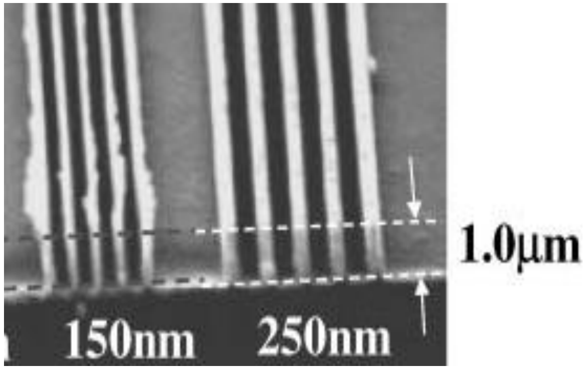
A Locking-free Smoothed Finite Element Formulation (Modified Selective FS/NS-FEM-T4) with Tetrahedral Mesh Rezoning for Large Deformation Problems

Yuki ONISHI, Kenji AMAYA
Tokyo Institute of Technology (Japan)

Motivation & Background

Motivation

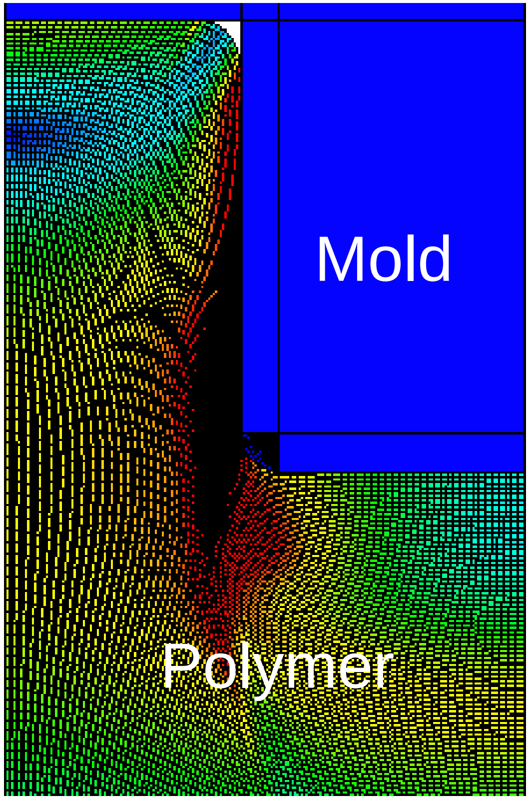
We want to analyze **severely large deformation** problems in solids ***accurately and stably!***
(Target: automobile tire, thermal nanoimprint, etc.)



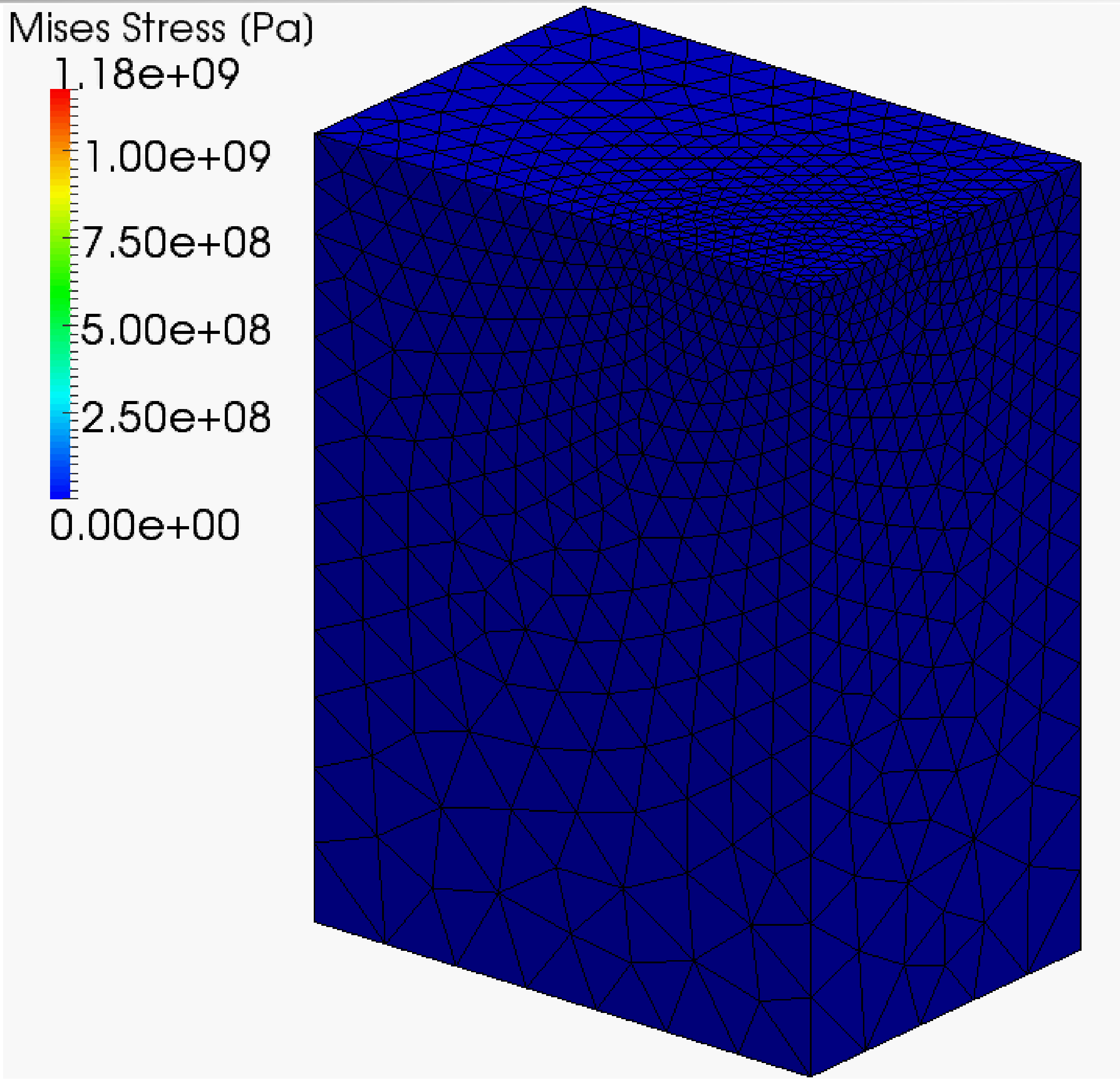
Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.

Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.



Our First Result in Advance



What we want to do:

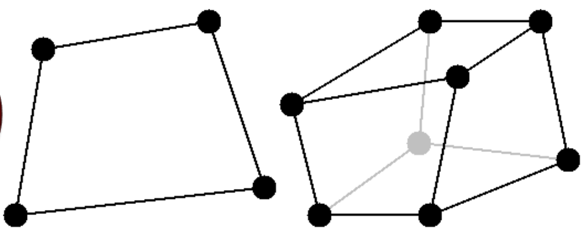
- Static
- Implicit
- Large deformation
- Mesh rezoning

Issues

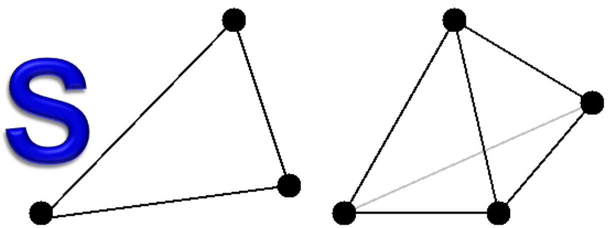
The biggest issue
in large deformation mesh rezoning

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexahedral elements.

NO



YES



We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Not effective in large deformation due to intermediate nodes.
- EAS elements:
  - ✗ Unstable.
- B-bar, F-bar and selective integration elements:
  - ✗ Not applicable to triangular/tetrahedral mesh.
- F-bar patch elements:
  - ✗ Difficult to construct good patches
- u/p hybrid (mixed) elements
  - ✗ No sufficient formulation for triangular/tetrahedral mesh is presented so far. (There are almost acceptable hybrid elements such as C3D4H or C3D10H of ABAQUS.)
- Selective smoothed finite elements:
  - ? Unknown potential (since 2009~). ***Let's try!***

Objective

Develop a locking-free
modified selective S-FEM
for large deformation problems
with mesh rezoning

Table of Body Contents

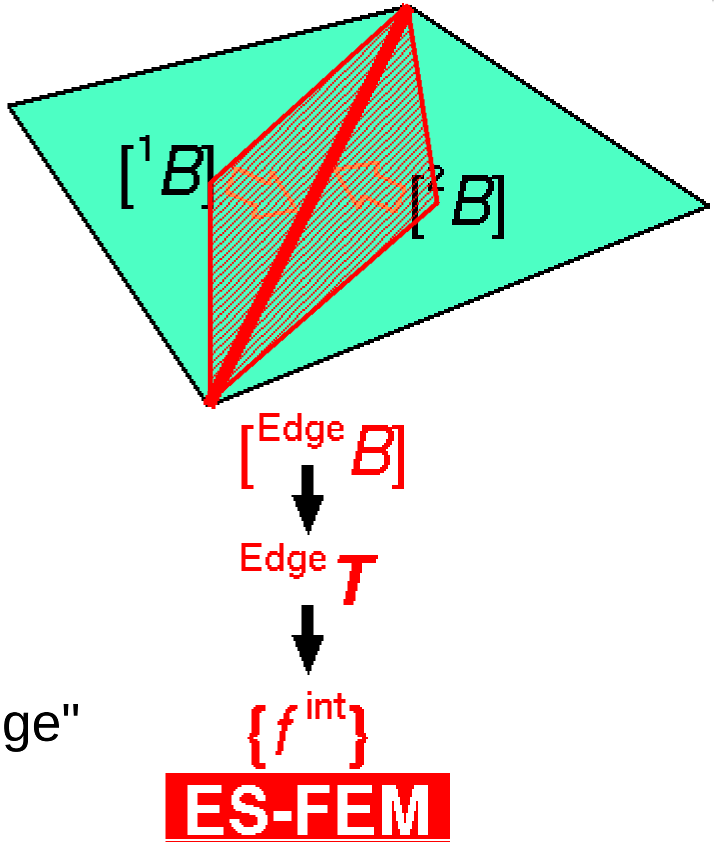
- Part 1: Introduction to our *modified* selective S-FEMs
- Part 2: Demonstration of our methods
  with mesh rezoning
- Summary

Part 1:
Introduction to Our *Modified* selective S-FEM

Review of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting edges and make $[\,{}^{\mathrm{Edge}}B]$.
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.

Generally accurate but induces volumetric locking.



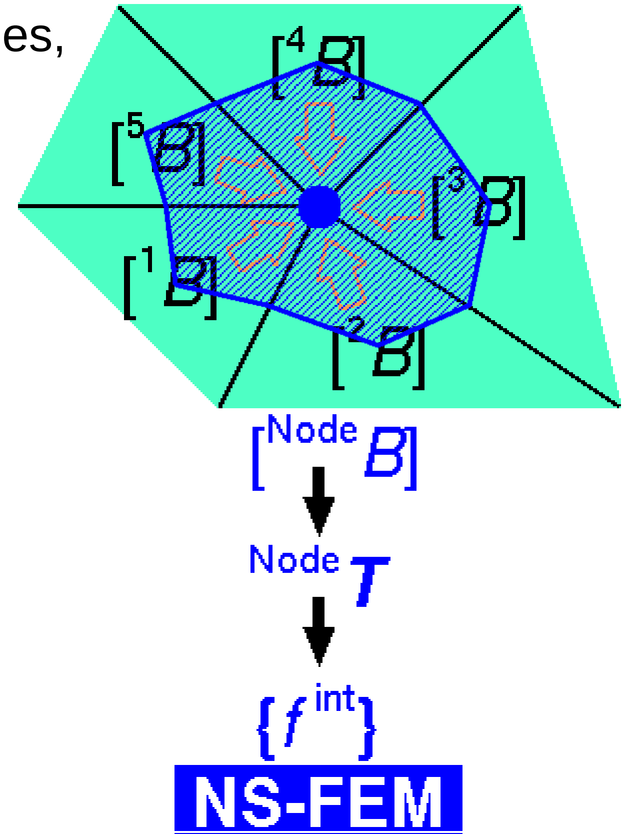
Substituting "face" for "edge" gives FS-FEM for 3D

Review of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\mathrm{Node}}B]$
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed node domains.

