in the previous talk)
2. Accurate T4 element (method in THIS talk)

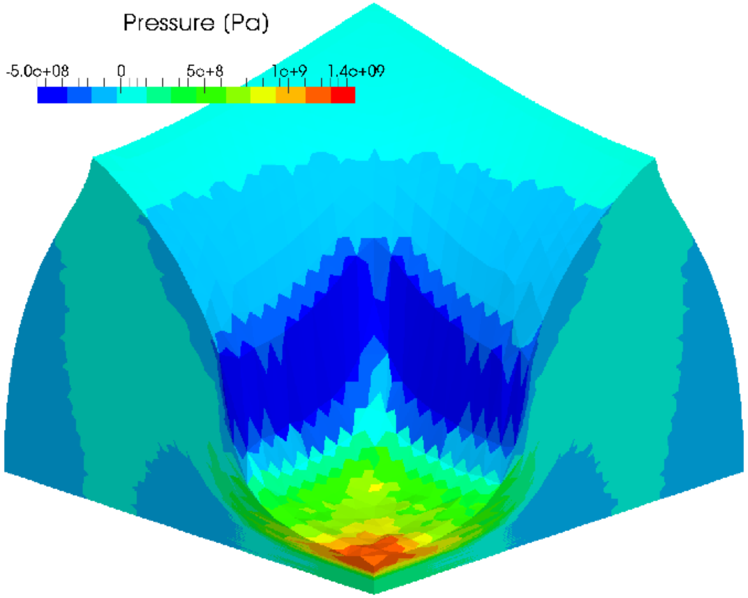
Our T4 Method for Static Problems

Our group has proposed a new S-FEM-T4 formulation, **F-barES-FEM-T4**, detailed later.



ABAQUS C3D4H

✗ pressure oscillation



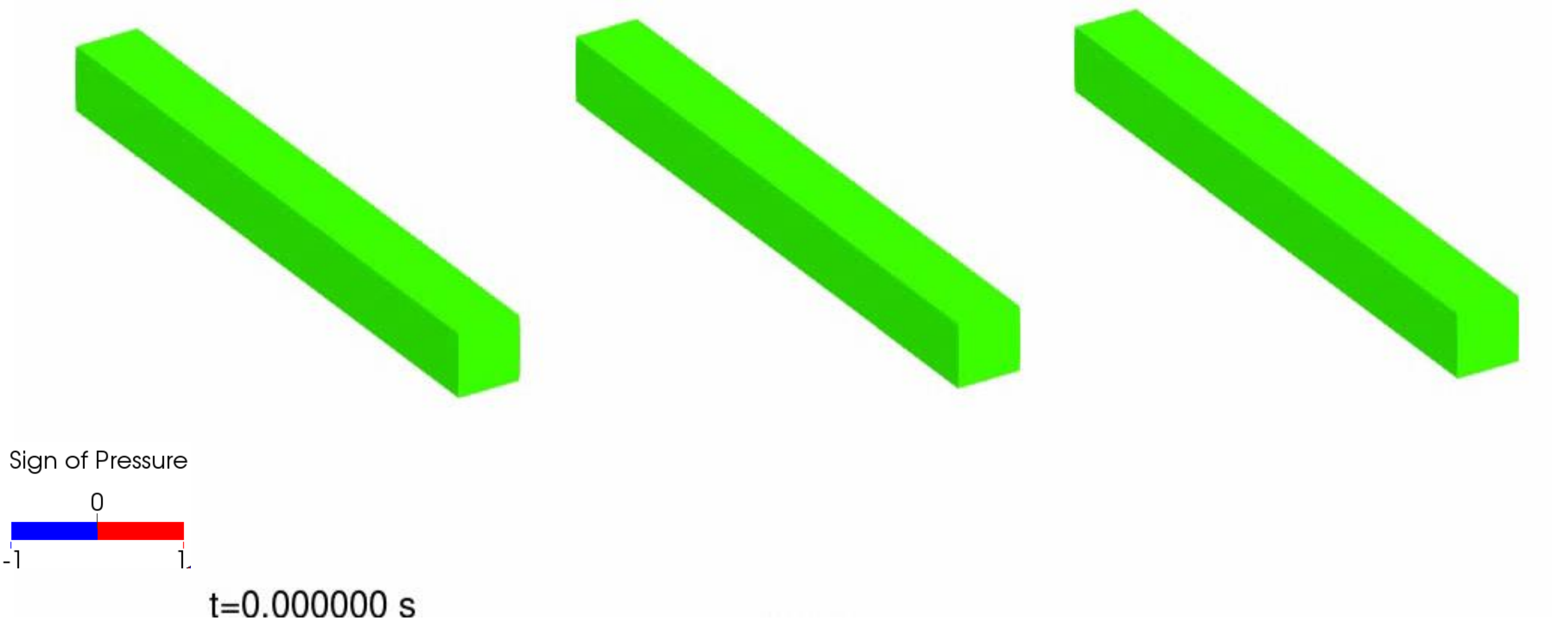
F-barES-FEM-T4(3)

↗ # of cyclic smoothings

F-barES-FEM-T4 shows excellent accuracy in ***static*** problems!

Our T4 Method for Dynamic Problems

Our group has proposed another S-FEM-T4 formulation, **SymF-barES-FEM-T4**, detailed later.



ABAQUS/Explicit C3D8
(H8-SRI element)

F-barES-FEM-T4(2)
✗ Energy divergence

SymF-barES-FEM-T4(2)

SymF-barES-FEM-T4 shows good accuracy and stability in ***dynamic*** problems!

Objective

FAQ

How about the stability in dynamic contact problems?

Objective

To evaluate the stability of SymF-barES-FEM-T4
in explicit dynamics with contact.

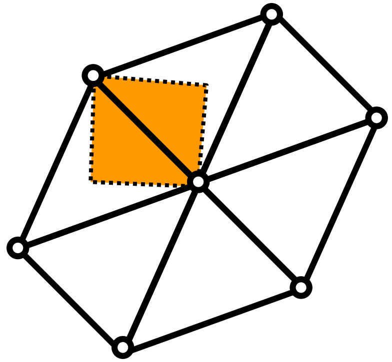
Table of Body Contents

- Methods: Quick introduction of SymF-barES-FEM-T4
- Results & Discussion: A few verification analyses
- Summary

Methods

Formulation of F-barES-FEM (1 of 2)

Deformation gradient of each edge ($\bar{\boldsymbol{F}}$) is derived as $\bar{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$



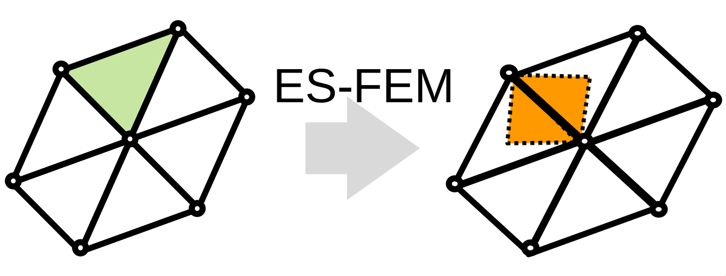
ES−FEM

Formulation of F-barES-FEM (2 of 2)

Each part of $\bar{\boldsymbol{F}}$ is calculated as follows.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part

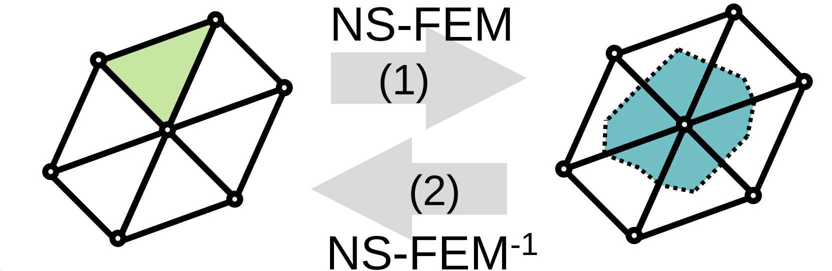


Smoothing ***F***s of adjacent elements at each edge.

