Application of Preconditioned Iterative Methods in ~~Selective~~ Smoothed Finite Element Methods F-bar with Tetrahedral Elements for Nearly Incompressible Materials

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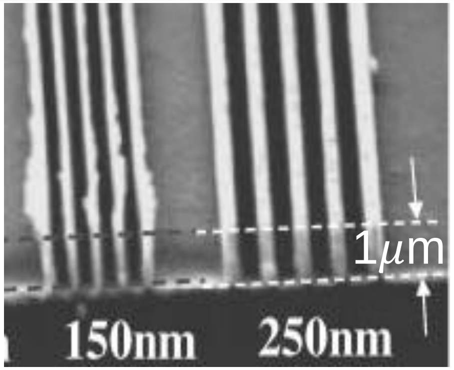
Motivation & Background

Motivation

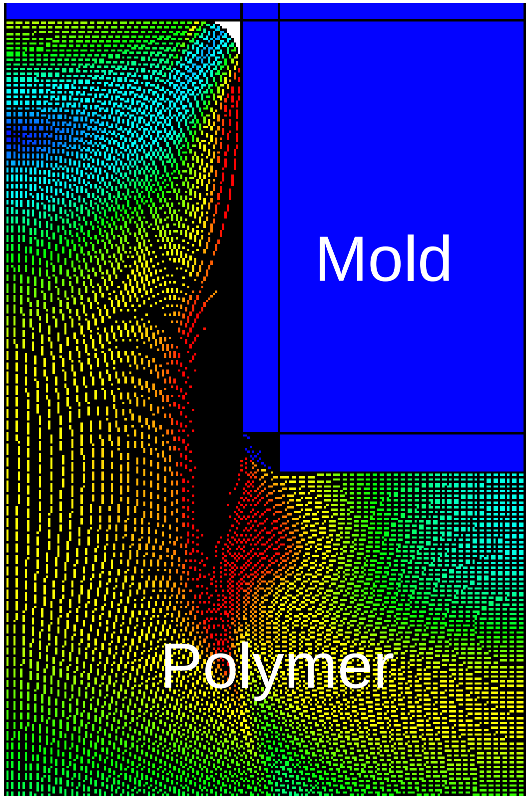
We want to analyze **severe large deformation** of nearly incompressible solids ***accurately and stably***!

(Target: automobile tire, thermal nanoimprint, etc.)



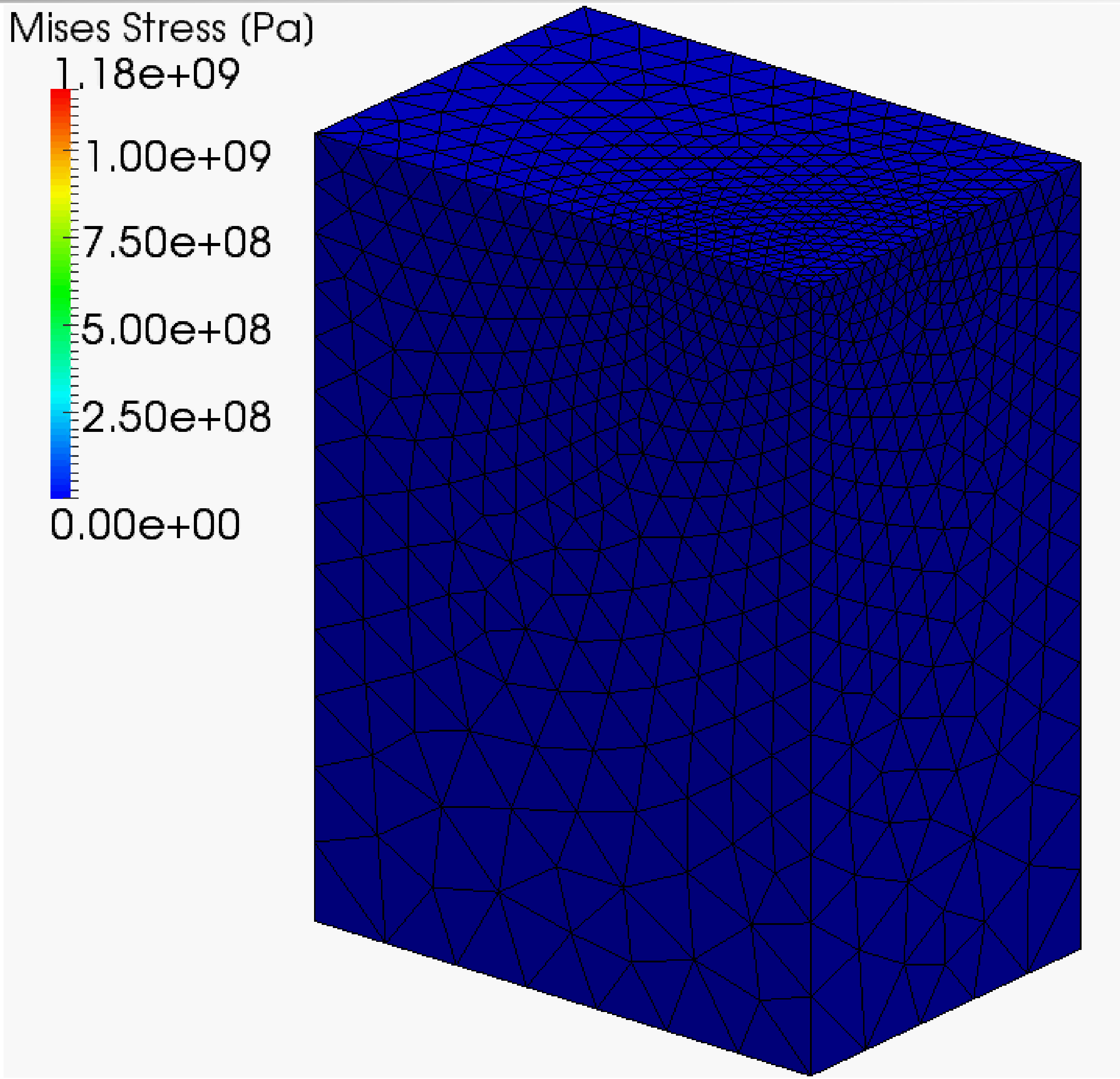
Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.



Mesh rezoning method is indispensable.