Generally not accurate but volumetric locking free.

(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)



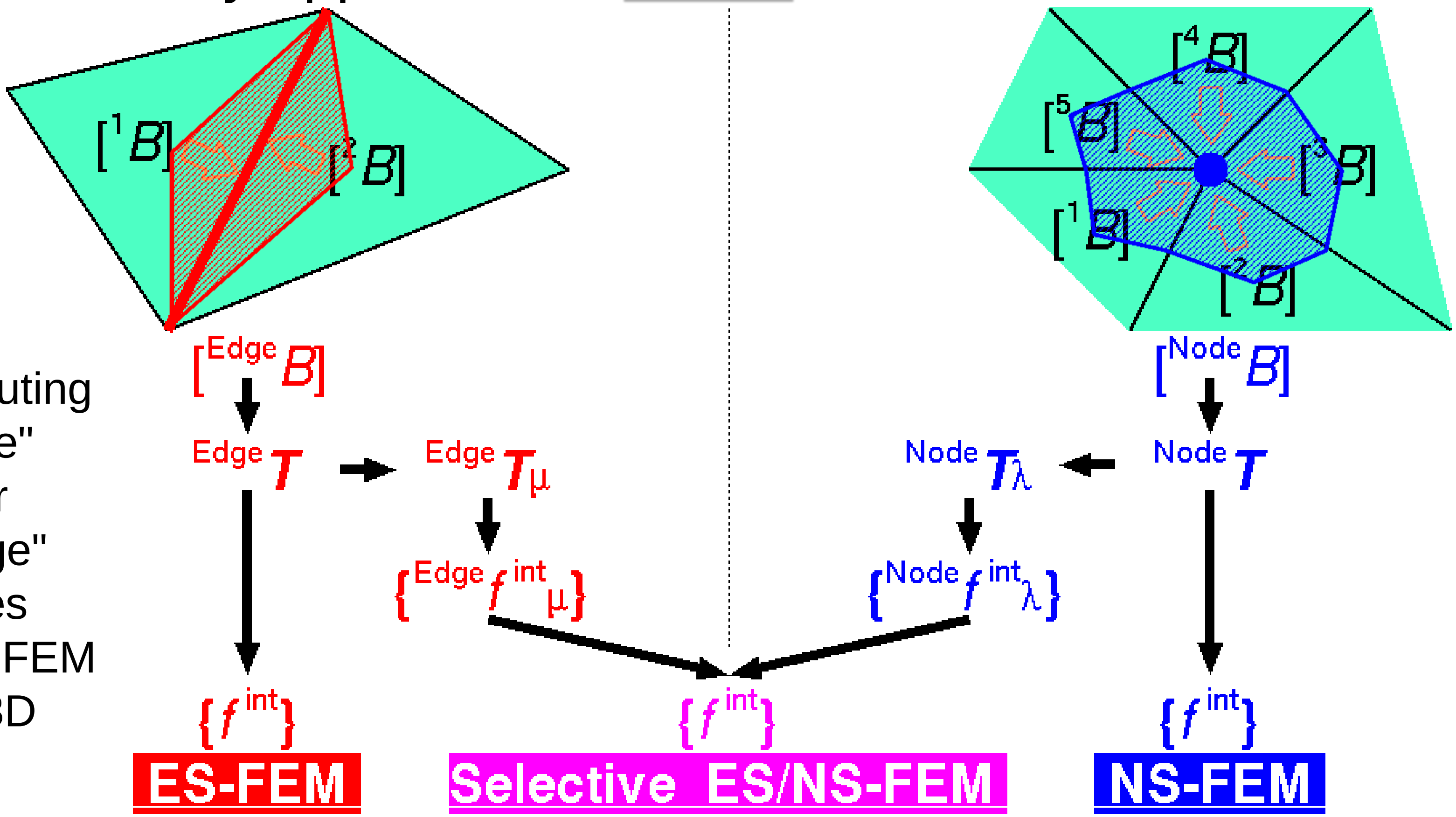
close to FVM with vertex-based control volume

Review of Selective ES/NS-FEM

- Separate stress into "μ part" and "λ part", where μ and λ are the Lame's parameters.
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

<u>Only applicable to **elastic** constitutive models.</u>

Substituting "face" for "edge" gives FS/NS-FEM for 3D

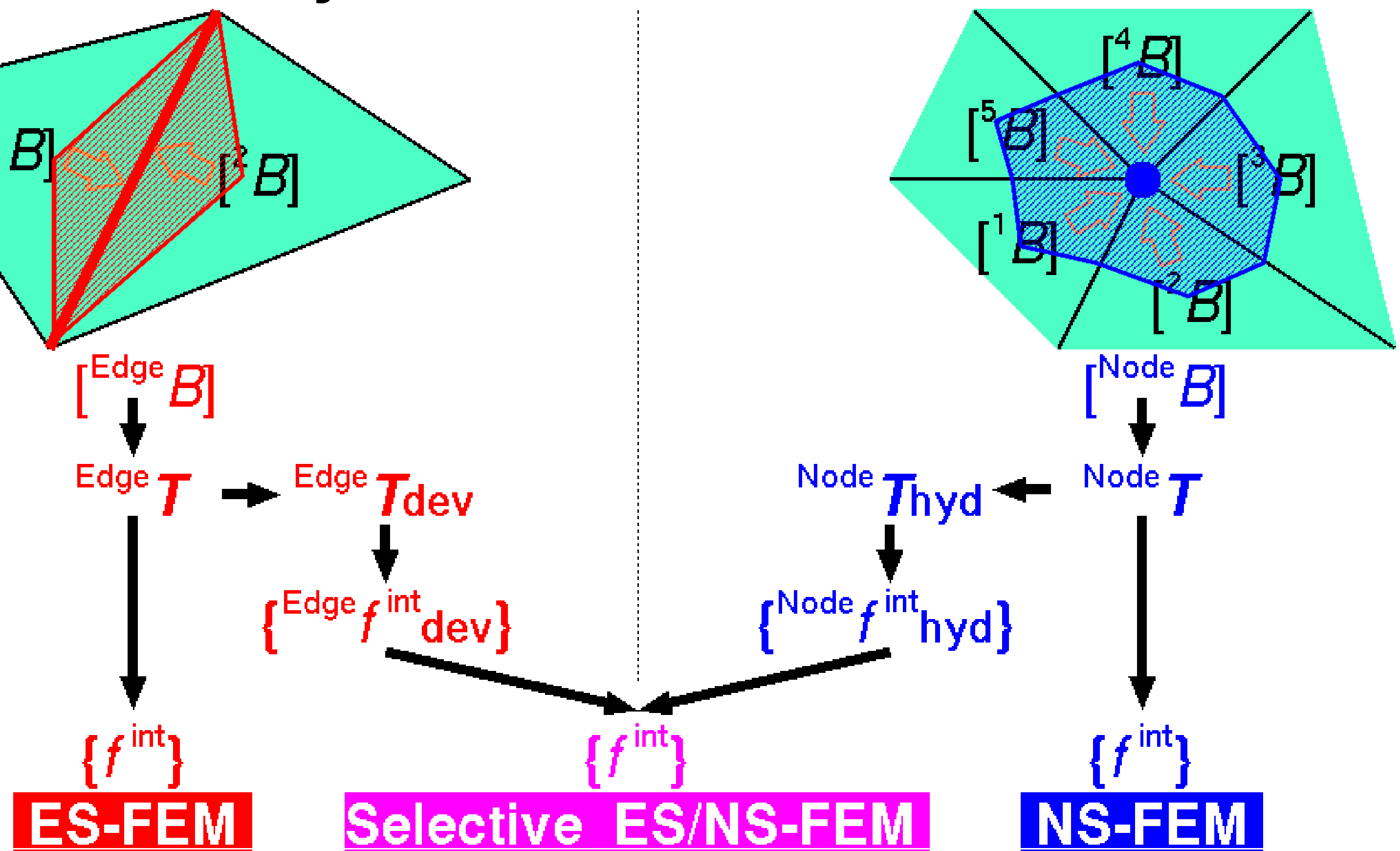


Modified Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "μ part" and "λ part".
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

<u>Applicable to any kind of material constitutive models.</u>

Substituting "face" for "edge" gives FS/NS-FEM for 3D



3 Types of Selective S-FEMs

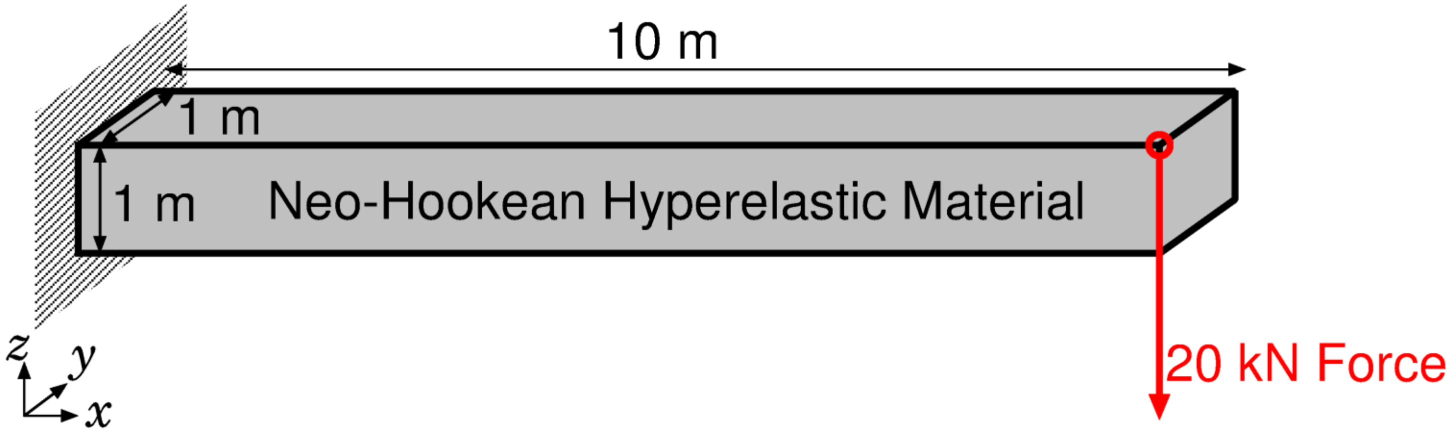
Method	Deviatoric Part	Hydrostatic Part
2D ES/NS-FEM-T3	ES-FEM	NS-FEM
3D **ES**/NS-FEM-T4	**ES**-FEM	NS-FEM
3D **FS**/NS-FEM-T4	**FS**-FEM	NS-FEM

No increase in DOF!!

Displacement vector $\{u\}$ is only the unknown.