↓

The same manner as ES-FEM.

Volumetric part



(1)Calculating node's ***F*** by smoothing ***F***s of adjacent elements.

(2)Calculating elements' ***F*** by smoothing ***F***s of adjacent nodes.

(3)Repeating (1) and (2) a few times.

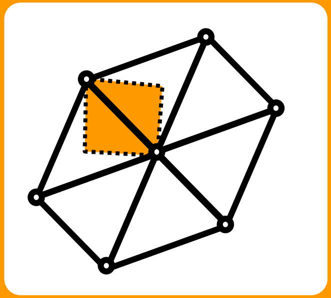
(This is named "cyclic smoothing".)

Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

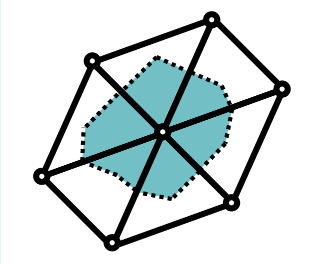
Isovolumetric part



Like ES-FEM

1. Shear locking free

Volumetric part



Like NS-FEM

2. Little pressure oscillation

3. Volumetric locking free
with the aid of F-bar method

Issue of F-barES-FEM in Dynamics

Issue: Energy divergence occurs.

∵ Due to the adoption of F-bar method,
the stiffness system is no longer symmetric.

In F-barES-FEM, the internal force vector is calculated as

Volume of ES-FEM

$$\{f^{\mathrm{int}}\} = \sum [\tilde{B}]\{\bar{T}\}\tilde{V}$$

B-matrix of ES-FEM

Stress derived from $\bar{\boldsymbol{F}}$

Defenition of $\{f^{\mathrm{int}}\}$ in the same fashion as the original **F-bar method**

Combination of $[\tilde{B}]$, $\{\bar{T}\}$ and $\tilde{V}$ (mixture of $\sim$ and $-$) causes asymmetry of the dynamic system.

Formulation of SymF-barES-FEM

In SymF-barES-FEM, the derivation of the internal force vector is slightly modified.

F-barES-FEM: $\{f^{\mathrm{int}}\} = \sum [\tilde{B}]\{\bar{T}\}\tilde{V}$

SymF-barES-FEM: $\{f^{\mathrm{int}}\} = \sum [\bar{B}]\{\bar{T}\}\bar{V}$

Volume derived as $\det(\bar{\boldsymbol{F}})V^{\mathrm{ini}}$

B-matrix derived from $\bar{\boldsymbol{F}}$

Stress derived from $\bar{\boldsymbol{F}}$

The replacements ($[\tilde{B}]$ to $[\bar{B}]$ & $\tilde{V}$ to $\bar{V}$) help to preserve the symmetry of the stiffness system.

Result & Discussion

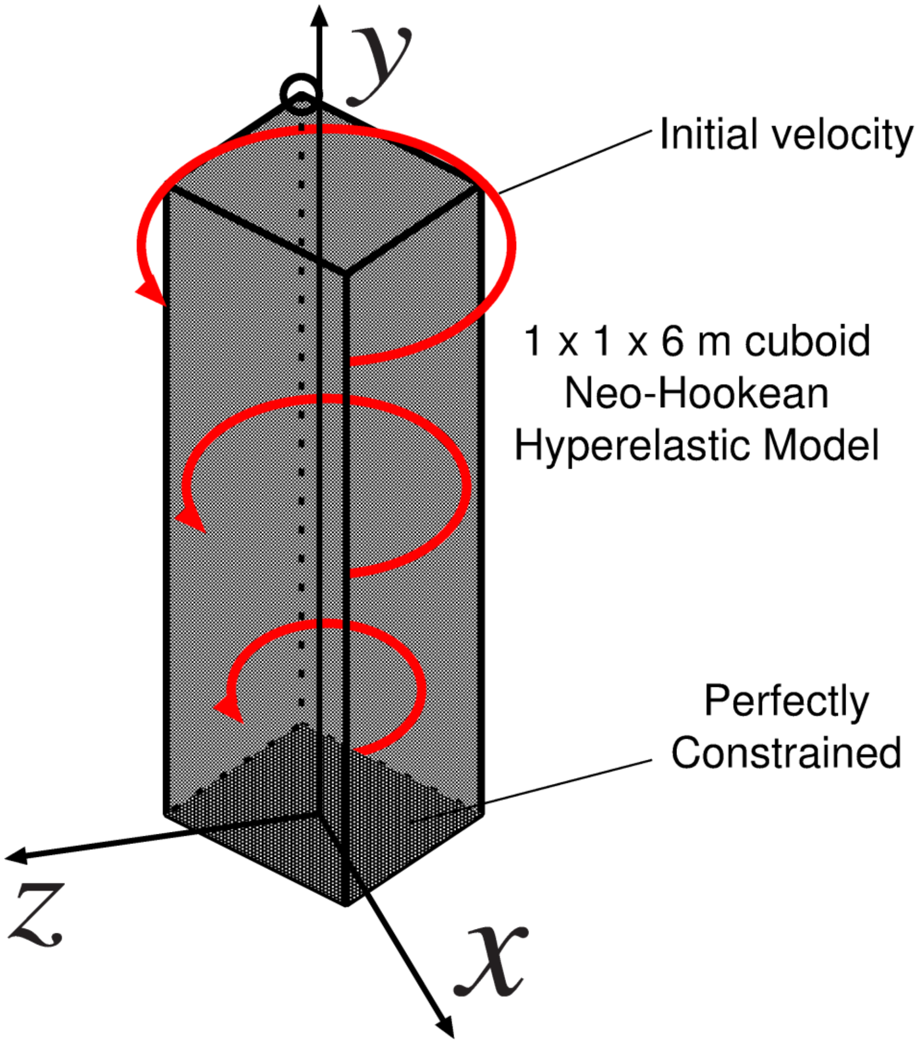
Twist of Rubber Cantilever

Outline

- No contact.
- Dynamic explicit analysis.
- Twisting initial velocity fields:

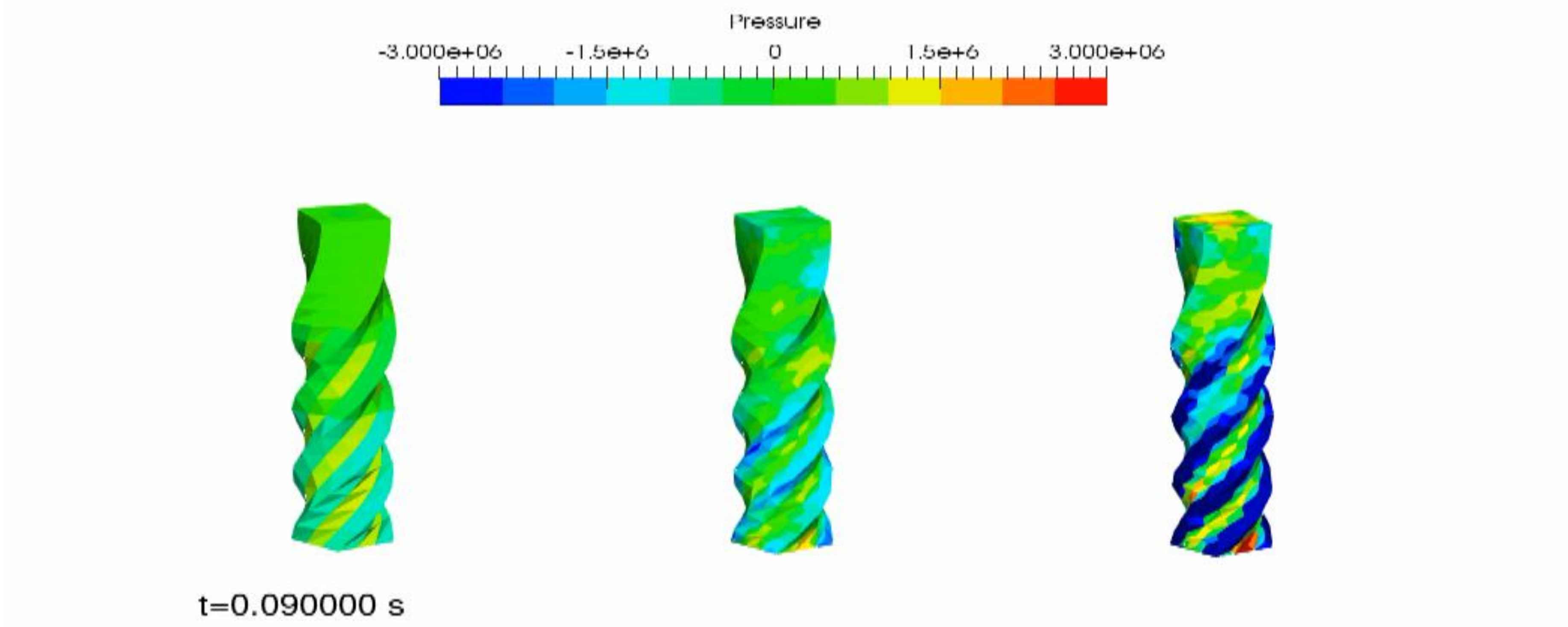
$$\boldsymbol{v}_0(x,y,z) = 100\,s$$

- Neo-Hookean material:
  Initial Young's modulus: 17.0 MPa,
  Initial Poisson's ratio: 0.49,
  Density: 1100 kg/m^3.
- Compare the results of SymF-barES-FEM-T4, F-barES-FEM-T4, and ABAQUS/Explicit C3D8.



Twist of Rubber Cantilever

Comparison of Pressure and Shape



ABAQUS/Explicit C3D8
(H8-SRI)

F-barES-FEM-T4(2)

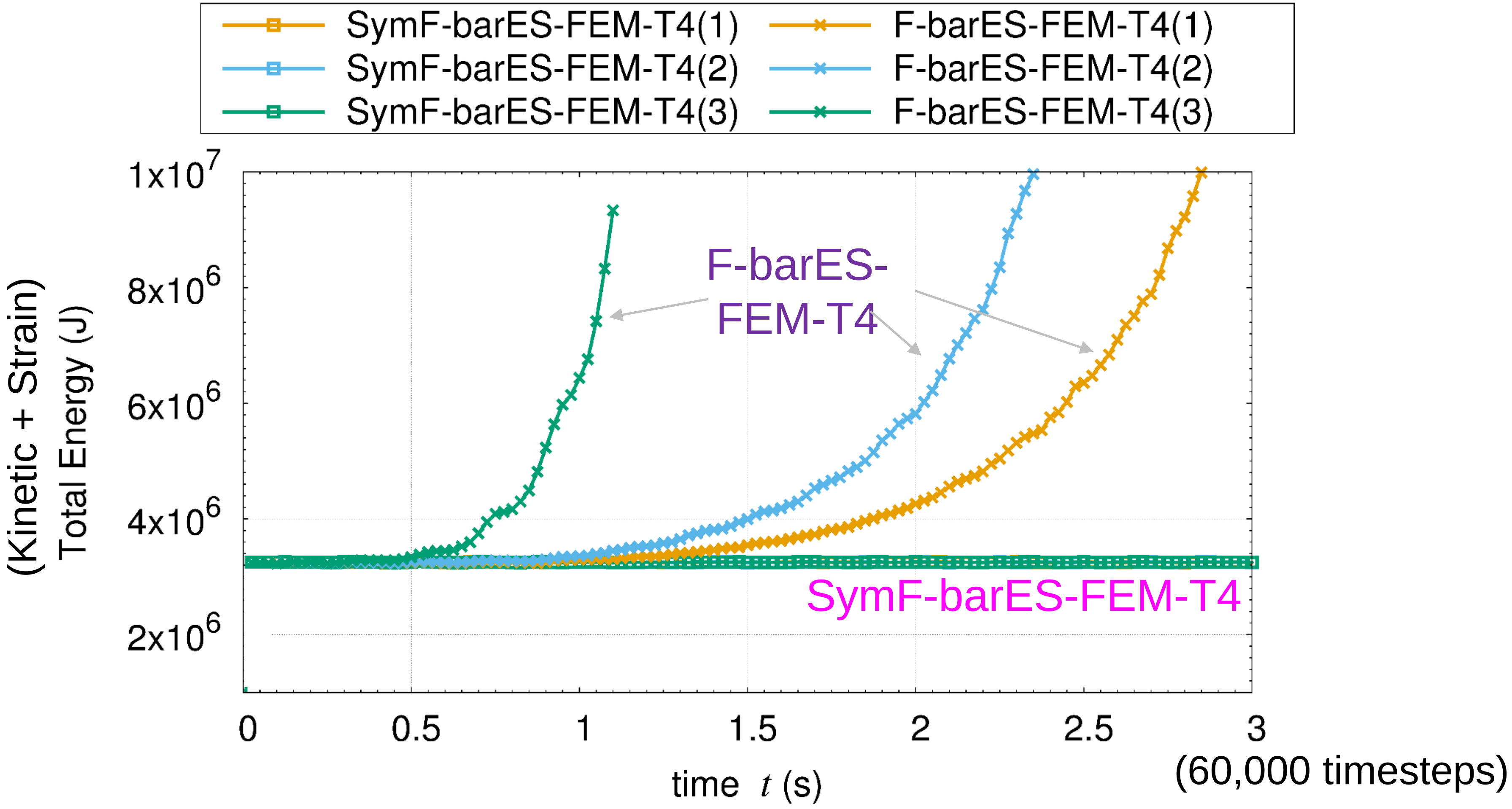
- ✔ No pressure oscillation
- ✔ No locking
- ✗ Energy divergence

SymF-barES-FEM-T4(2)