Our First Result in Advance



What we want to do:

- Static
- Implicit
- Large deformation
- Mesh rezoning

with locking-free T4 elements

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Unstable in contact analysis; No good in large deformation due to intermediate nodes.
- EAS method:
  - ✗ Unstable due to spurious zero-energy modes.
- B-bar, F-bar and selective integration method:
  - ✗ Not applicable to T4 mesh directly.
- F-bar patch method:
  - ✗ Difficult to construct good patches. Not shear locking free.
- u/p hybrid (mixed) elements:
  - ✗ No sufficient formulation for T4 mesh so far. (There are almost acceptable hybrid elements such as C3D4H of ABAQUS.)
- Smoothed finite element method (S-FEM):
  - ? Unknown potential (since 2009~). ***It's worth trying!!***

Various Types of S-FEMs

- **Basic type**
  - Node-based S-FEM (NS-FEM) — ✗ Spurious zero-energy
  - Face-based S-FEM (FS-FEM)
  - Edge-based S-FEM (ES-FEM)
  - (FS-FEM, ES-FEM) — ✗ Volumetric Locking
- **Selective type**
  - Selective FS/NS-FEM
  - Selective ES/NS-FEM
  - ✗ Limitation of constitutive model, Pressure oscillation, Corner locking
- **Bubble-enhanced or Hat-enhanced type**
  - bFS-FEM, hFS-FEM
  - bES-FEM, hES-FEM
  - ✗ Pressure oscillation, Short-lasting
- **F-bar type**
  - NEW F-barES-FEM — ? Unknown potential

Objective

Develop a new S-FEM, F-barES-FEM-T4,
by combining F-bar method and ES-FEM-T4
for large deformation problems
of nearly incompressible solids

Table of Body Contents

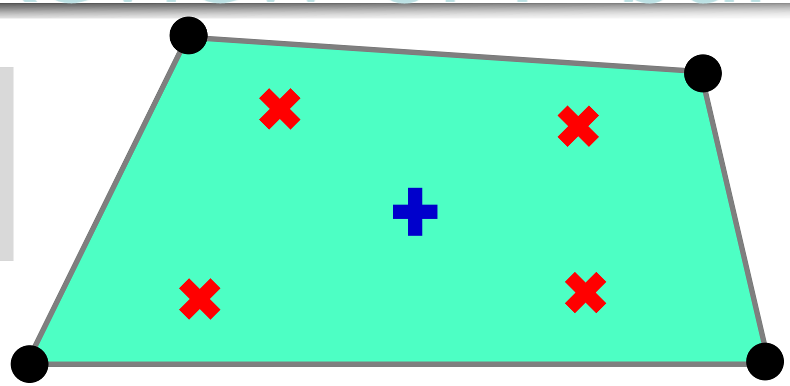
Method

Formulation of F-barES-FEM-T4 & Introduction of AMG-GMRES

(F-barES-FEM-T3 in 2D is explained for simplicity.)

Quick Review of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}$ $(= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\mathrm{iso}}$ $(= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point as

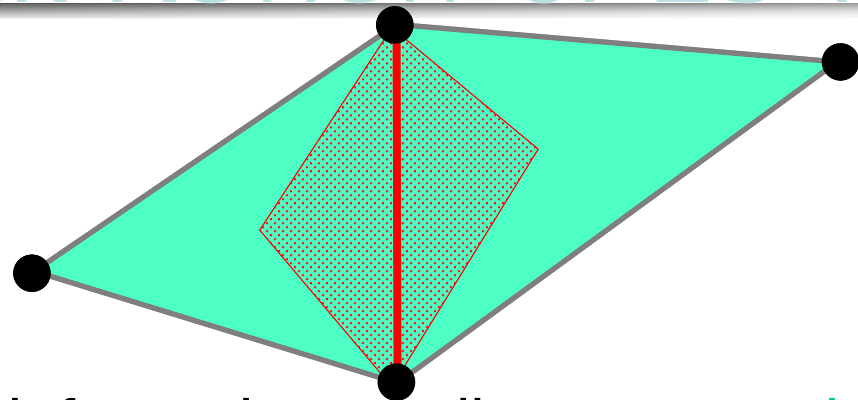
$$\bar{\boldsymbol{F}} = \bar{J}^{\,1/3}\, \boldsymbol{F}^{\mathrm{iso}}.$$

4. Use $\bar{\boldsymbol{F}}$ to calculate the stress, nodal force and so on.

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.

Quick Review of ES-FEM

For triangular (T3) or tetrahedral (T4) elements.



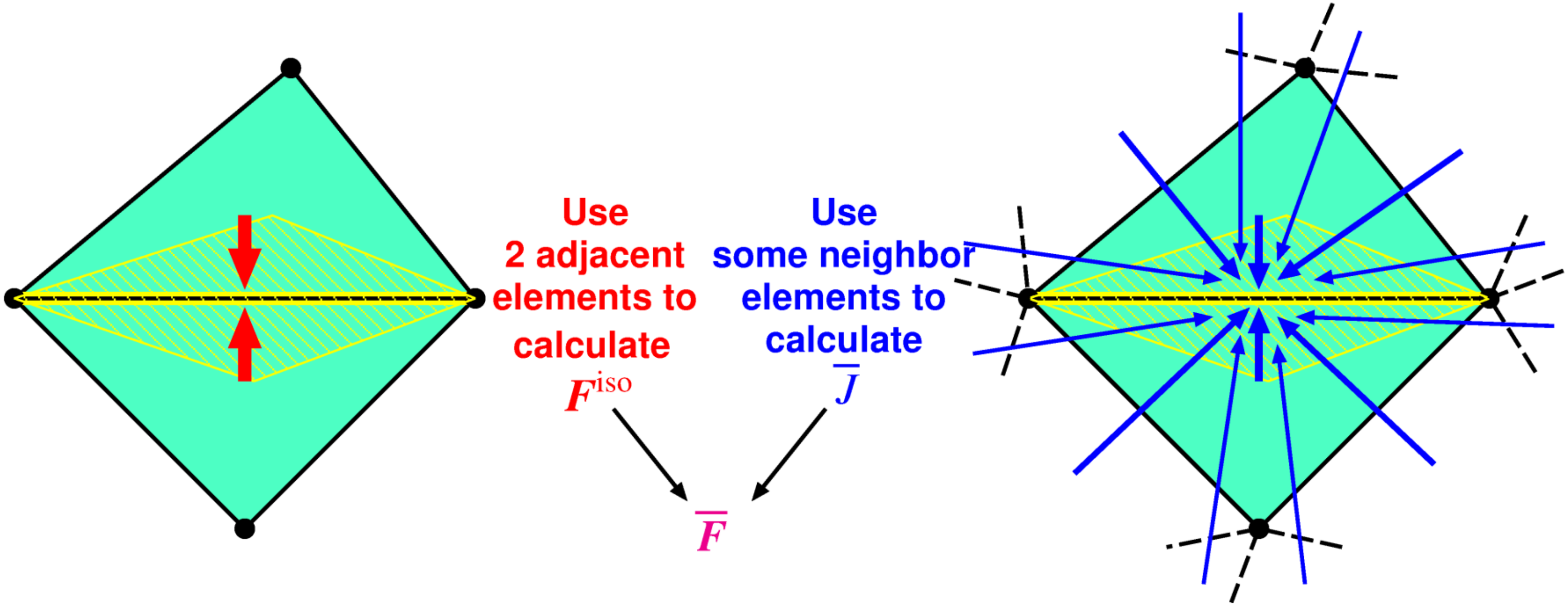
Algorithm:

1. Calculate the deformation gradient $\boldsymbol{F}$ at each element as usual.
2. Distribute the deformation gradient $\boldsymbol{F}$ to the connecting edges with area weights to make ${}^{\text{Edge}}\boldsymbol{F}$ at each edge.
3. Use ${}^{\text{Edge}}\boldsymbol{F}$ to calculate the stress, nodal force and so on.

ES-FEM is used to **avoid shear locking** in T3 or T4 elements. Yet, it **cannot avoid volumetric locking**.

Outline of F-barES-FEM

Concept Combination of F-bar method and ES-FEM



- ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\overline{J}$ is given by Cyclic Smoothing (detailed later).
- ${}^{\mathrm{Edge}}\overline{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

$${}^{\mathrm{Edge}}\overline{\boldsymbol{F}} = {}^{\mathrm{Edge}}\overline{J}^{1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}.$$

Outline of F-barES-FEM

Brief Formulation

1. Calculate $^{\text{Elem}}J$ as usual.

2. Smooth $^{\text{Elem}}J$ at nodes and get $^{\text{Node}}\tilde{J}$.
3. Smooth $^{\text{Node}}\tilde{J}$ at elements and get $^{\text{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

⋮ (c layers of ~)

5. Smooth $^{\text{Elem}}\tilde{\tilde{J}}$ at edges to make $^{\text{Edge}}\bar{J}$.
6. Combine $^{\text{Edge}}\bar{J}$ and $^{\text{Edge}}\boldsymbol{F}^{\text{iso}}$ of ES-FEM as

$$^{\text{Edge}}\bar{\boldsymbol{F}} = {^{\text{Edge}}\bar{J}}^{1/3}\ {^{\text{Edge}}\boldsymbol{F}^{\text{iso}}}.$$

Hereafter, F-barES-FEM-T4 with c-time cyclic smoothing is called "F-barES-FEM-T4(c)".

Quick Introduction of AMG-GMRES

Preconditioner: AMG

- **A**lgebraic **M**ulti-**G**rid.
- A framework of stationary iterative methods.
- Mainly comprised of 3 parts:
  Smoothing, Restriction, and Prolongation.
- Can be used as a preconditioner.

Solver: GMRES

- **G**eneralized **M**inimal **RES**idual.
- One of the non-stationary iterative methods.
- Usually used with a restart parameter *r* as GMRES(*r*).

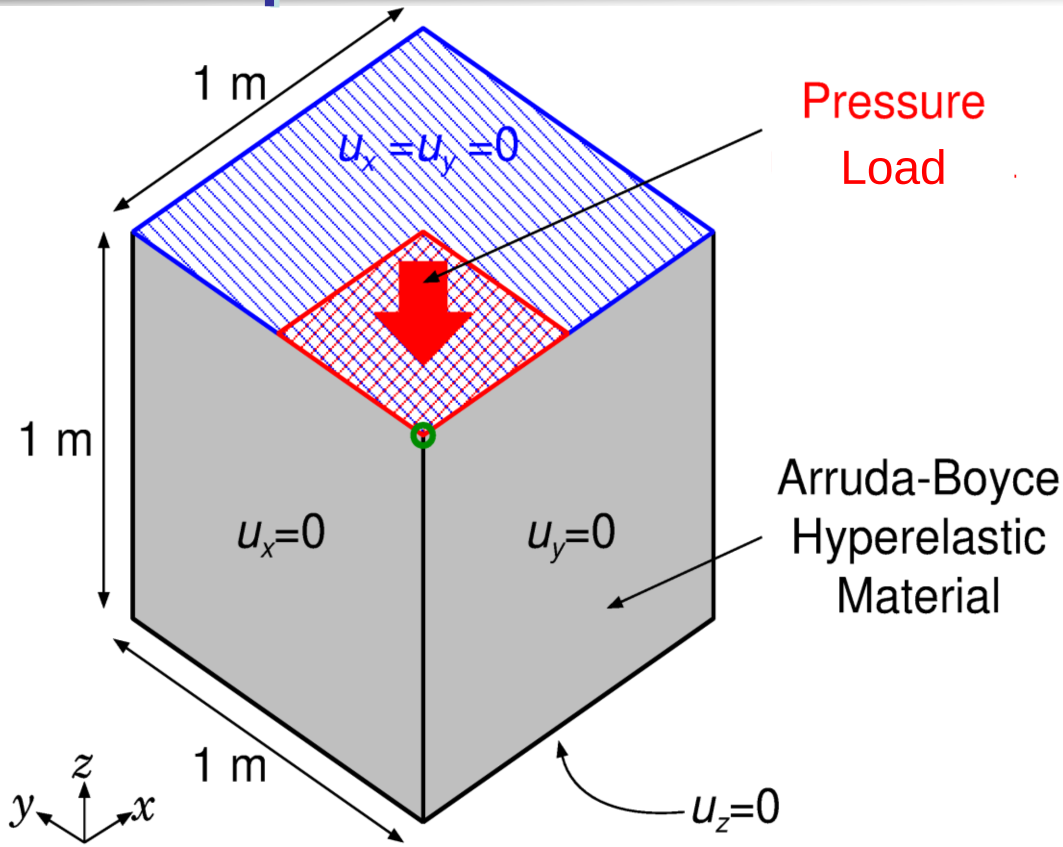
Result

Verification of F-barES-FEM-T4

(Analyses without mesh rezoning are presented for pure verification.)