Verification ~ Bending of Cantilever ~

Outline



- Neo-Hookean hyperelastic material

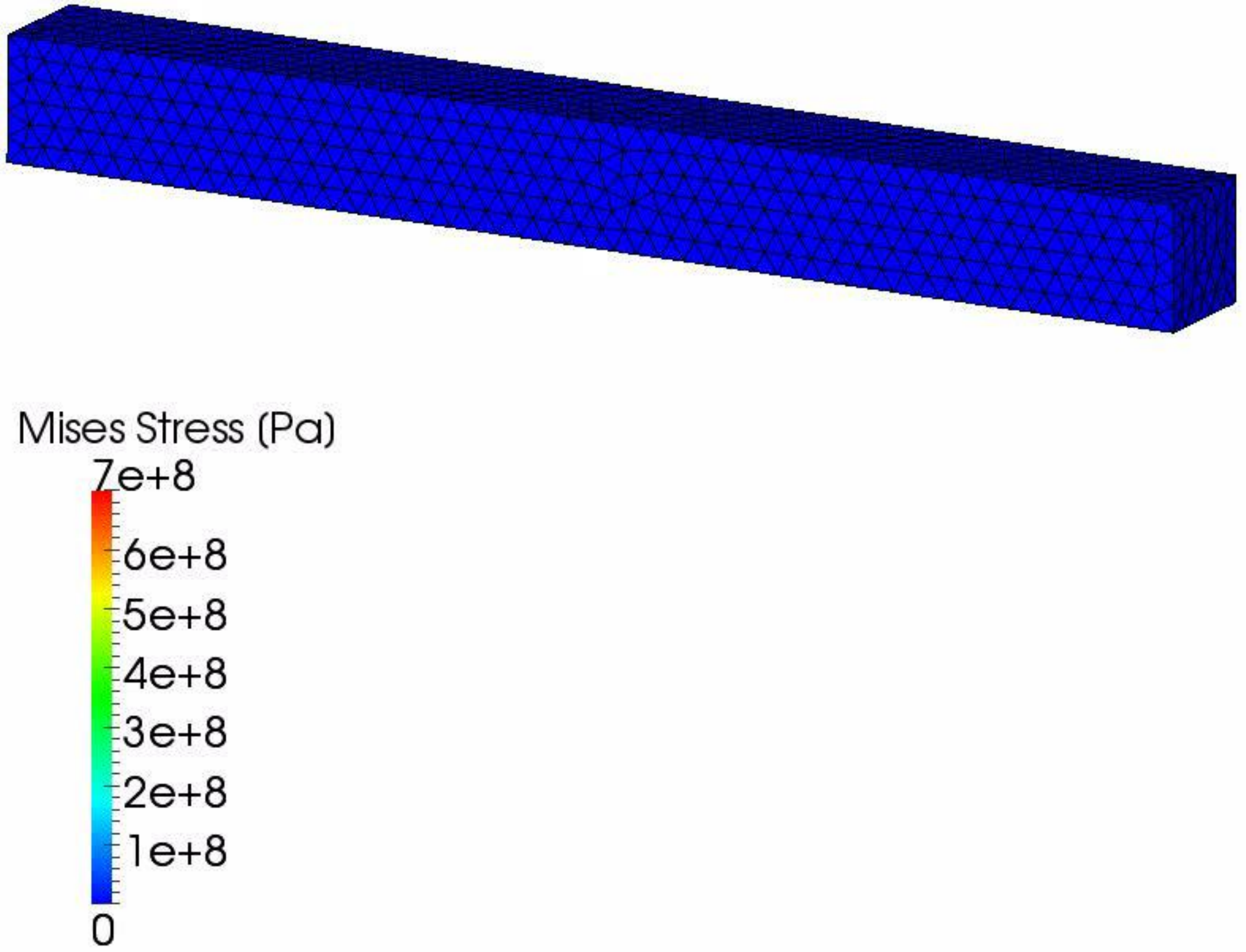
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s.

- Compared to ABAQUS/Standard with *C3D20H* (2nd-order hybrid hexahedral) elements.
- No mesh rezoning is taken place for this test.

Verification ~ Bending of Cantilever ~

Results with $D_1 = 2 \times 10^{-15}\ [\mathrm{Pa}^{-1}]$ (ν_{ini}=0.499999)

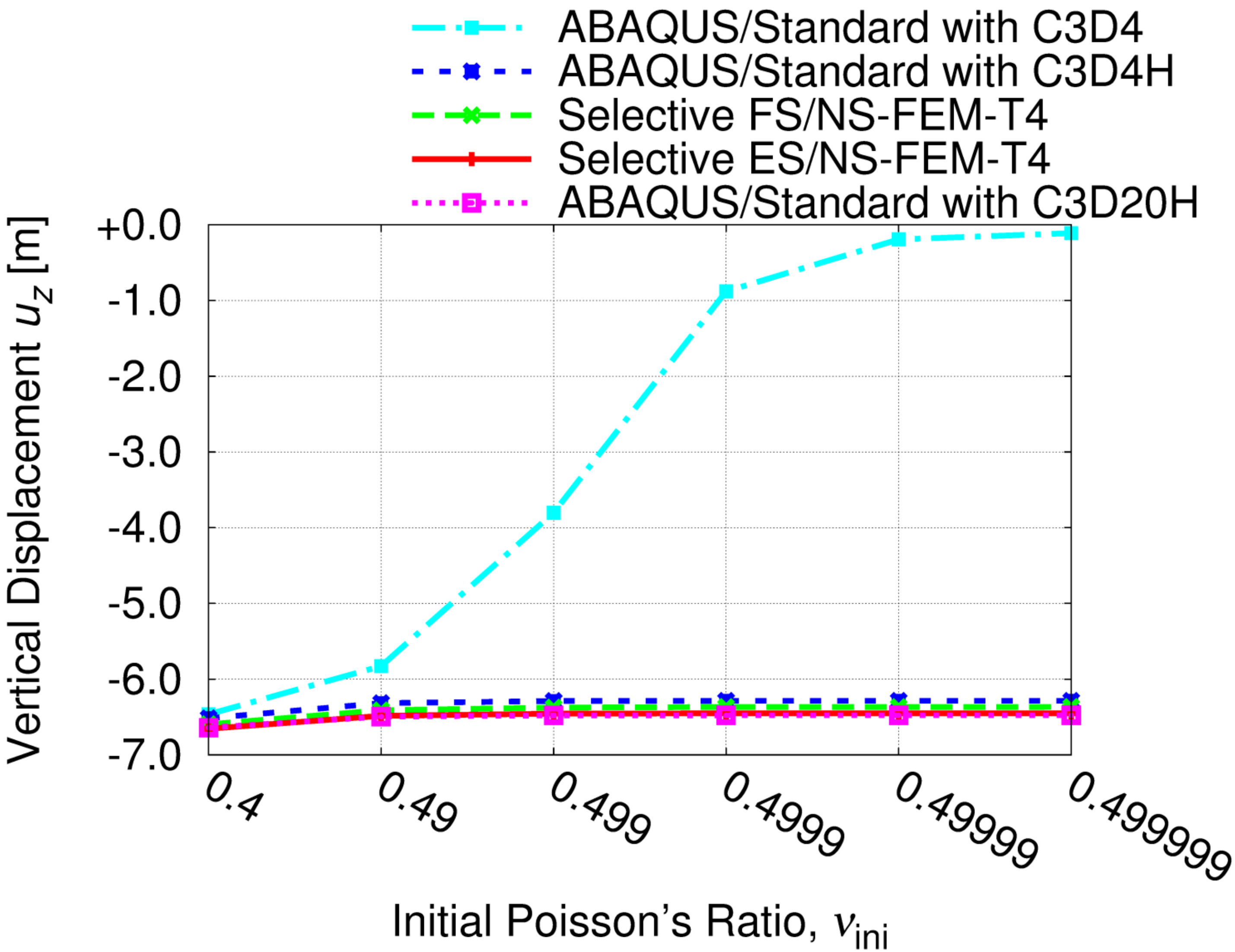


The amount of vertical deflection is about 6.5 m.

If we use constant strain tetrahedral, the amount of vertical deflection is about only 0.1 m.