- ✔ Less pressure oscillation
- ✔ No locking
- ✔ No energy divergence

Twist of Rubber Cantilever

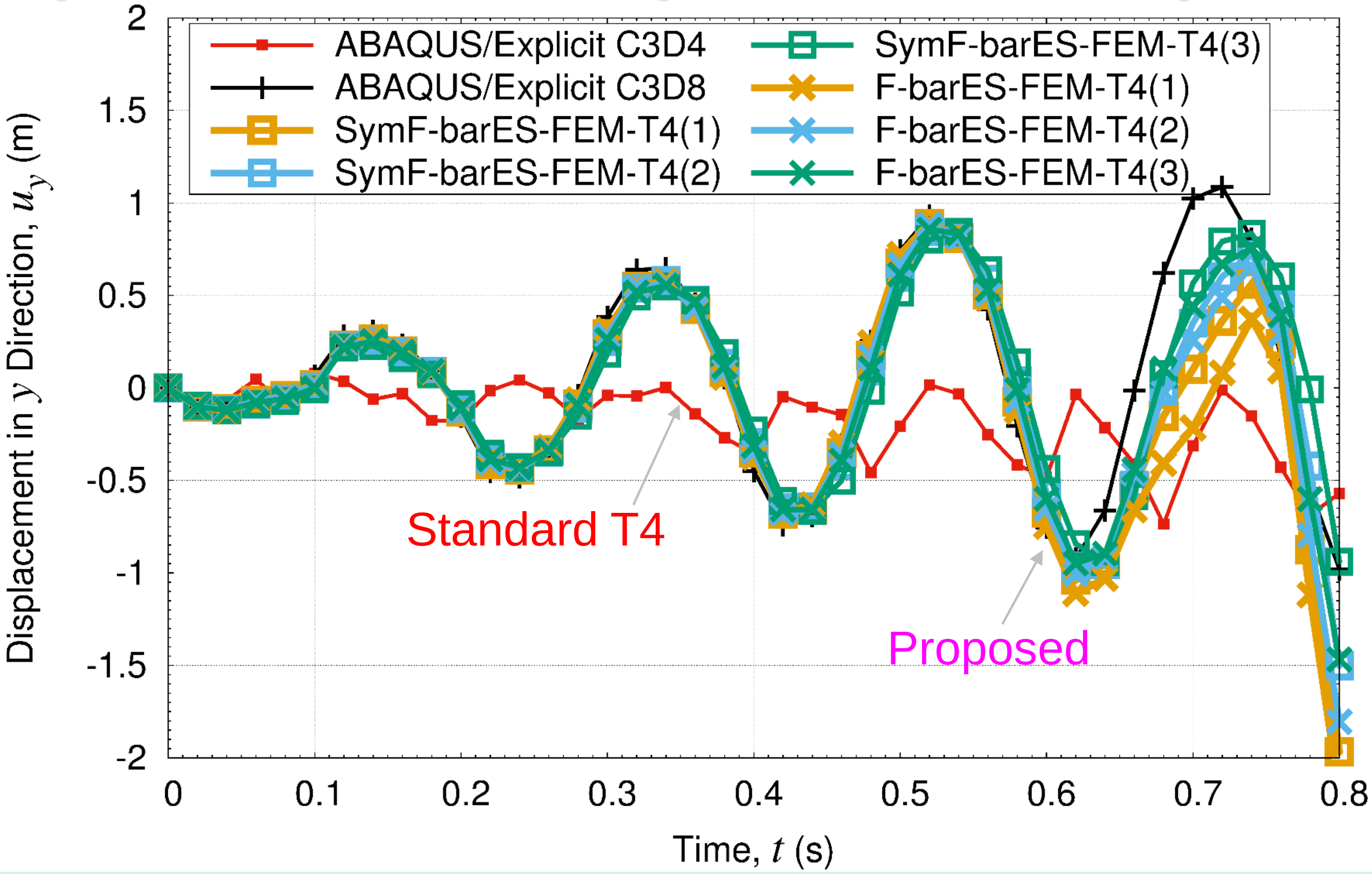
Comparison of total energy



SymF-barES-FEM-T4 can suppress energy divergence!

Time history of displacement

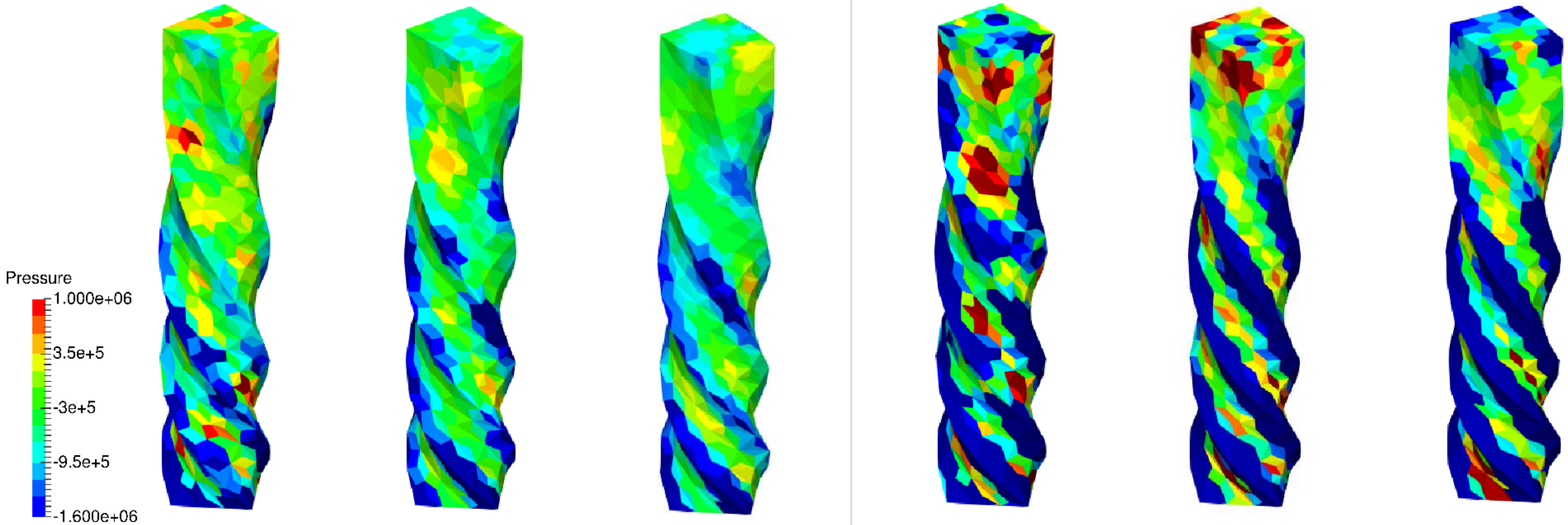
Comparison of axial displacement at a tip node



Proposed methods can show good results as H8-SRI.

Twist of Rubber Cantilever

Effect of the number of cyclic smoothing



#cyclic smoothings	1	2	3	1	2	3

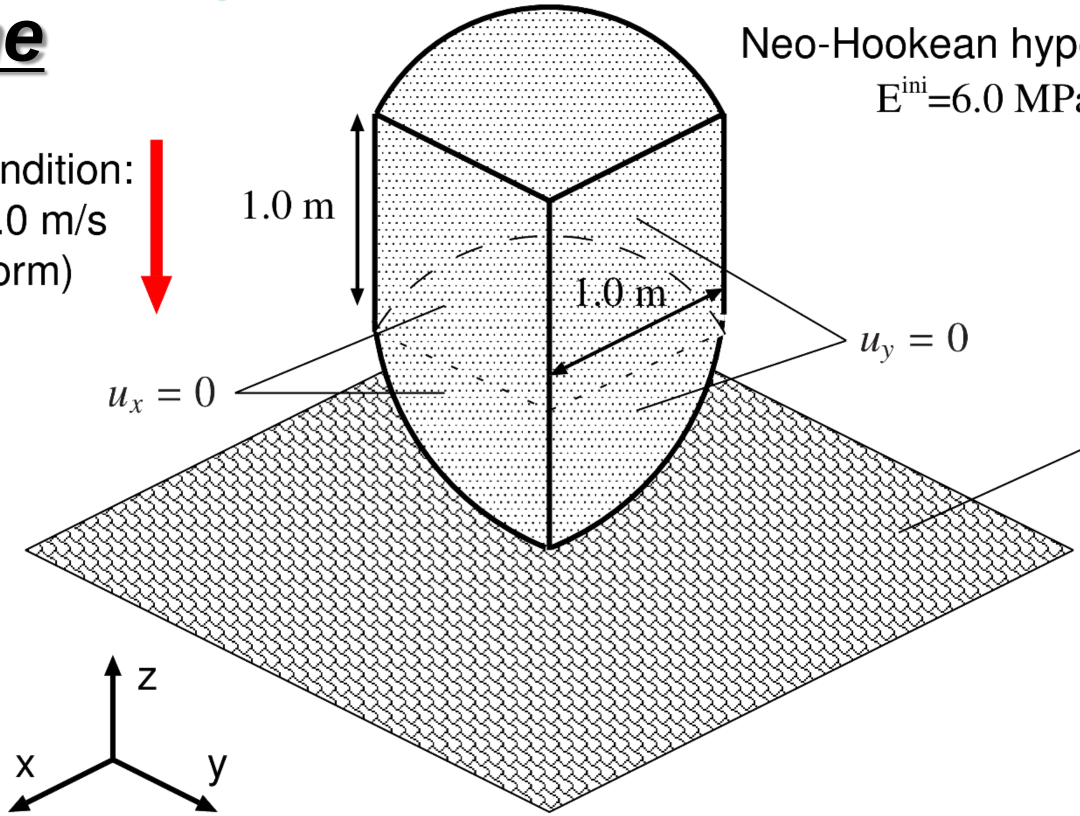
F-barES-FEM-T4

SymF-barES-FEM-T4

In SymF-barES-FEM-T4, the increase in cyclic smoothings no longer improves the accuracy, unlike F-barES-FEM-T4.