#1: Compression of a Block

Outline



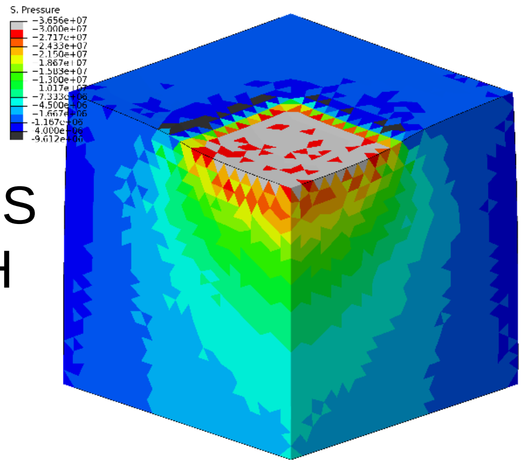
- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured tetra mesh.

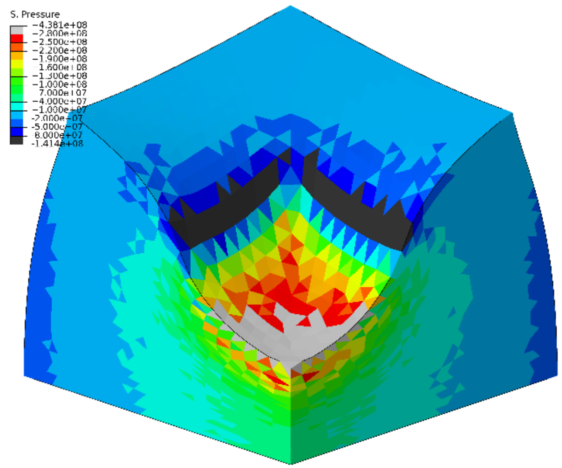
#1: Compression of a Block

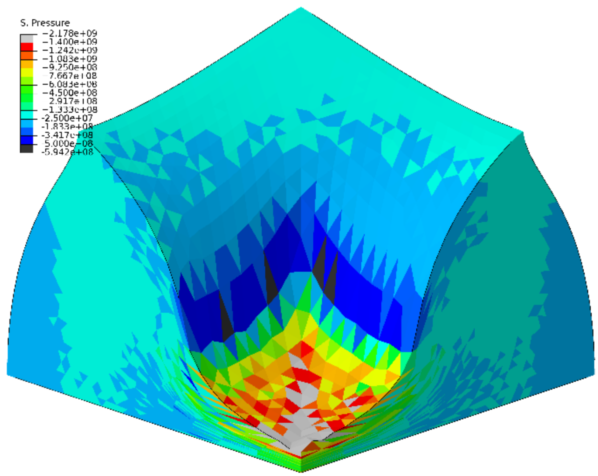
Pressure Distribution

Early stage | Middle stage | Later stage

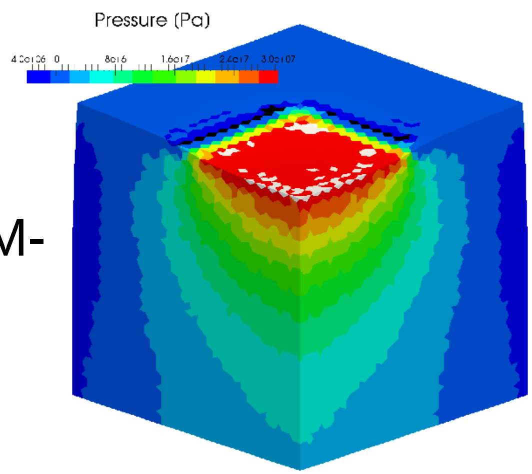
ABAQUS C3D4H

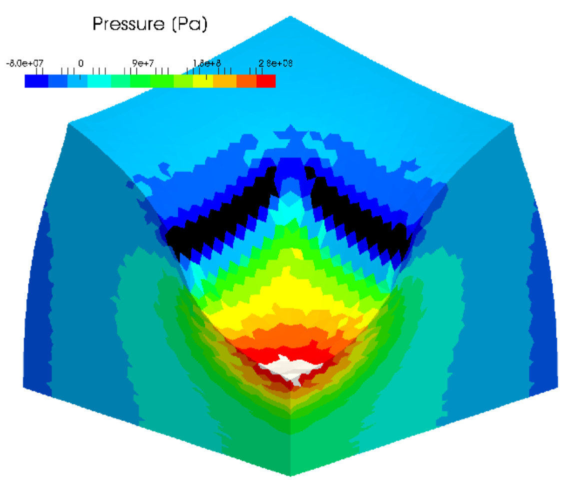


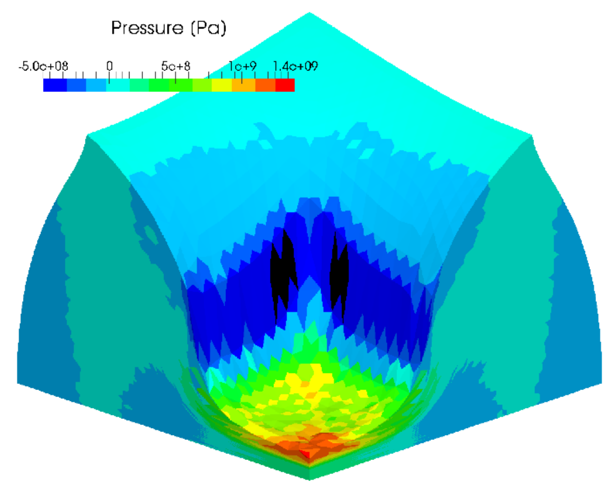




F-bar ES-FEM-T4(2)