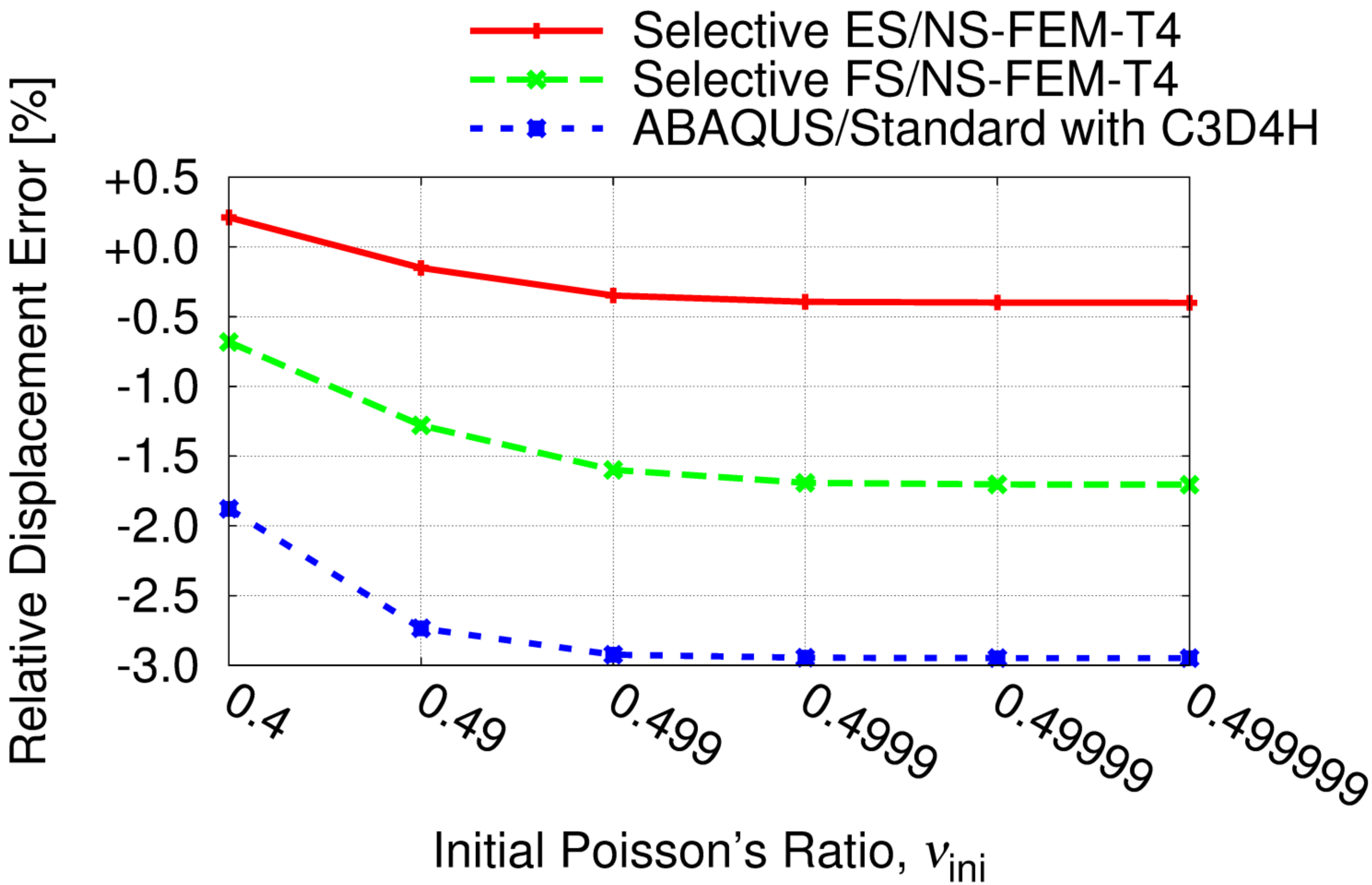
Verification ~ Bending of Cantilever ~

Comparison of Deflection Displacements



Verification ~ Bending of Cantilever ~

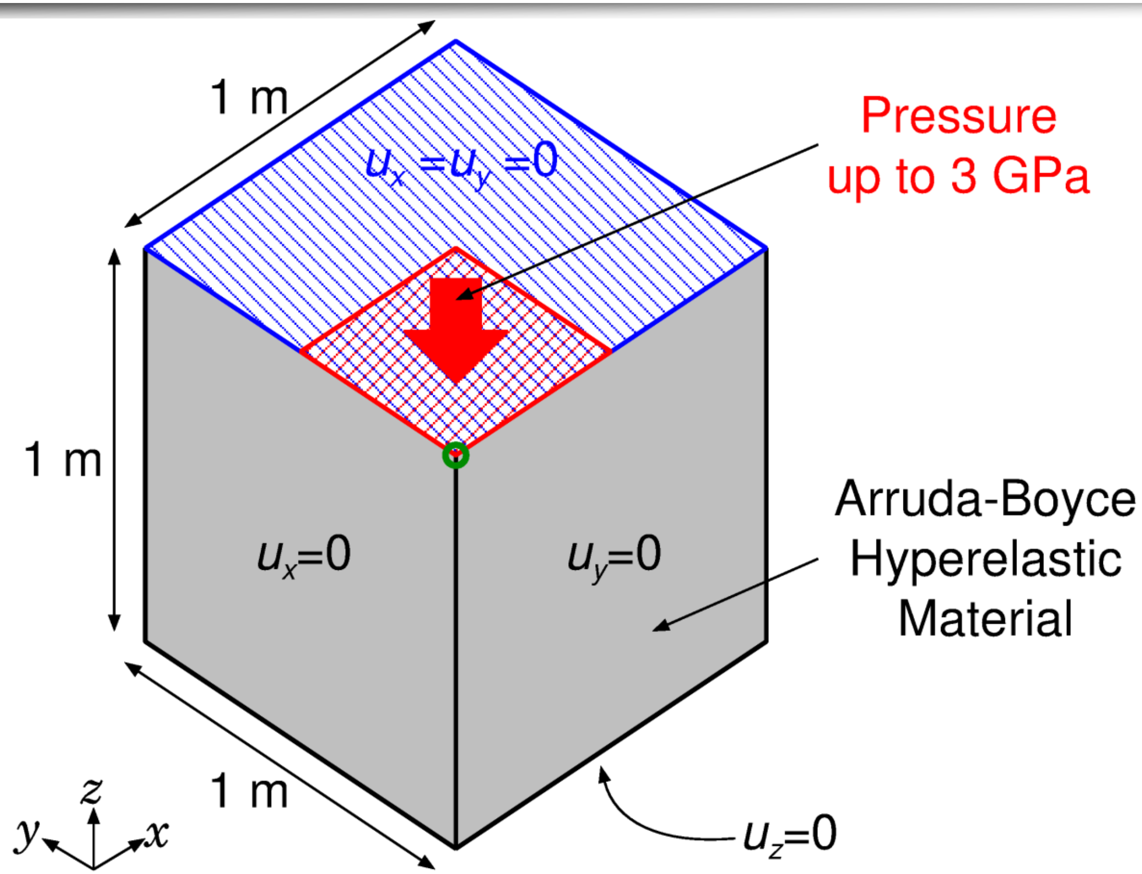
Comparison to 2nd-order Hybrid Hex Element (C3D20H)



Selective S-FEM is ***locking-free***
in large deflection analysis!!

Verification ~Partial Compression of Block ~

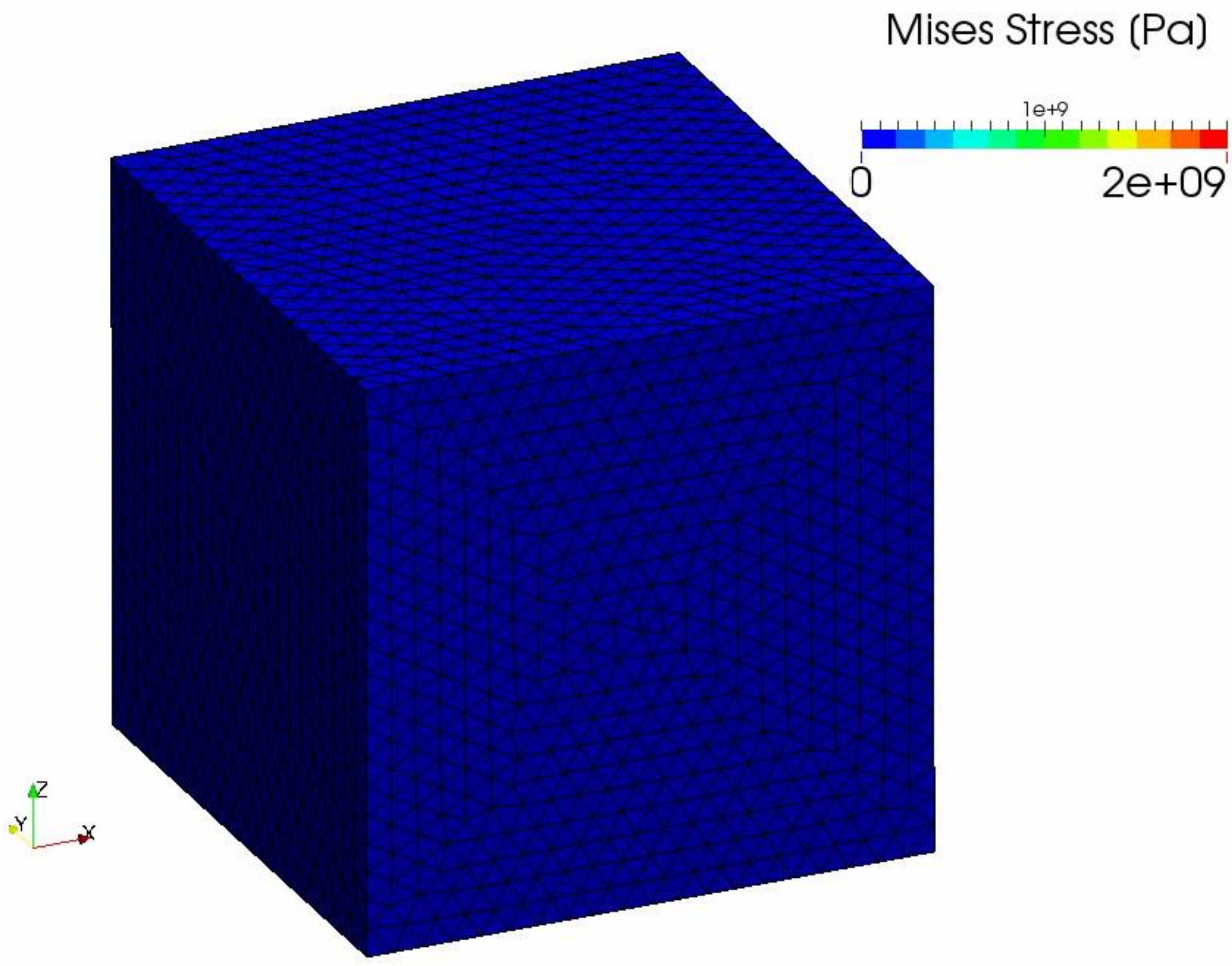
Outline



- Arruda-Boyce Hyper elastic Material with $\nu_{\mathrm{ini}} = 0.4999$
- Applying pressure on ¼ of the top face

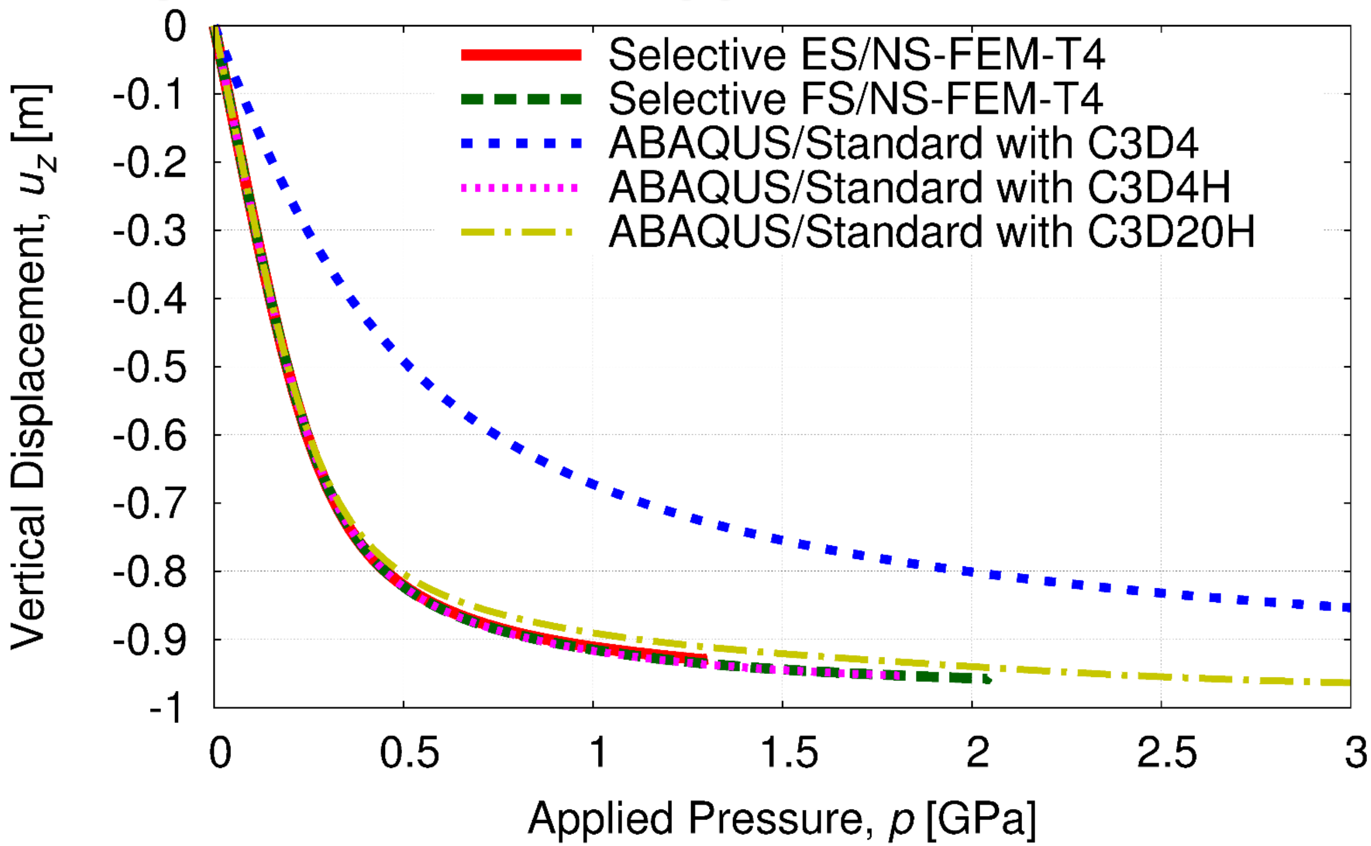
Verification ~Partial Compression of Block ~