Impact of Rubber Bullet

Outline



Neo-Hookean hyper elestic material
E^{ini}=6.0 MPa. ν^{ini}=0.49

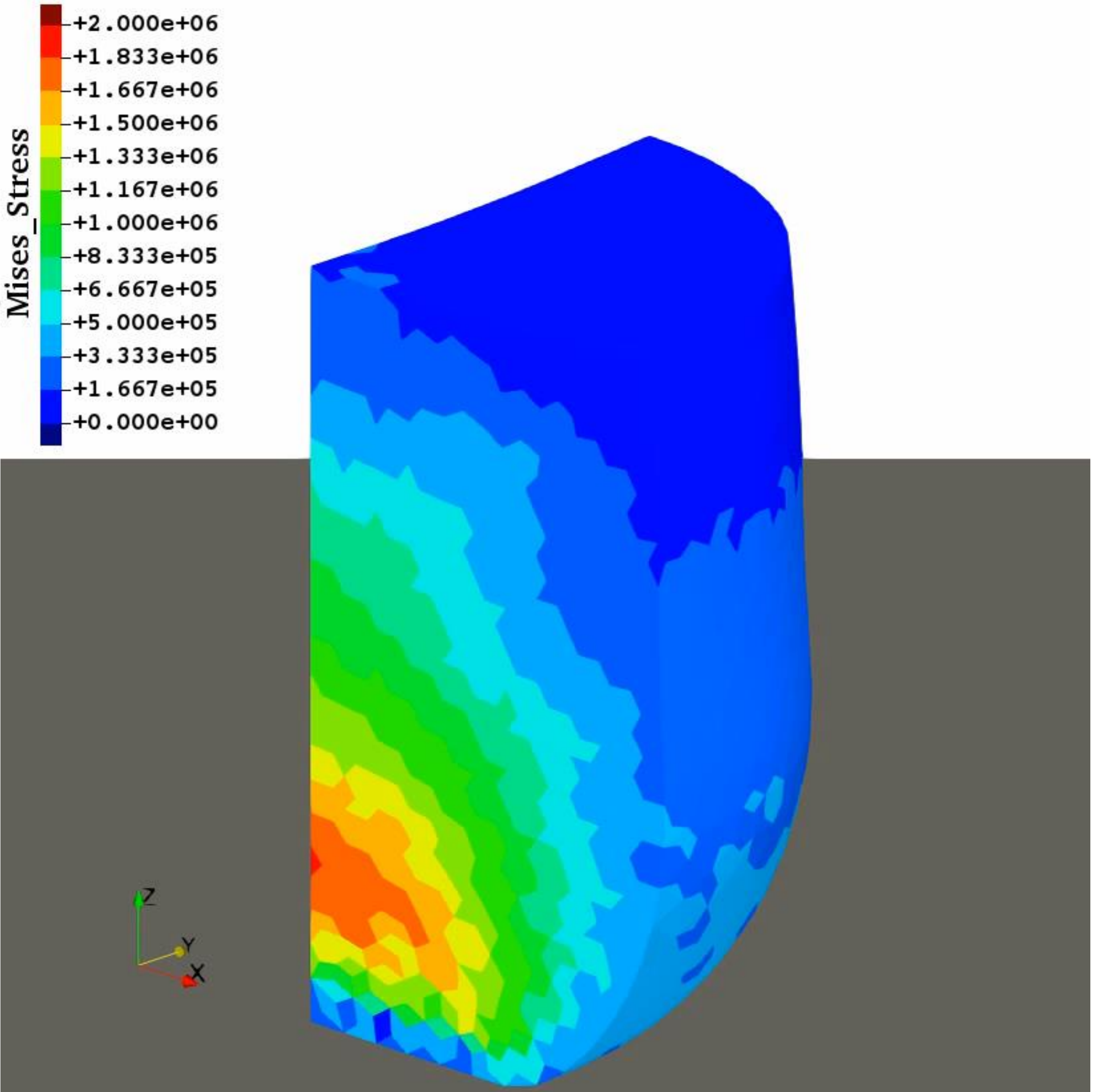
initial condition:
$\dot{u}_z$ = - 10.0 m/s
(uniform)

Rigid plane
(perfectly constrained)
Contact condition:
free-slip, free-separation

- ¼ bullet made of nearly incompressible rubber.
- Impacting the bullet to a slippery rigid wall with a uniform initial velocity.
- Compared to ABAQUS/Explicit C3D4 with a same mesh.

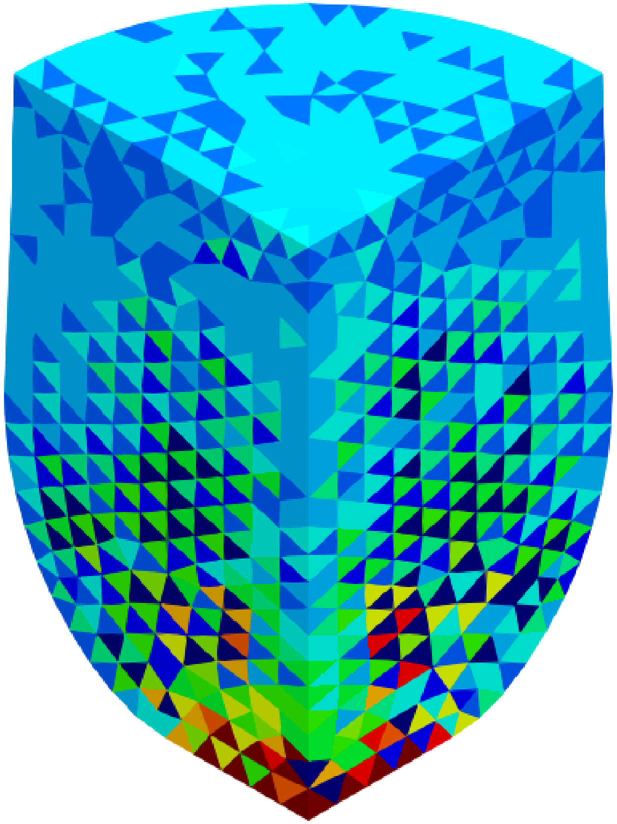
Impact of Rubber Bullet

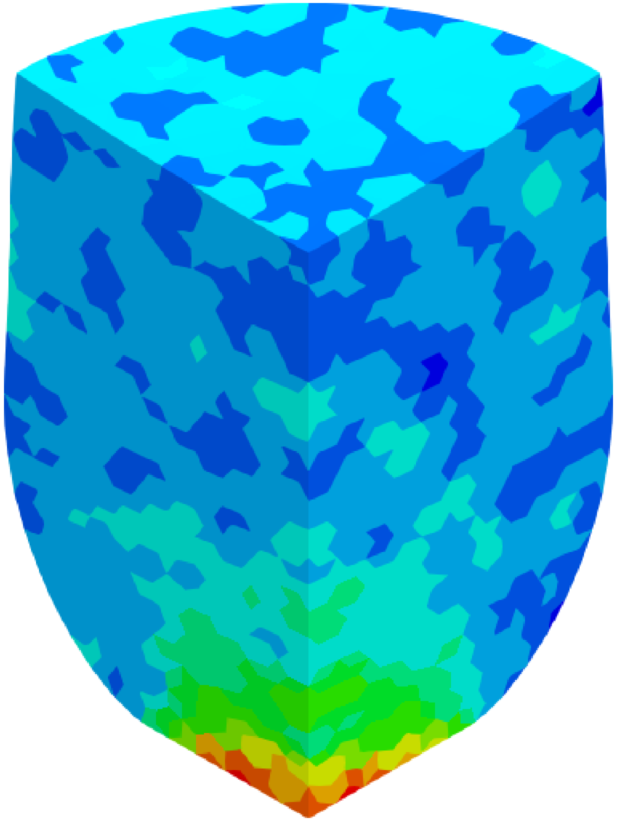
Mises stress Anim. of *SymF-bar ES-FEM -T4(1)*