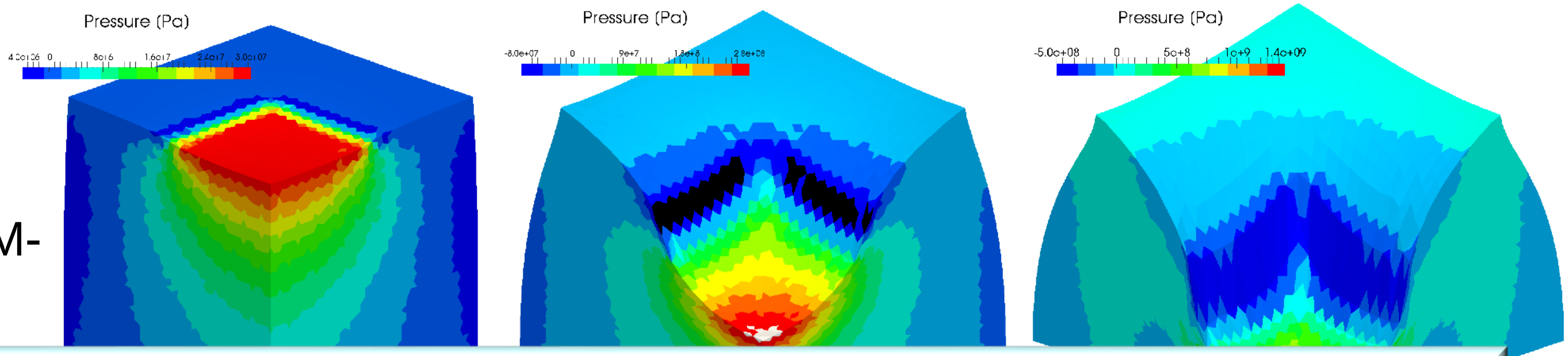


#1: Compression of a Block

Pressure Distribution

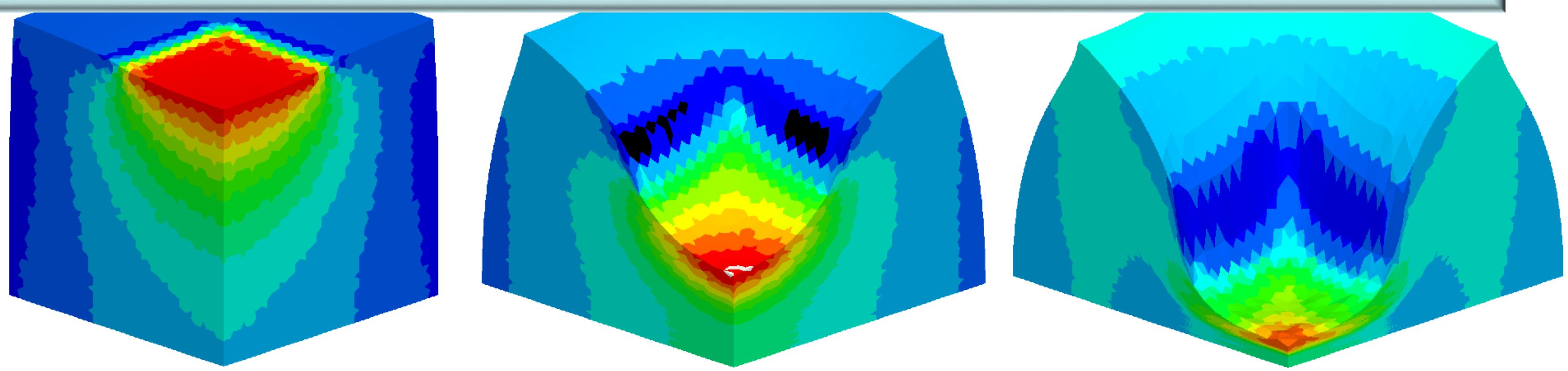
Early stage | Middle stage | Later stage

F-bar ES-FEM-T4(3)



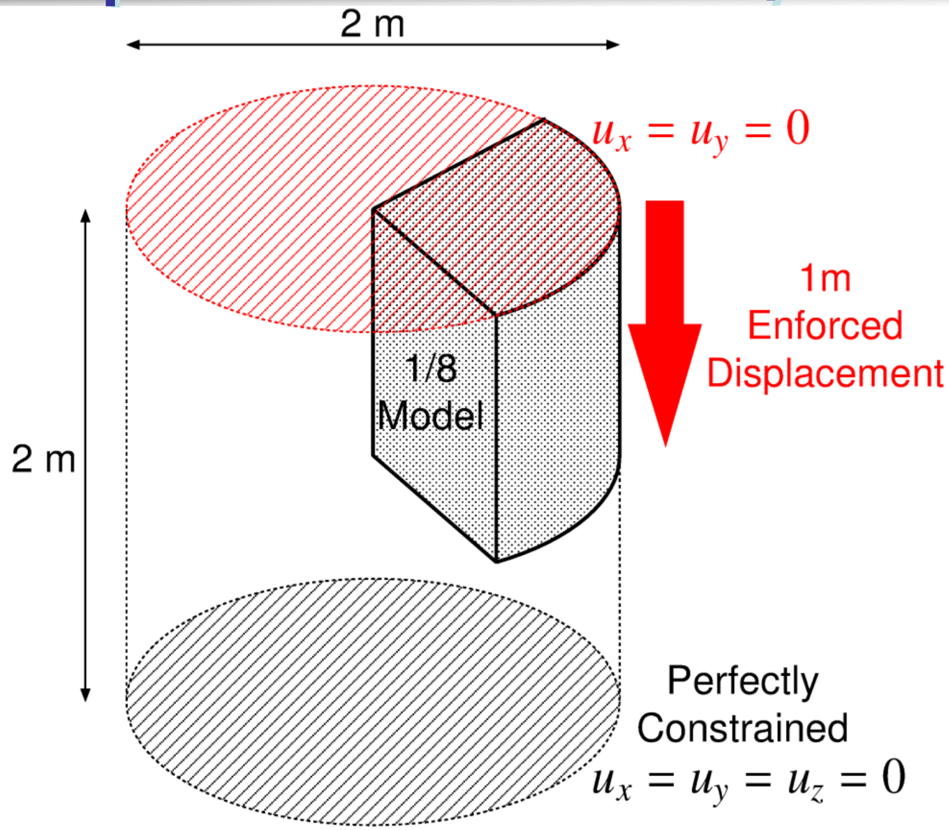
In case the Poisson's ratio is 0.499,
F-barES-FEM-T4(2) or later resolves the pressure oscillation issue.