Result of Selective FS/NS-FEM-T4



Verification ~Partial Compression of Block ~

Vertical Displacements vs. Applied Pressure

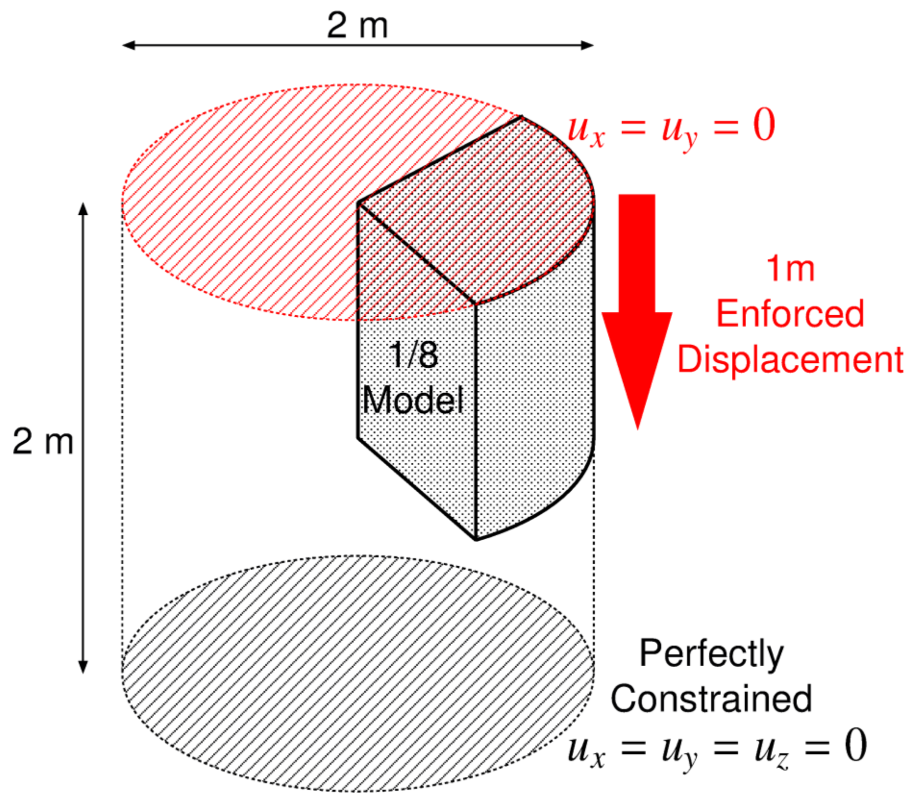


- Constant strain element (C3D4) locks quickly.
- Other elements including selective S-FEMs do not lock.

Selective S-FEMs are **locking-free** in large strain analysis!!

Verification ~ Compression of 1/8 Cylinder ~

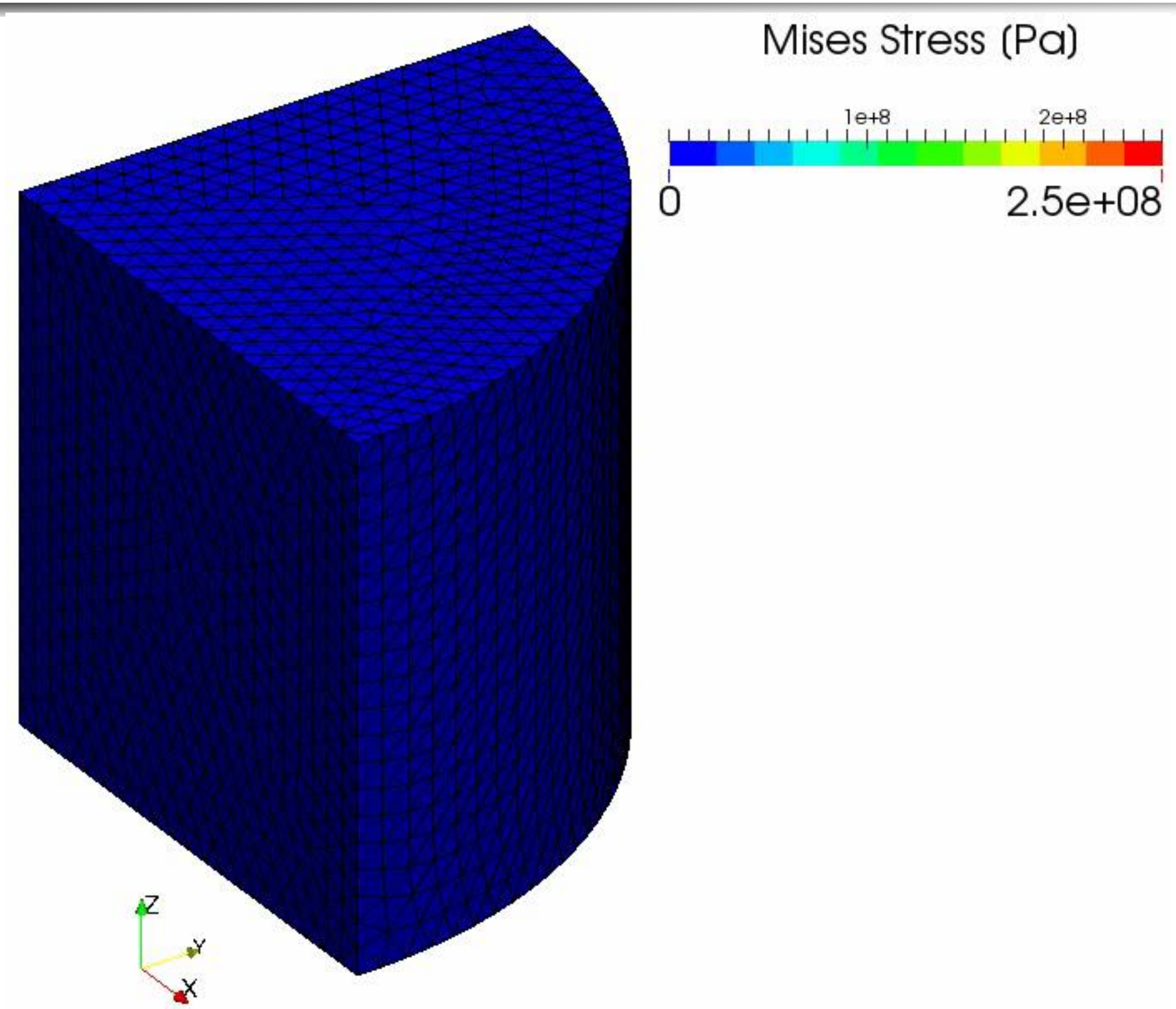
Outline



- 50% axial compression.
- Neo Hookean hyper elastic material of $C_{10} = 40 \times 10^6$ Pa, $D = 5 \times 10^{-12}$ Pa^{-1} (i.e., $\nu_{\mathrm{ini}} = 0.4999$).
- Compared to C3D4H element of ABAQUS/Standard with exactly the same mesh.

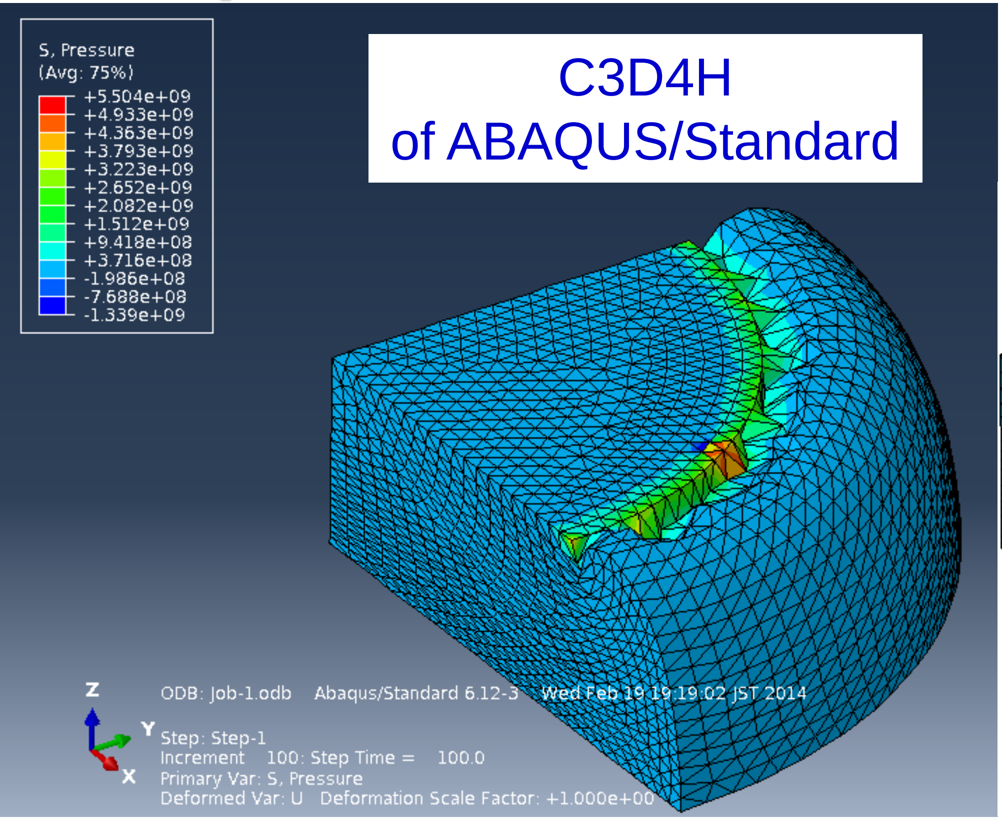
Verification ~ Compression of 1/8 Cylinder ~