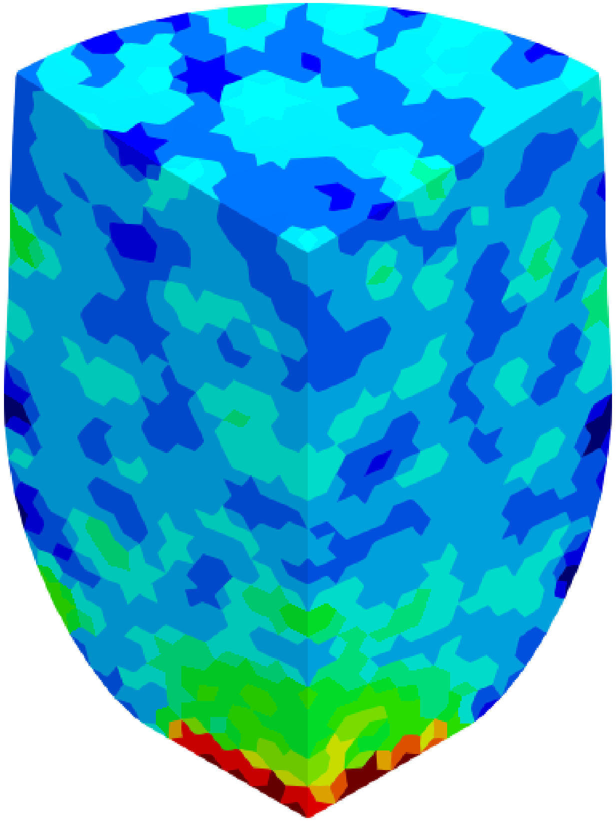


Impact of Rubber Bullet

Comparison of pressure dist. in a contact state


ABAQUS/Explicit C3D4

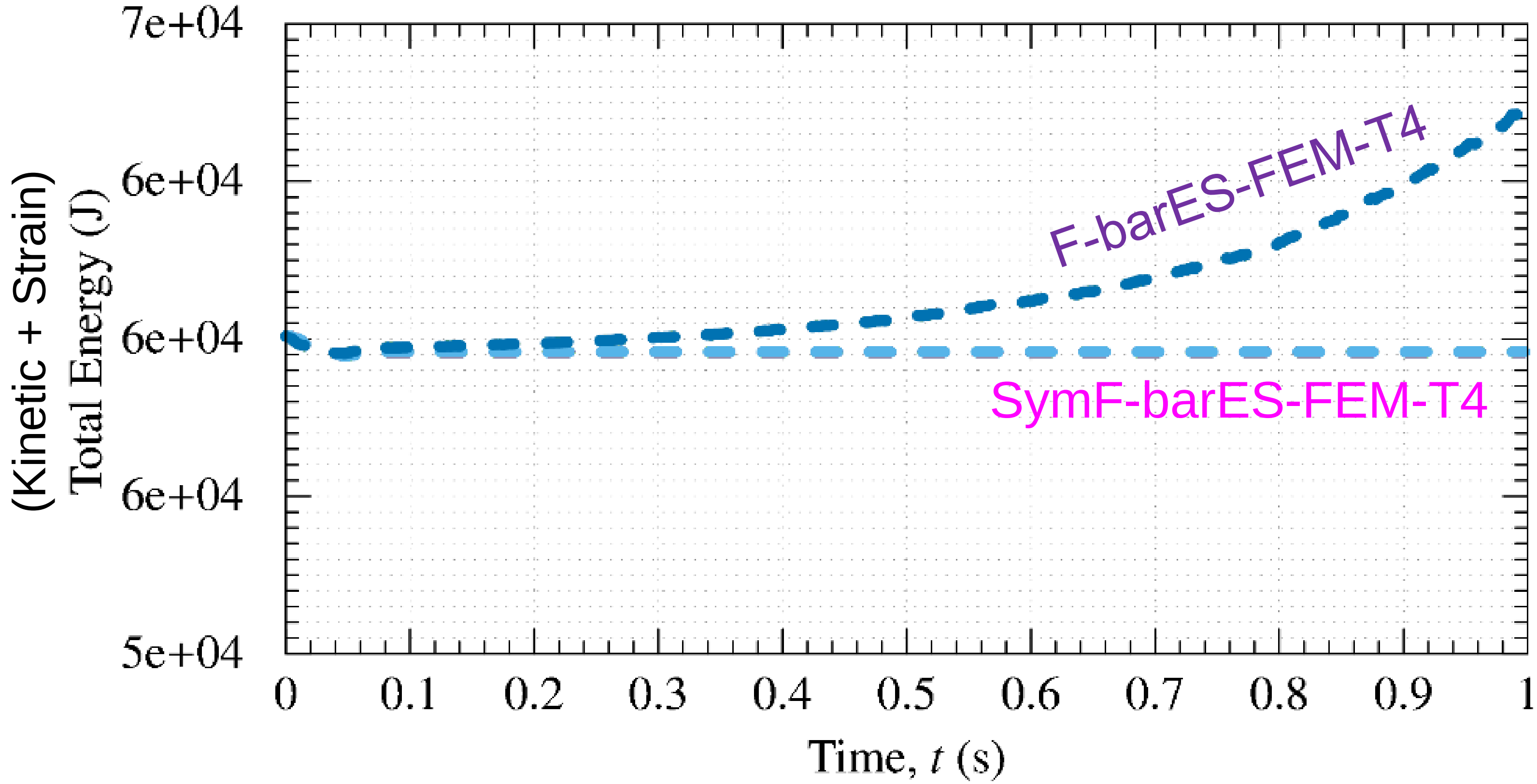

F-barES-FEM-T4(1)


SymF-barES-FEM-T4(1)

Our methods represent far better pressure distributions without major checkerboarding.

Impact of Rubber Bullet

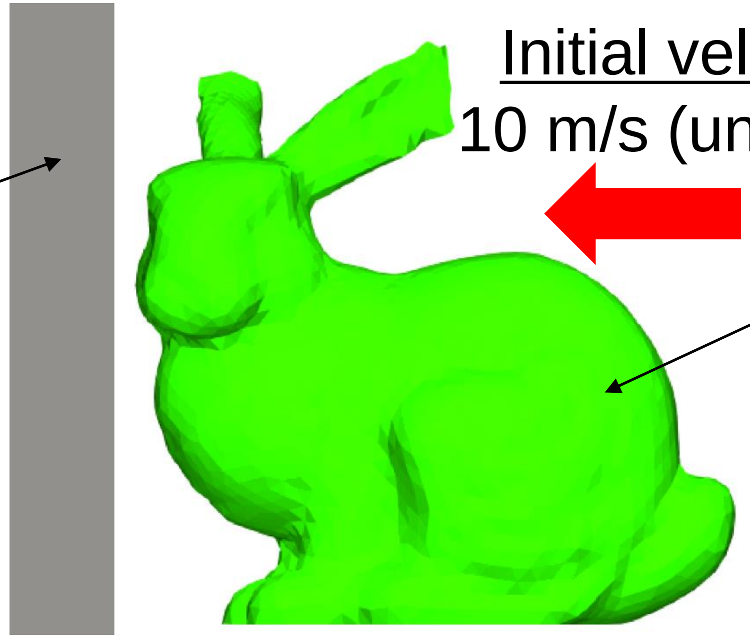
Comparison of total energy over time



SymF-barES-FEM-T4 conserves the total energy even in a contact problem.

Impact of Rubber Bunny

Outline



Rigid Wall
contact condition:
free-slip &
free-separation

Initial velocity
10 m/s (uniform)

Rubber body
$E_{\mathrm{ini}} = 6.0\ \mathrm{MPa}$
$\nu_{\mathrm{ini}} = \mathbf{0.49}$
$\rho = 920\ \mathrm{kg/m^3}$

- A Stanford bunny made of nearly incompressible rubber (neo-Hookean hyperelastic body with $\nu_{\mathrm{ini}} = \mathbf{0.49}$.)
- Impacting the bunny to a slippery rigid wall with a uniform initial velocity.
- Compared to ABAQUS/Explicit C3D4 with a same mesh.