F-bar ES-FEM-T4(4)



#2: Compression of 1/8 Cylinder

Outline

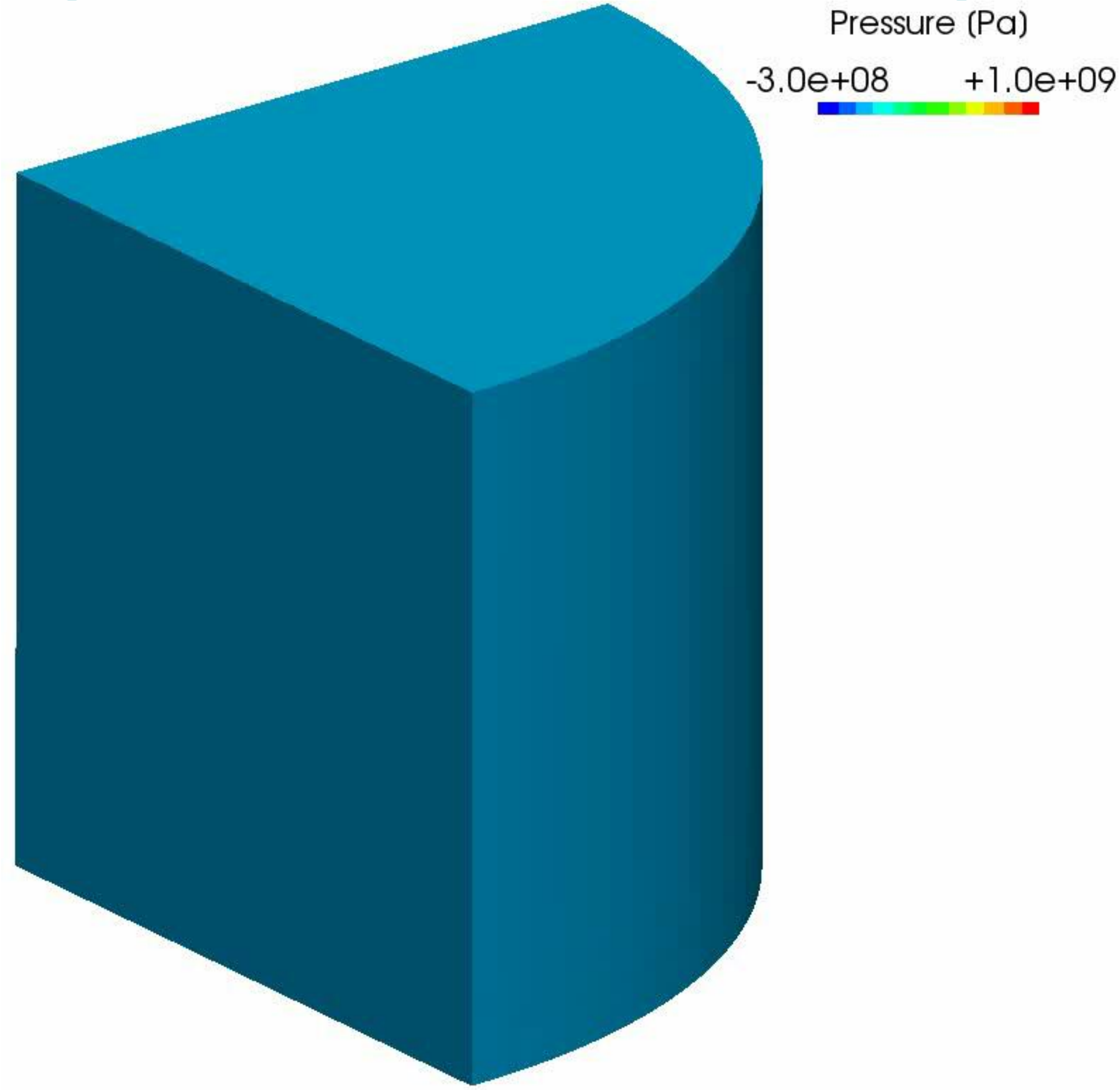


- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Enforced displacement is applied to the top surface.
- Compared to ABAQUS C3D4H with the same unstructured tetra mesh.

#2: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

50% nominal compression

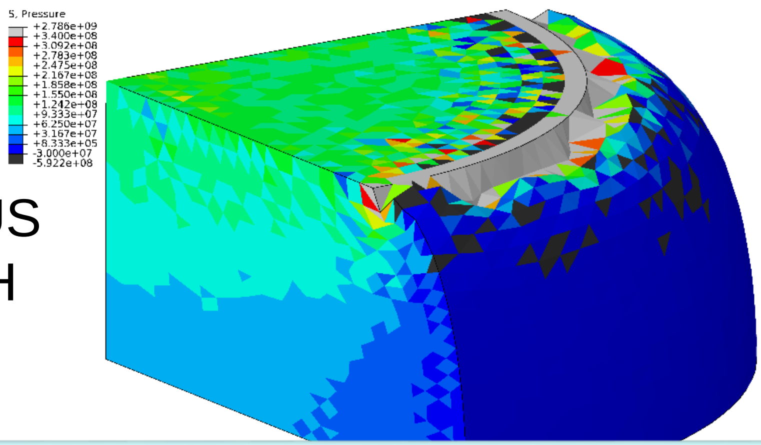


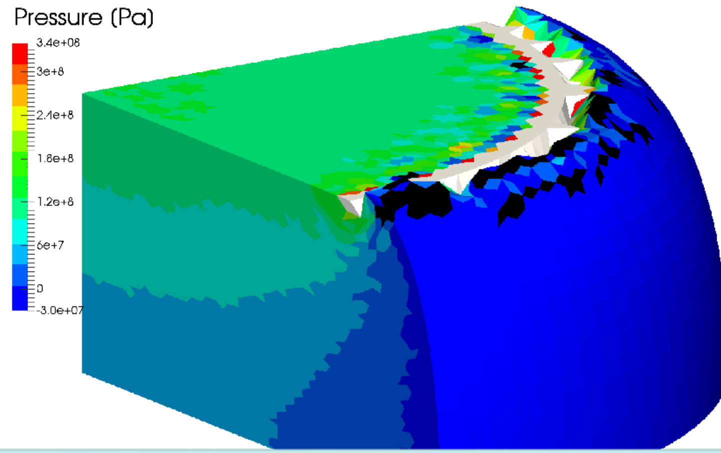
Almost smooth pressure distribution is obtained except just around the rim.