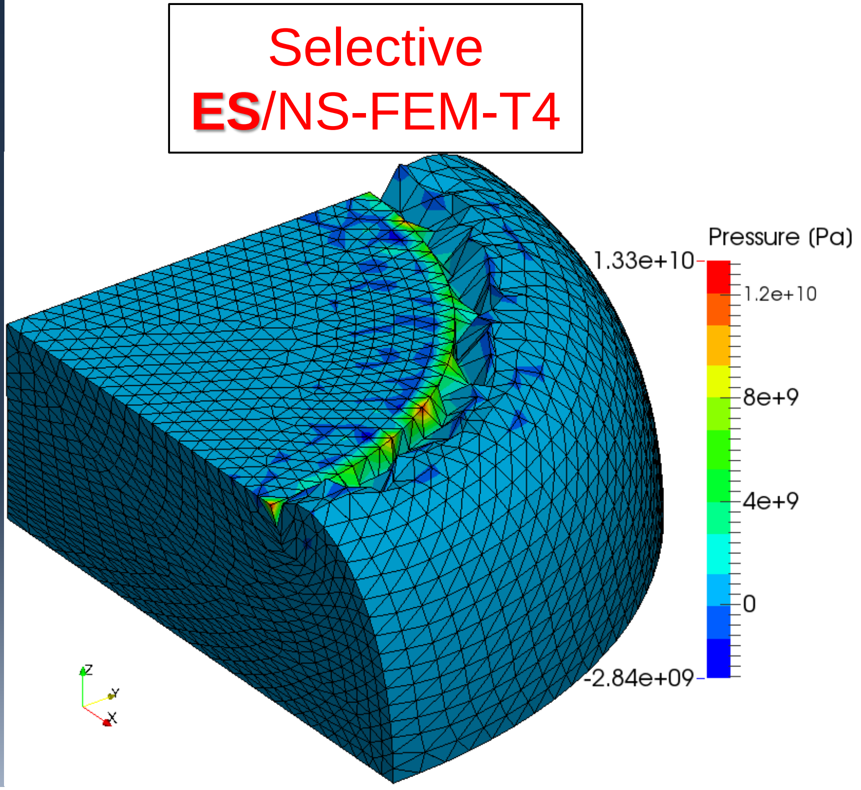
Result of Selective FS/NS-FEM-T4



Verification ~ Compression of 1/8 Cylinder ~

Comparison to ABAQUS

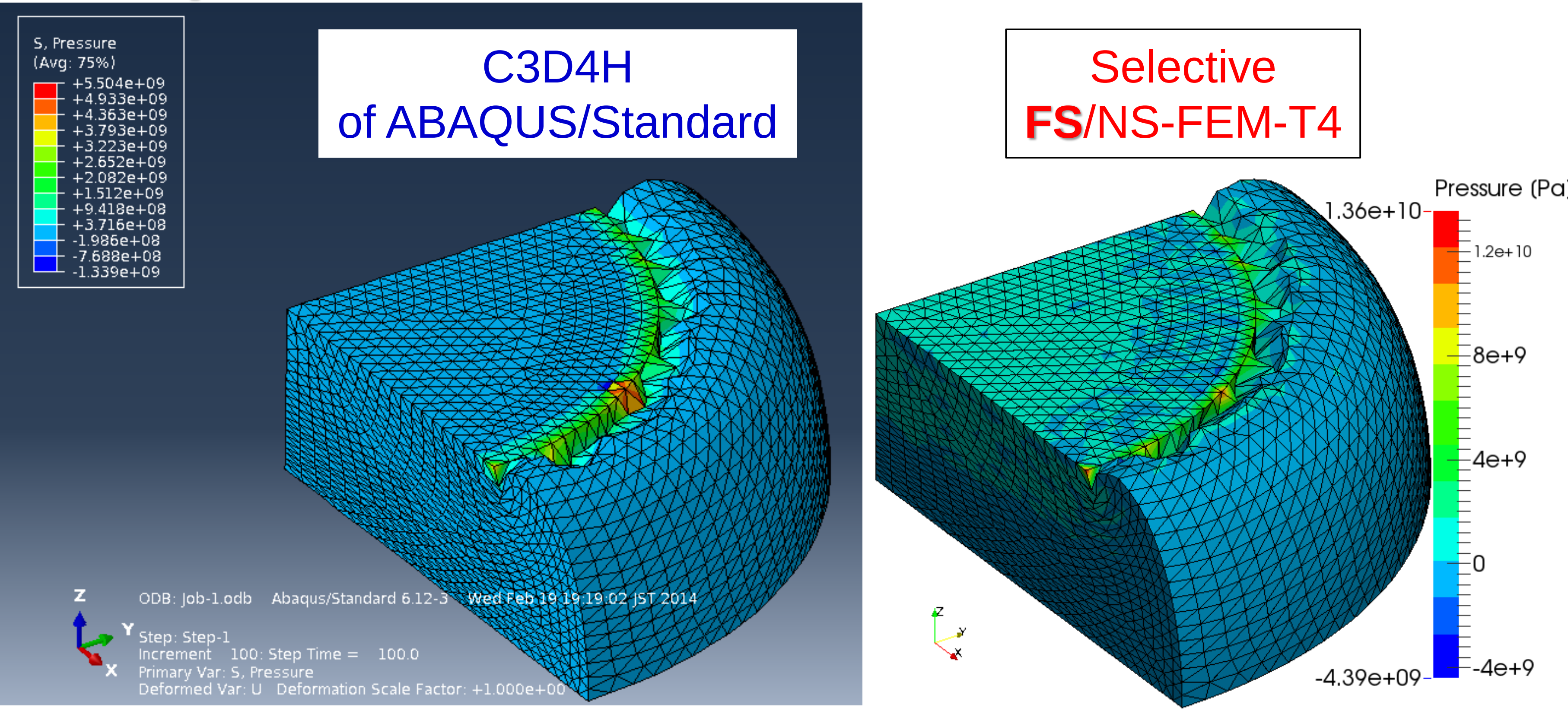




- Deformation is almost the same each other.
- Pressure oscillation is about double in our result.
- Locking of corner/edge elements is observed.

Verification ~ Compression of 1/8 Cylinder ~

Comparison to ABAQUS

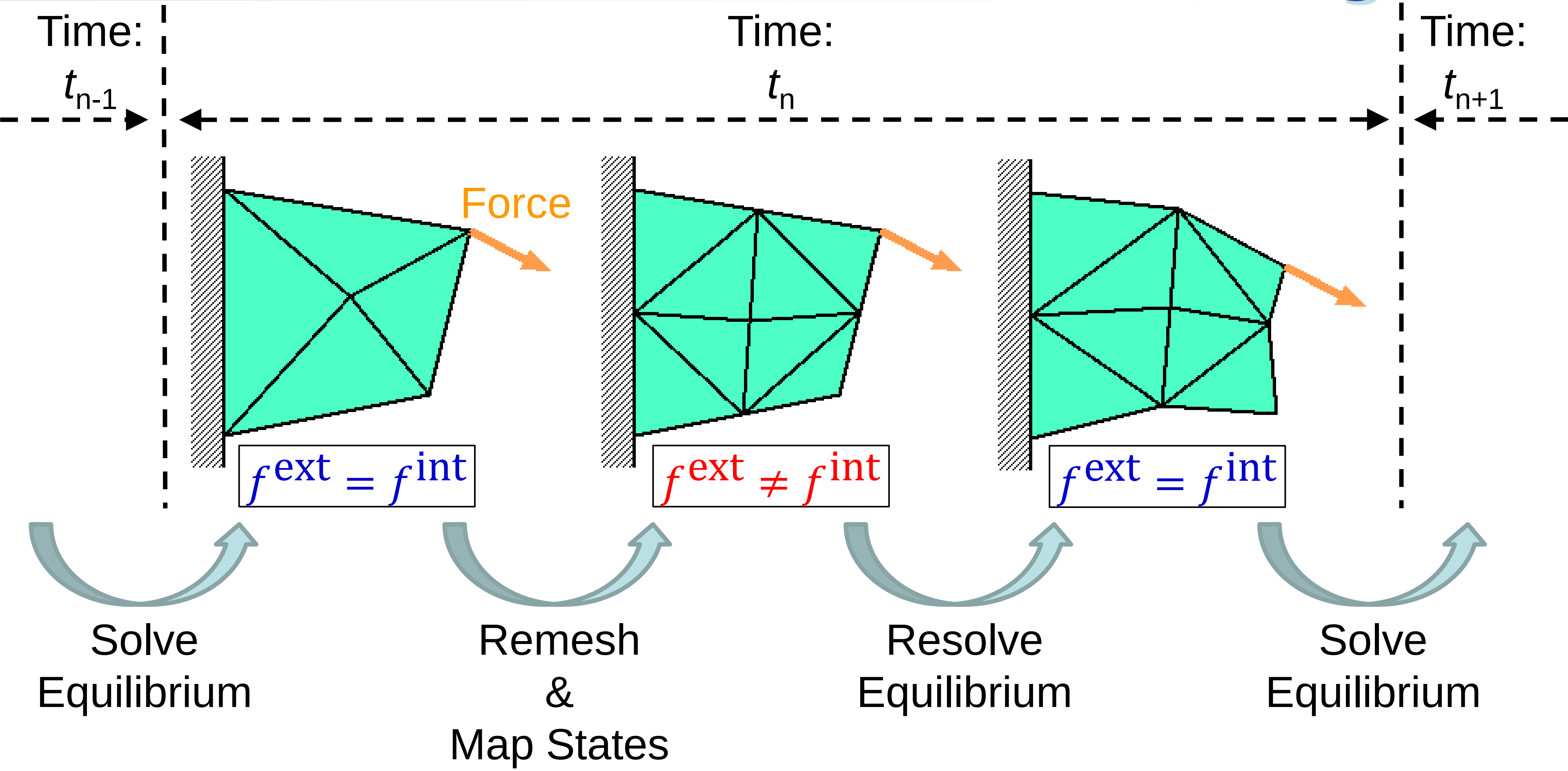


- Deformation is almost the same each other.
- Pressure oscillation is about <u>double</u> in our result.
- Locking of corner/edge elements is observed.

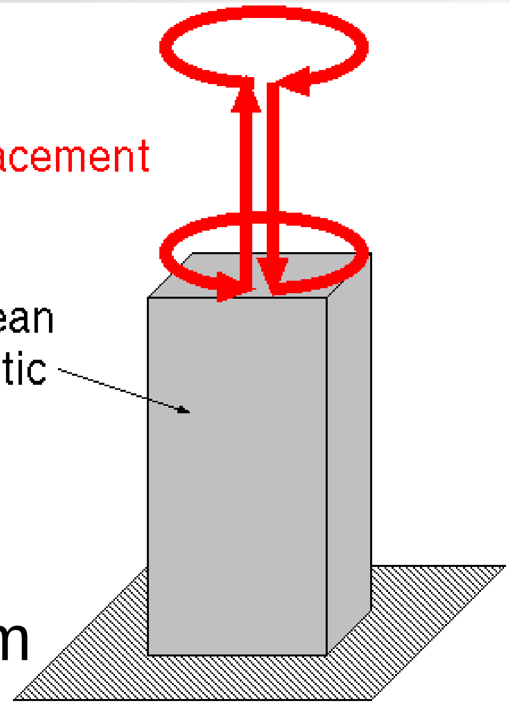
Part 2:

Demonstration of our methods with mesh rezoning

Procedure of Mesh Rezoning



Demo ~ Twist & Stretch of Rubber Cuboid ~

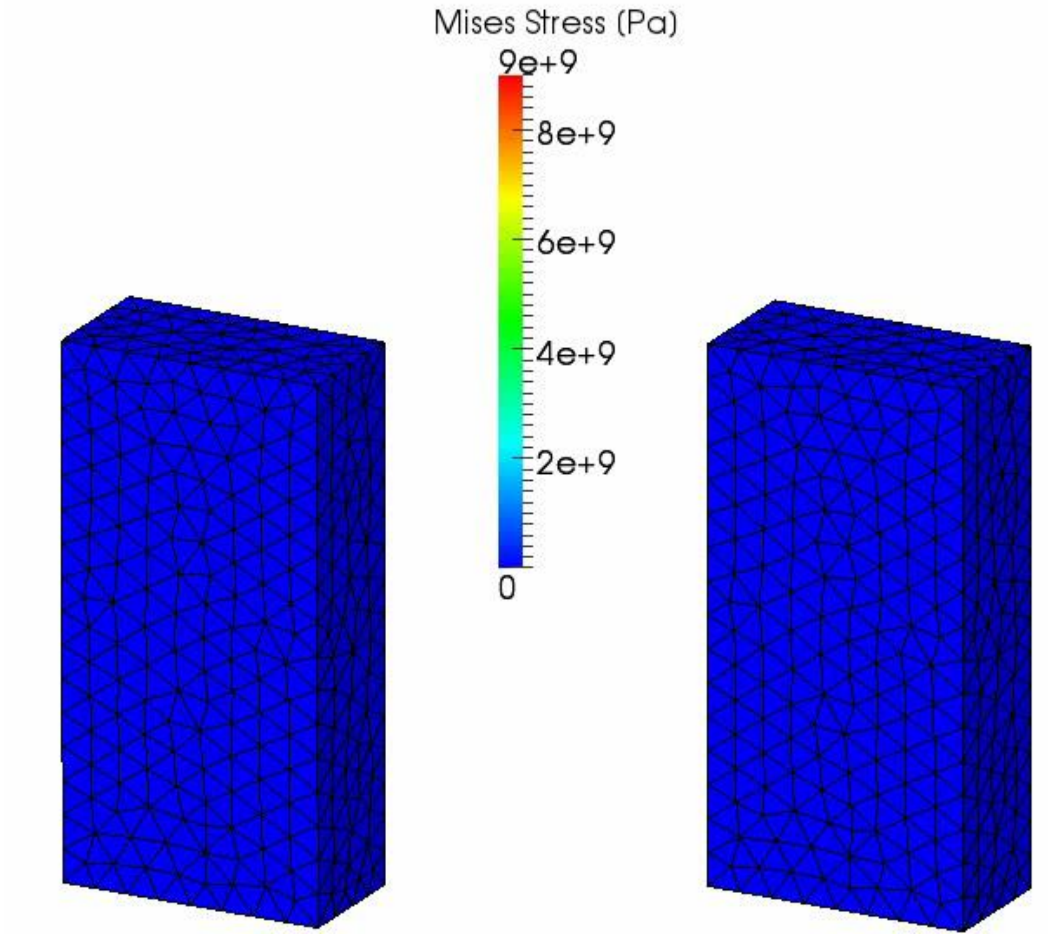


- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ ($\nu_0 = 0.48$)
- Twist up to 360 deg. ⟹ Stretch up to 100% nominal strain ⟹ Twist back ⟹ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