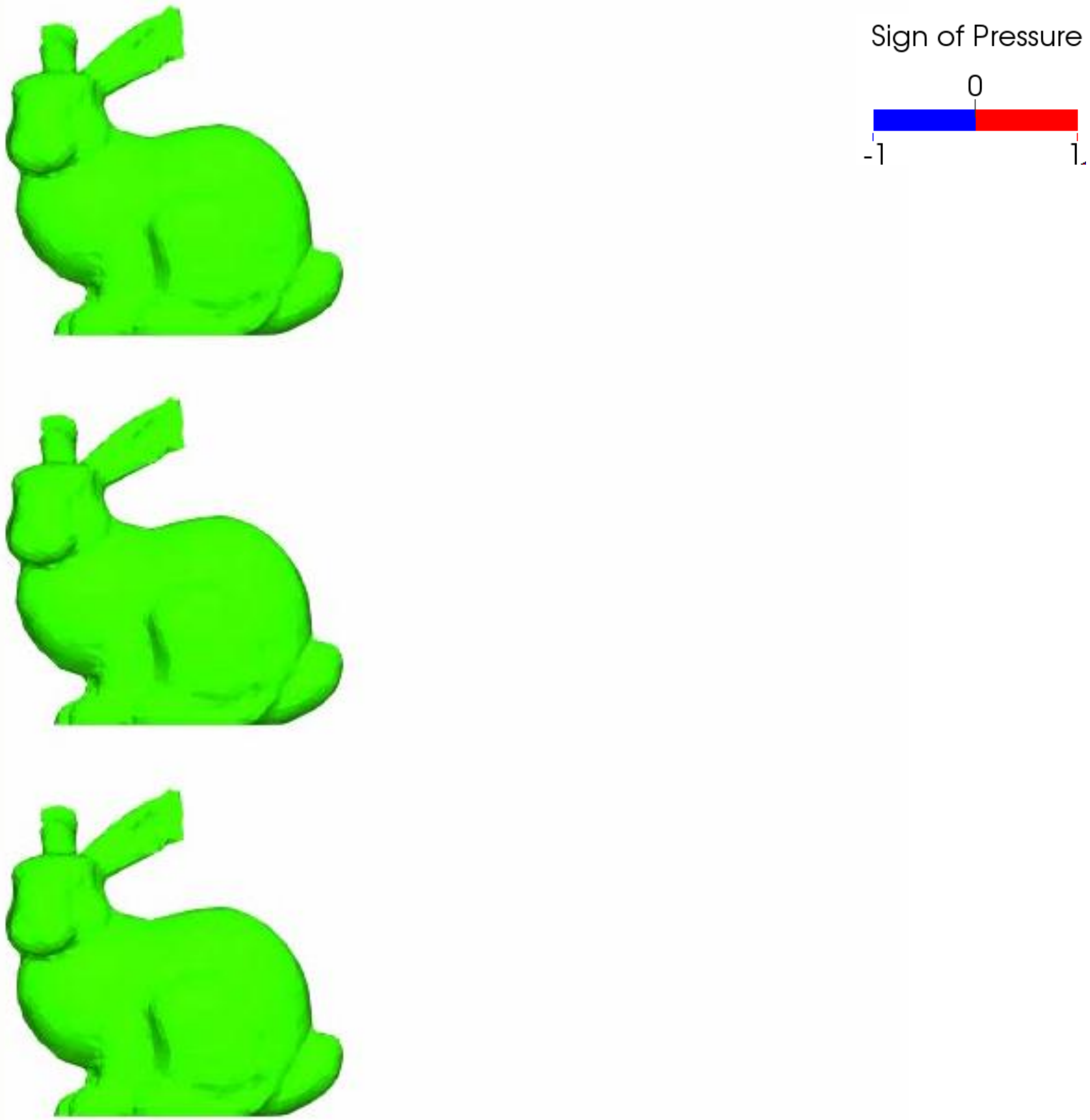
Impact of Rubber Bunny

Pressure sign anim.

ABAQUS/Explicit C3D4
✗ Pressrue checkerboard
✗ Locking
✔ No energy divergence

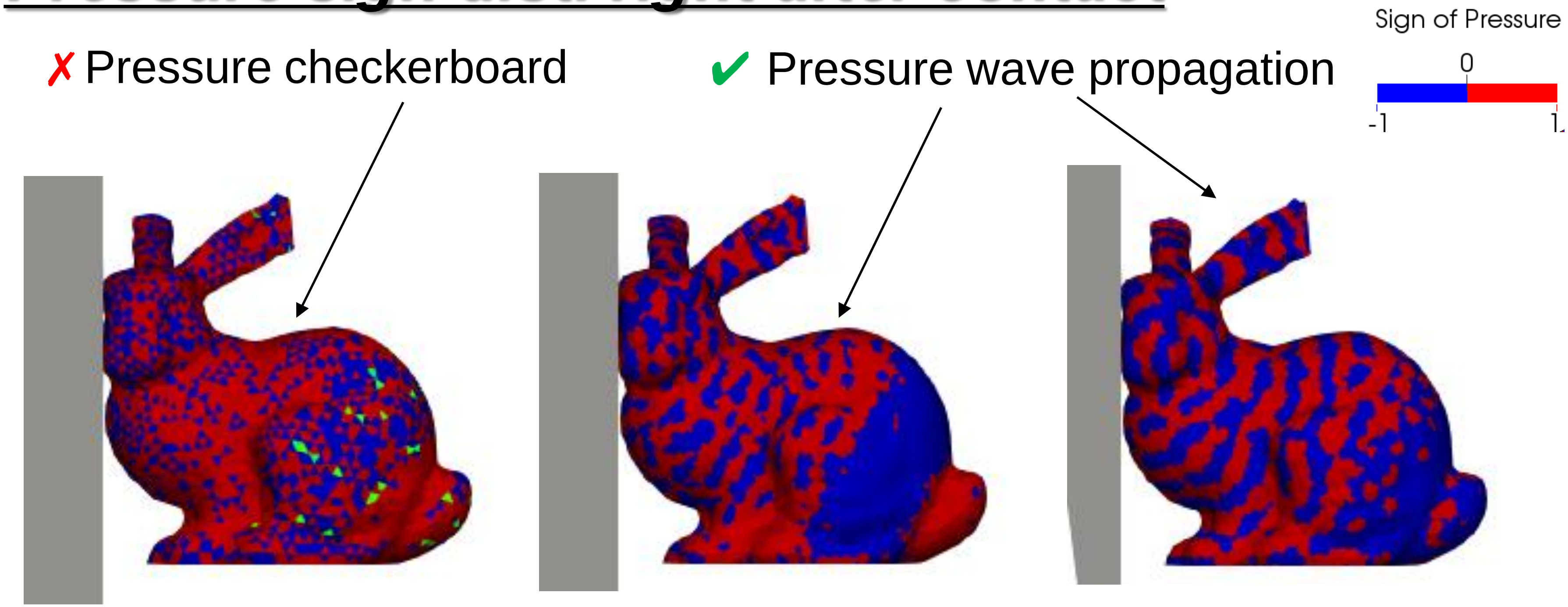
F-barES-FEM-T4(1)
✔ No pressure checker
✔ No locking
✗ Energy divergence