#2: Compression of 1/8 Cylinder

Pressure Distribution

ABAQUS
C3D4H

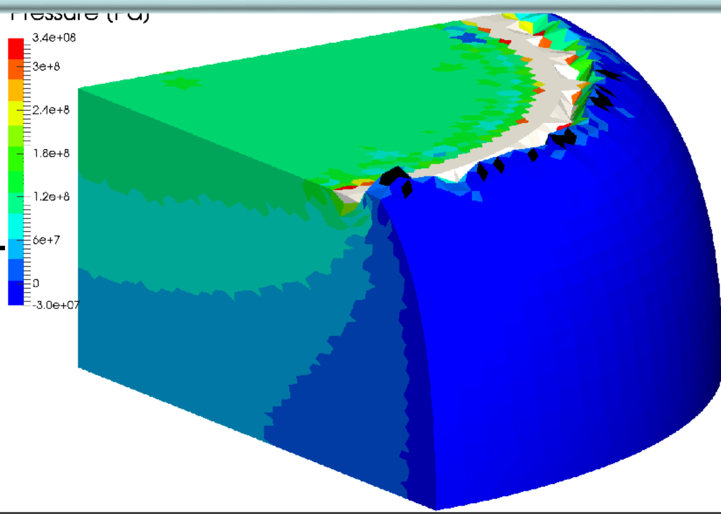


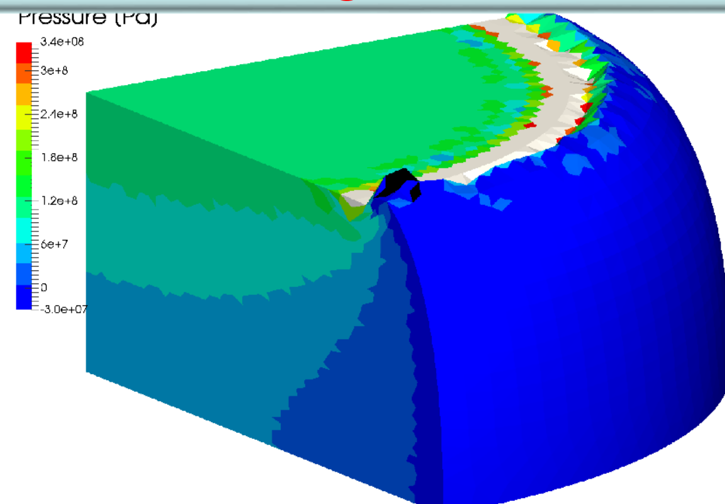


F-bar
ES-FEM-
T4(2)

F-barES-FEM-T4 with a sufficient cyclic smoothing
also resolves the corner locking issue.

F-bar
ES-FEM-
T4(3)





F-bar
ES-FEM-
T4(4)

Discussion

Application of AMG-GMRES to F-barES-FEM-T4

Characteristics of [K] in F-barES-FEM-T4

✓ No increase in DOF.
(No Lagrange multiplier. No static condensation.)

✓ Positive definite.

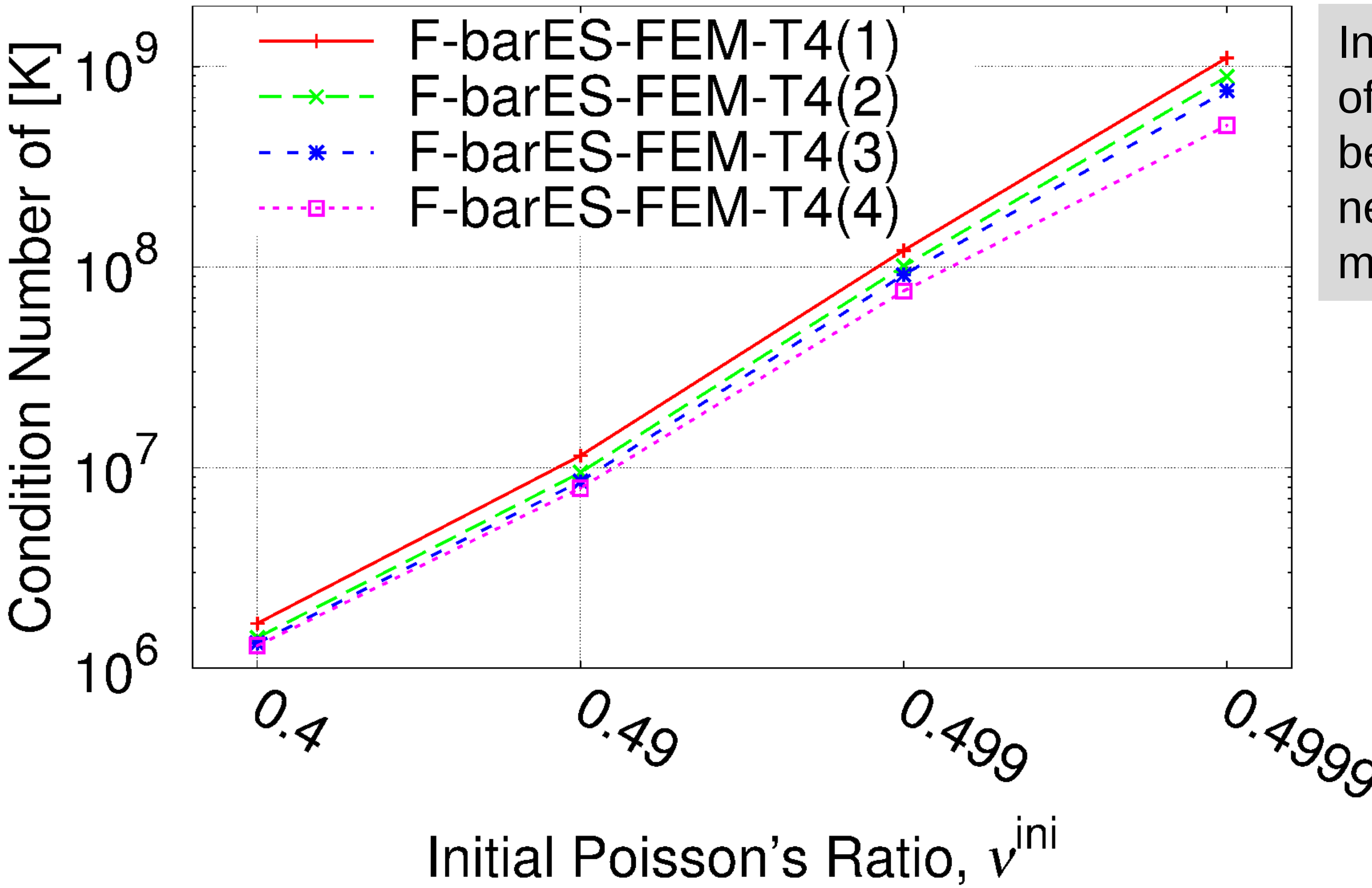
✗ Wide in bandwidth…
In case of standard unstructured T4 meshes,

Method	Approx. Bandwidth	Approx. Ratio
Standard FEM-T4	40	1
F-barES-FEM(1)	390	x10
F-barES-FEM(2)	860	x20
F-barES-FEM(3)	1580	x40
F-barES-FEM(4)	2600	x65

✗ Ill-posed…
(Relatively large condition number.)