Demo ~ Twist & Stretch of Rubber Cuboid ~

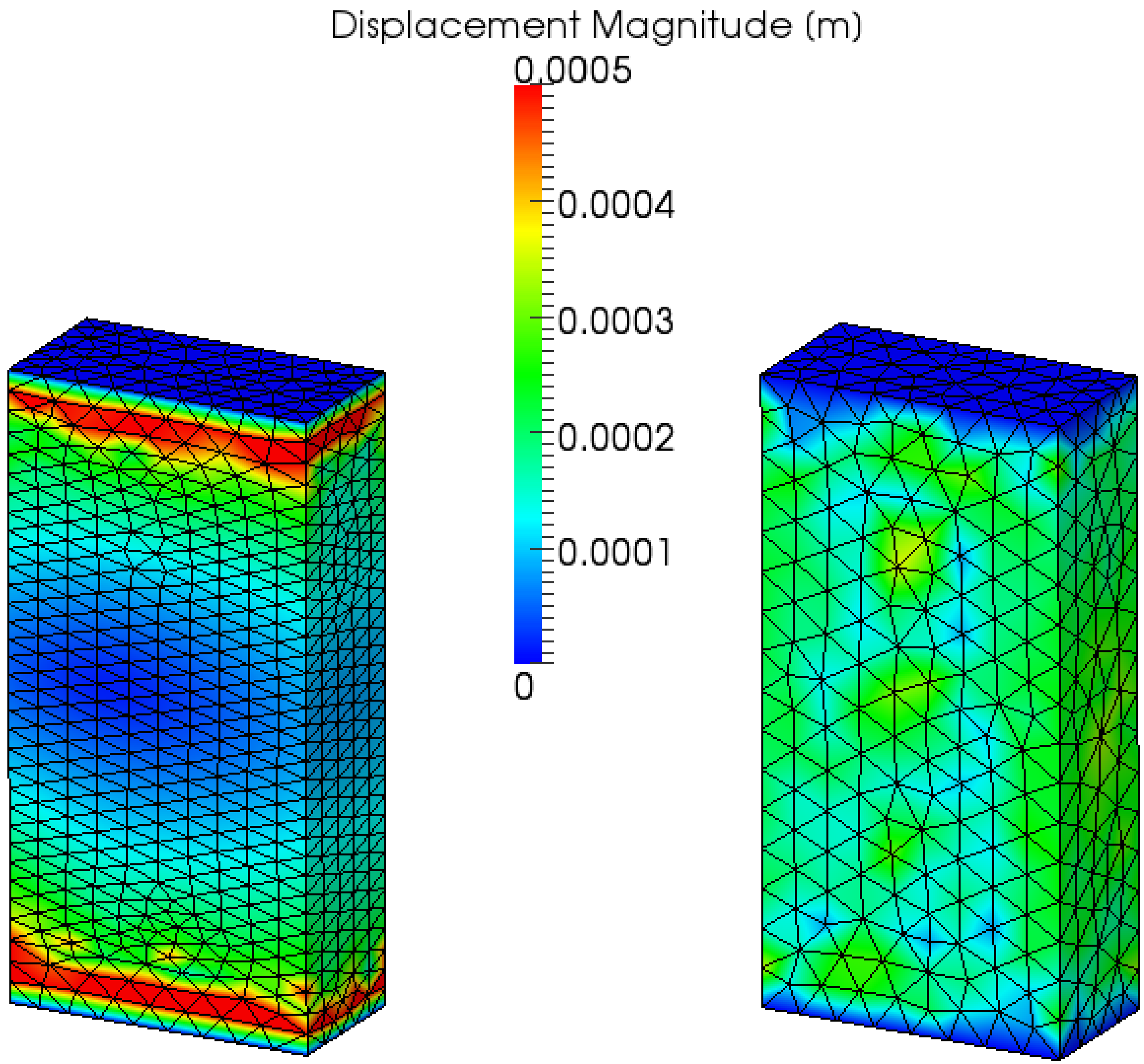
Our selective FS/NS-FEM with mesh rezoning

Our selective FS/NS-FEM without mesh rezoning



Demo ~ Twist & Stretch of Rubber Cuboid ~

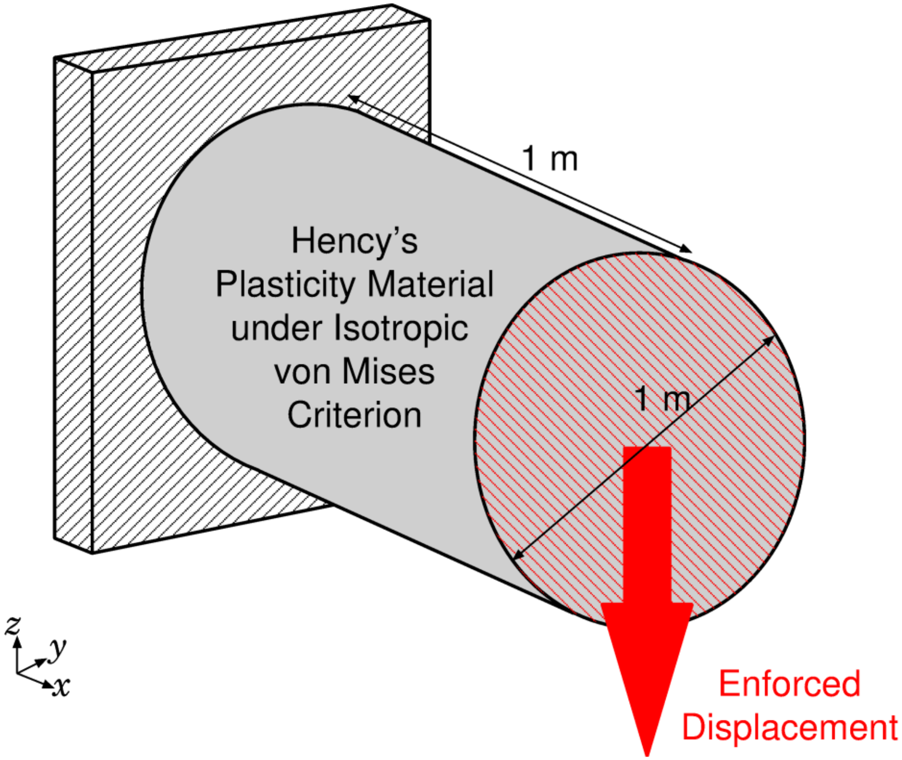
Residual Displacement



It
spring-
backed
almost
perfectly!!

Demo ~ Shearing & Necking of Plastic Rod ~

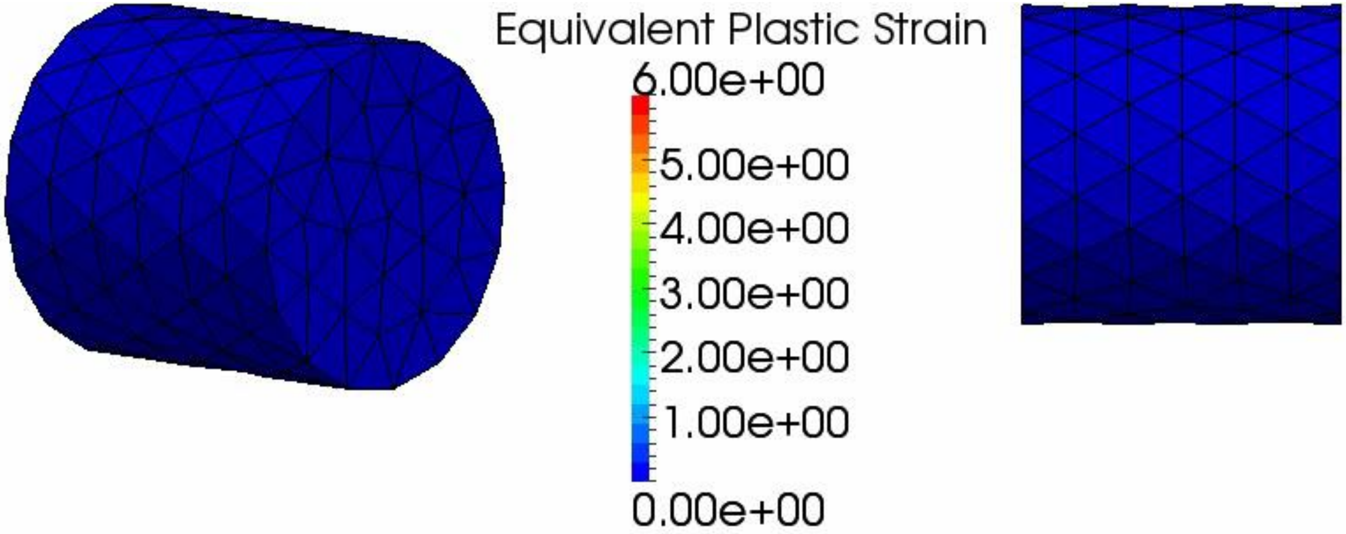
Outline



- Static, 3D
- Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}} / J$,
  with von Mises yield criterion and isotropic hardening.
  Young's Modulus: 1 GPa, Poisson's Ratio: 0.3,
  Yield Stress: 1 MPa, Hardening Coeff.: 0.5 MPa.