SymF-barES-FEM-T4(1)
✔ Less pressure checker
✔ No locking
✔ No energy divergence



Impact of Rubber Bunny

Pressure sign dist. right after contact



✗ Pressure checkerboard

✔ Pressure wave propagation

ABAQUS/Explicit C3D4

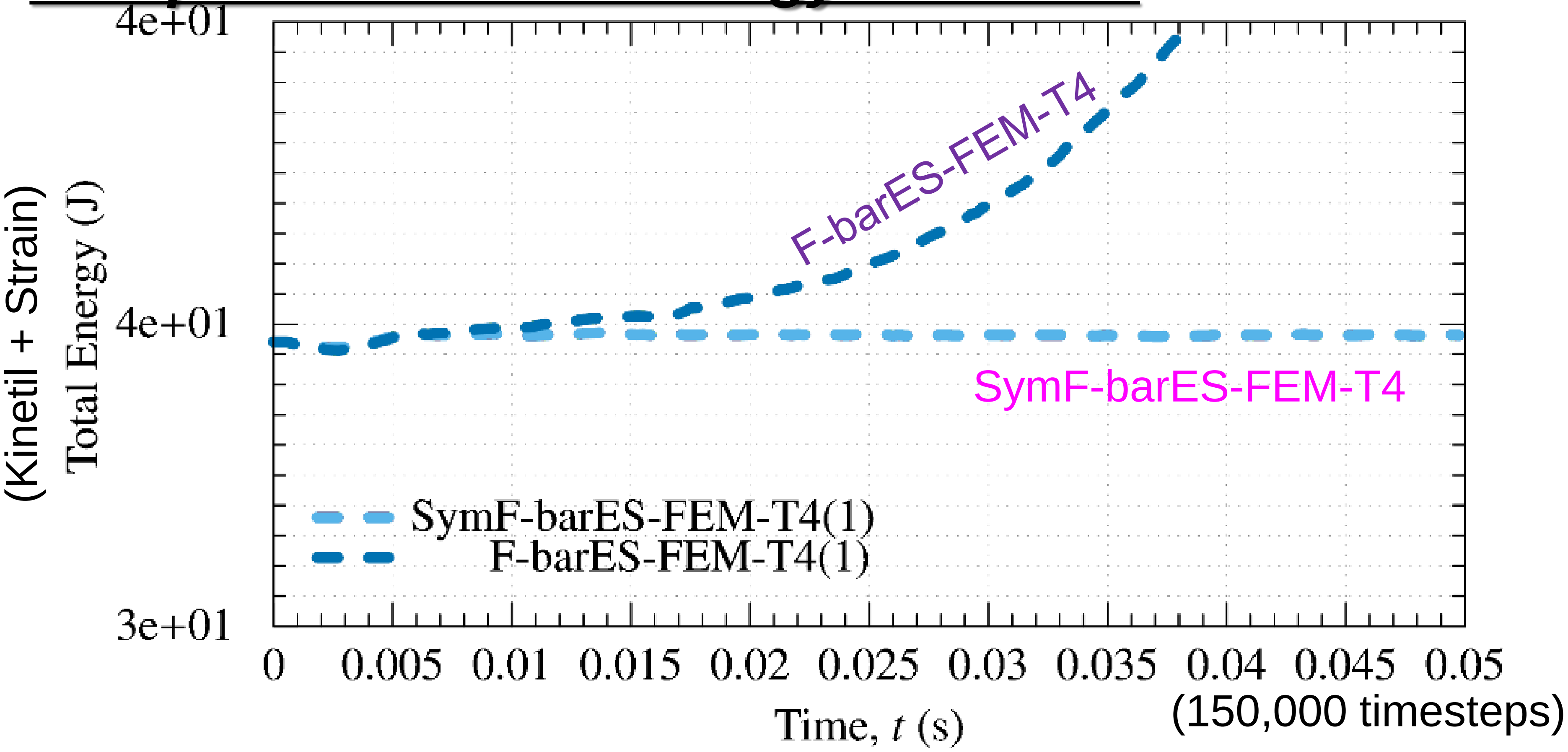
F-barES-FEM-T4(1)

SymF-barES-FEM-T4(1)

Our methods can capture the pressure wave propagation in a complex body.

Impact of Rubber Bunny

Comparison of total energy over time



SymF-barES-FEM-T4 conserves the total energy in a contact problem even with complex shapes.

Summary

Summary

- The accuracy and stability of SymF-barES-FEM-T4 in dynamic explicit contact problems was evaluated.
- SymF-barES-FEM-T4 realizes
  - ✓ Less pressure oscillation
  - ✓ No locking
  - ✓ No energy divergence
- Further improvement for perfect suppression of pressure oscillation is our future work.

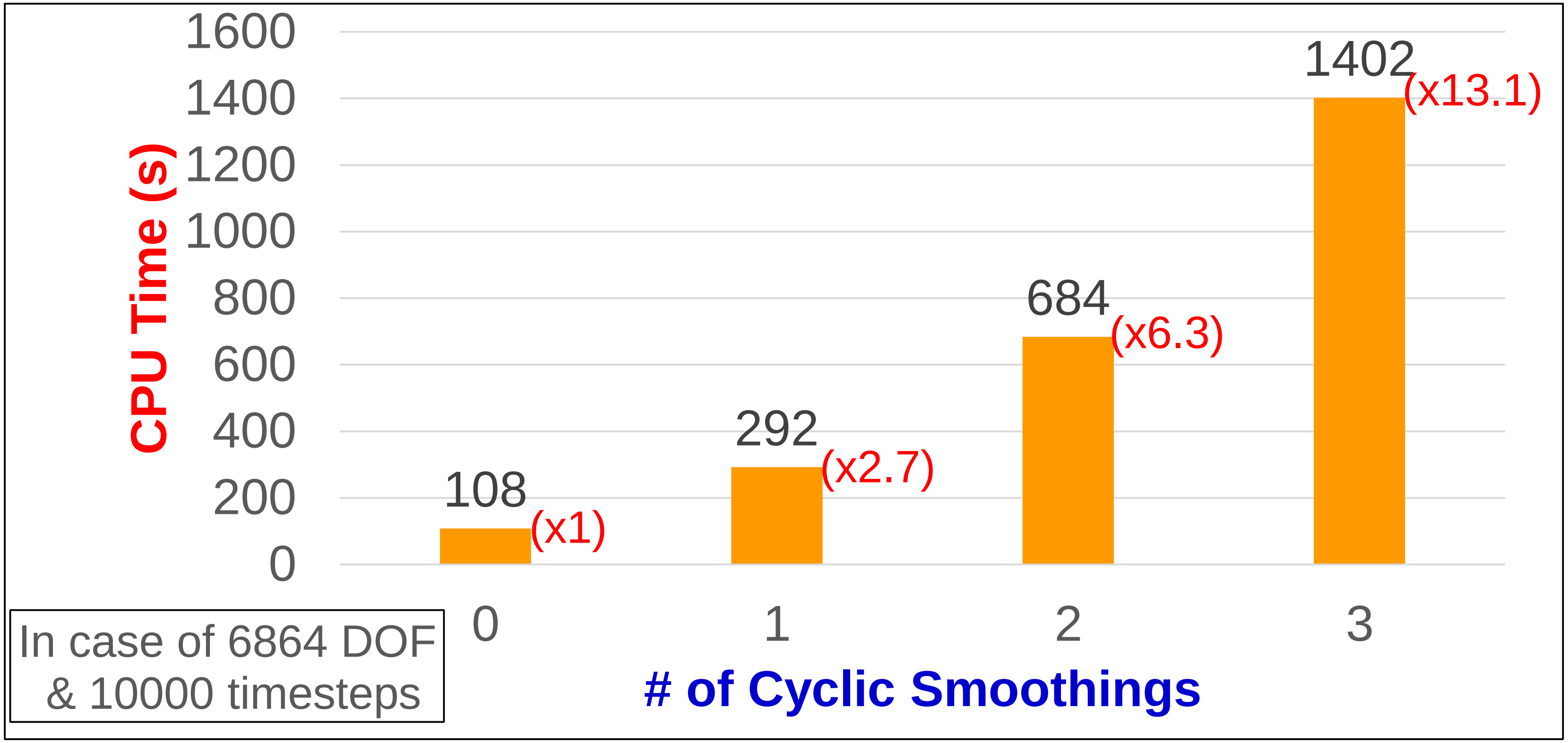
Thank you for your kind attention.

Appendix

Benefits and Drawbacks of F-barES-FEM-T4

Drawbacks

✗ Slow speed of calculation.



In ***explicit*** analyses, [K] is unnecessary; yet, CPU Time increases gradually with the # of cyclic smoothings.