Demo ~ Shearing & Necking of Plastic Rod ~

3D Result

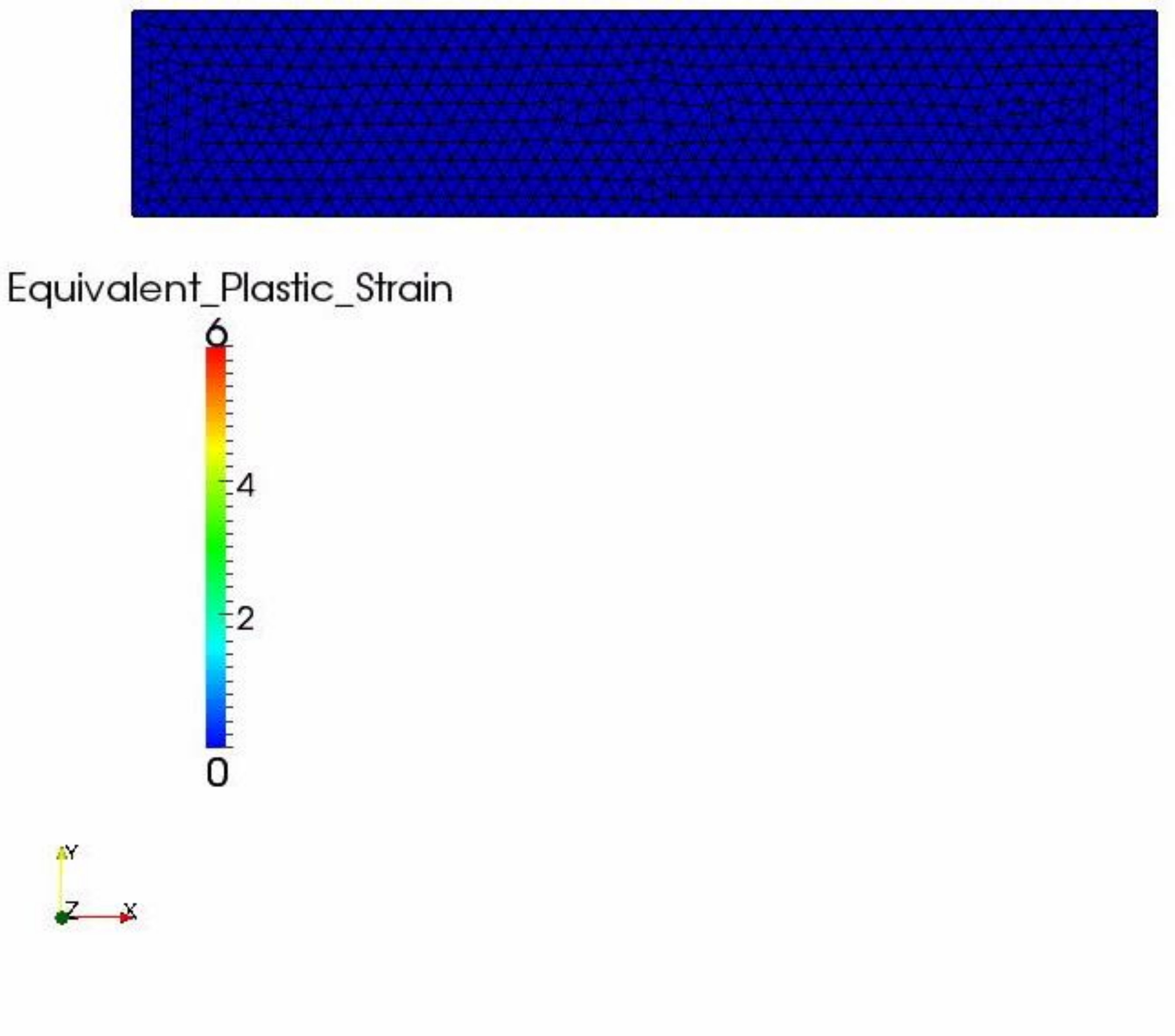


The deformation seems
to be valid.

After 2.8 m disp.,
mesh rezoning error
occurred.

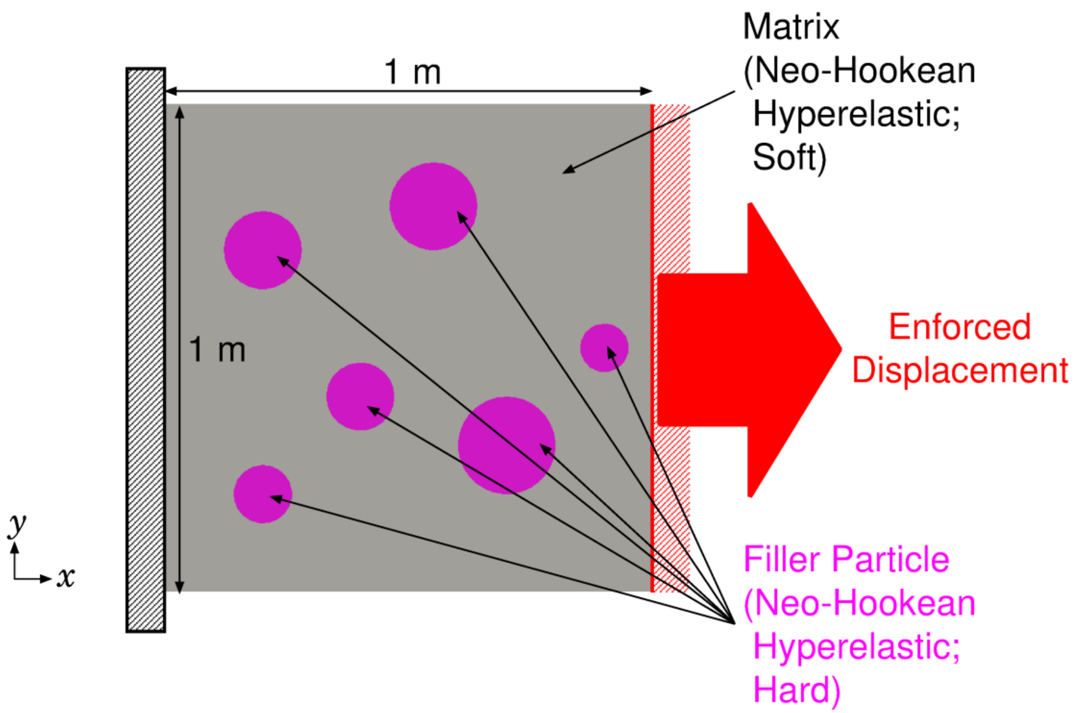
Demo ~ Shearing & Necking of Plastic Rod ~

Result of Similar Analysis in 2D with Selective ES/NS-FEM-T3



Demo ~Tension of Filler Particle Composite~

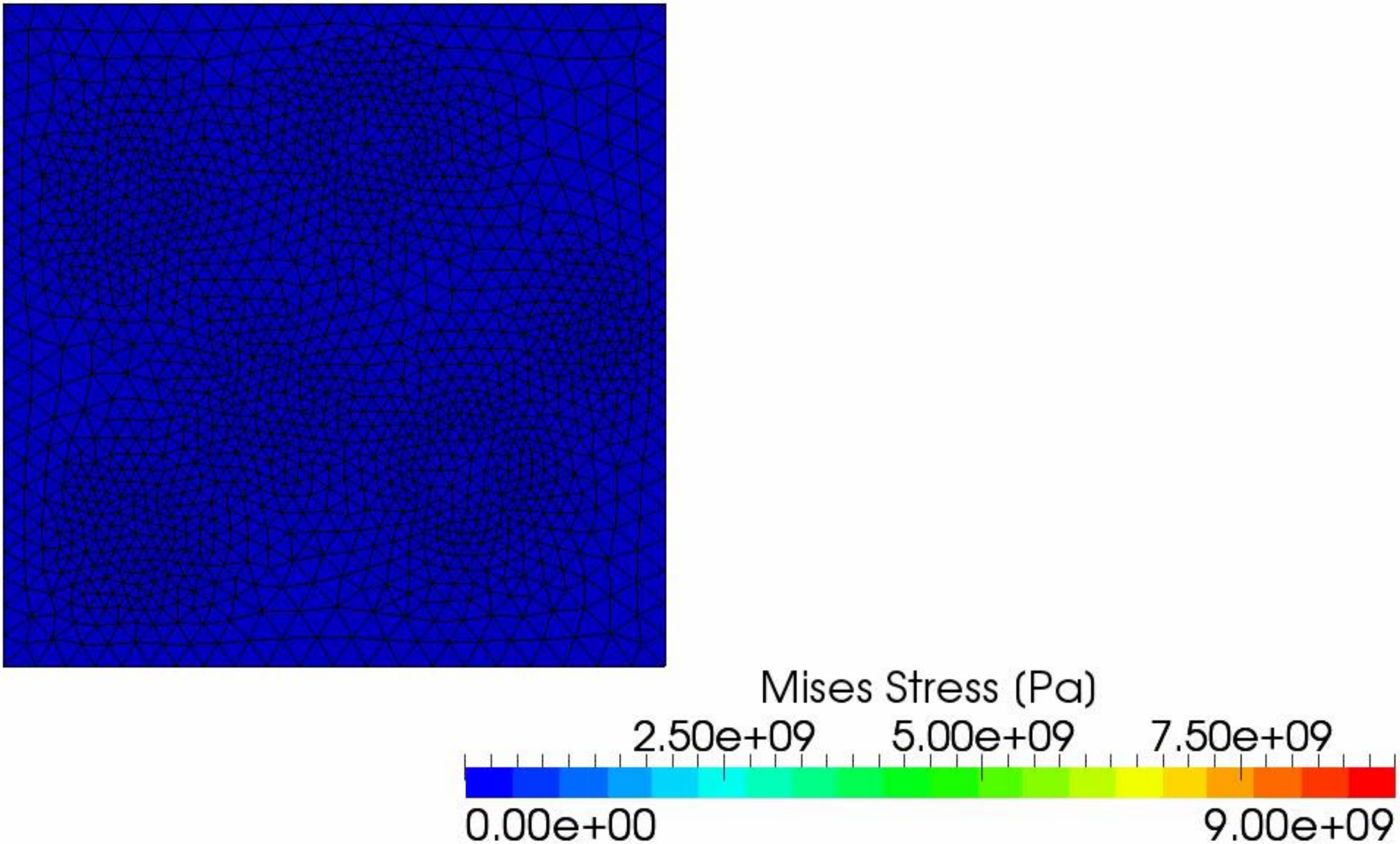
Outline



- 2D, plane-strain, static
- Neo-Hookean Hyperelastic
  - Filler: hard rubber ($E^{\text{initial}} = 100$ GPa, $\nu^{\text{initial}} = 0.49$)
  - Matrix: soft rubber ($E^{\text{initial}} = 1$ GPa, $\nu^{\text{initial}} = 0.49$)

Demo ~Tension of Filler Particle Composite~

Result of Selective ES/NS-FEM-T4



- The deformation seems to be valid.
- After 180% stretch, analysis stopped due to mesh rezoning error.

Summary

Characteristics of S-FEMs & C3D4H

	Shear Locking	Volumetric Locking	Zero Energy Mode	No Increase in DOF	Pressure Oscillation	Dev/Vol Coupled Material
Standard FEM-T4	✗	✗	✓	-	✗	✓
Selective FS/NS-FEM-T4 & ES/NS-FEM-T4	✓	✓	✓	✓	✗	✗
ABAQUS C3D4H	✓	✓	✓	✗	✗	✓

Issues in Future

Take-Home Messages

1. Selective S-FEMs with triangular or tetrahedral elements are **locking free** and easy to implement.

2. The accuracy of selective S-FEMs is almost the same as C3D4H of ABAQUS, which is one of the current best hybrid elements.

3. Selective S-FEMs go well together with mesh rezoning.

Summary and Future Work

Summary

- A new implicit static mesh rezoning method for severely large deformation analysis is proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its accuracy are verified with hyperelastic material and elasto-plastic material.

Future Work

- Resolve pressure oscillation issue
- Apply to contact forming, crack propagation, etc.
- Explicit dynamic formulation
- Local mesh rezoning

Thank you for your kind attention.
I appreciate your question in slow and easy English!!