Condition Number of [K]

Condition number vs. Initial Poisson's ratio



In one case of cantilever bending with neo Hookean model

Increase in *c* does not improve the ill-posedness of [K] much…
⟹ Application of iterative solver for [K] is difficult.

Capability of AMG-GMRES

- AMG-GMRES with
  - 5th order <u>Chebyshev polynomial smoother</u>
  - Jacobi smoothed aggregation
  - Restart number is fixed at $r = 150$
  - # of V-cycle is varied between 10 and 30.

	$\nu = 0.49$	$\nu = 0.499$	$\nu = 0.4999$
# of V−cycle = 10	✓	✗	✗
# of V−cycle = 20	✓	✓	✗
# of V−cycle = 30	✓	✓	✓

Increase in the # of V-cycles improves the condition number of [*K*] for GMRES and helps the convergence of AMG-GMRES.

CPU Time of AMG-GMRES

CPU time is compared between

- Direct solver: MKL PARDISO of Intel
- Iterative solver: AMG-GMRES(150)

(Note that it is not tuned yet...)

Currently,

- AMG-GMRES is faster only when $\nu \leq 0.49$.
- MKL PARDISO is faster when $\nu > 0.49$.

This is due to the increase of cost for many V-cycles and also the lack of tuning of AMG-GMRES.

In point of speed, F-barES-FEM-T4 needs some improvements.
e.g.) finding a good sparse approximation of $[K]$, generalization of $[K]$, and so on.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

- ✓ Locking-free with 1st -order tetra meshes.
  No difficulty in severe strain or contact analysis.
- ✓ No increase in DOF.
  No need of static condensation;
  Easy extension to dynamic explicit analysis.
- ✓ Suppression of pressure oscillation in nearly incompressible materials.
- ✓ Suppression of corner locking.

Drawbacks

- ✗ Increase in bandwidth of the exact tangent stiffness $[K]$.
- ✗ Relatively large condition number of $[K]$.

F-barES-FEM-T4 has excellent accuracy but needs some effort for speed-up.

Conclusion

- A new FE formulation named "F-barES-FEM-T4" is proposed.
- F-barES-FEM-T4 combines the F-bar method and ES-FEM-T4.
- Owing to the cyclic smoothing, F-barES-FEM-T4 is locking-free and also pressure oscillation-free with no increase in DOF.
- Only one drawback of F-barES-FEM-T4 is the decrease of calculation speed due to the increase in bandwidth of $[K]$, which is our future work to solve.

Thank you for your kind attention!
I appreciate your questions and comments in *easy* and *slow* English!

Appendix

Characteristics of FEM-T4s

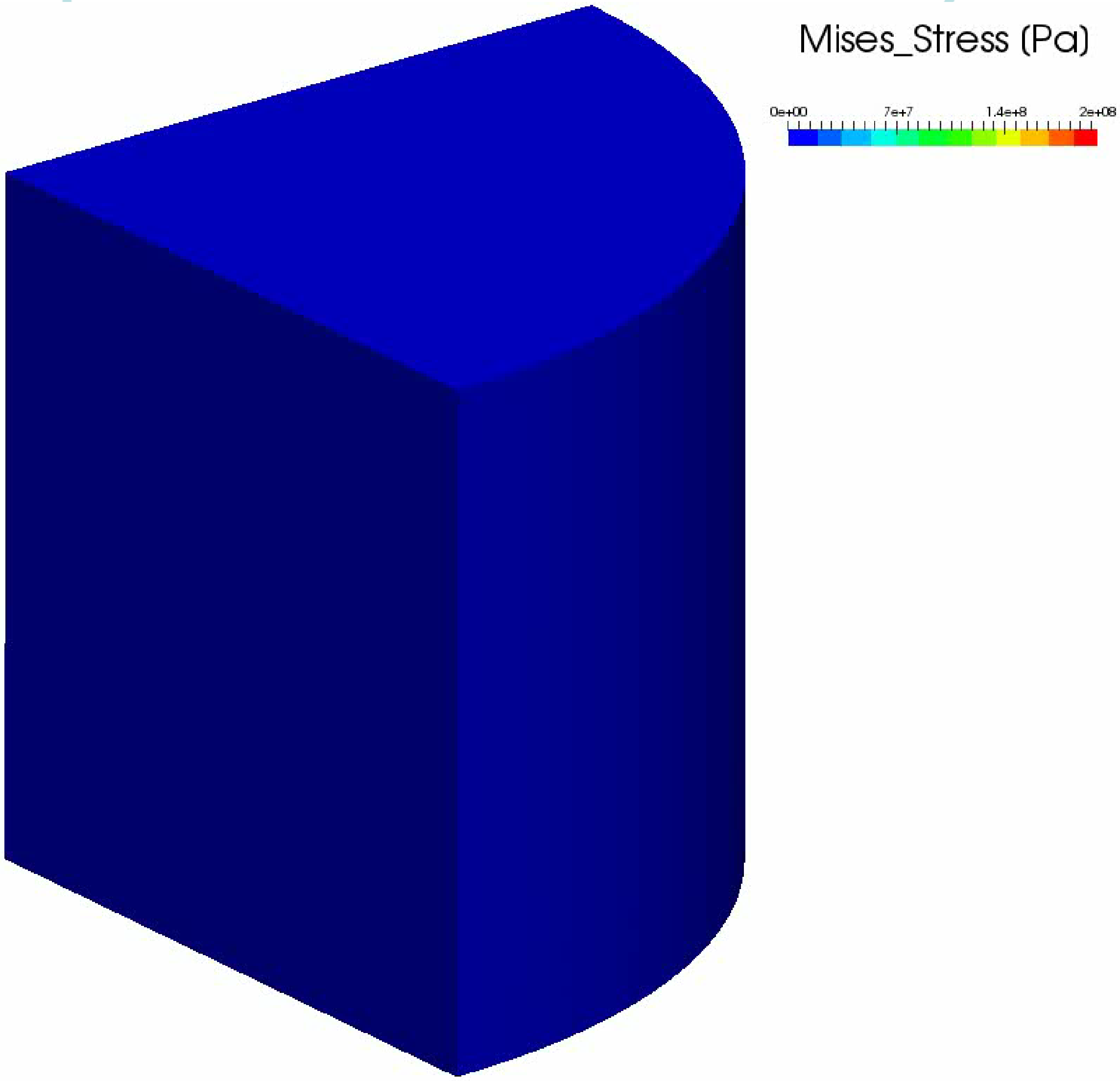
	Shear & Volumetric Locking	Zero-Energy Mode	Dev/Vol Coupled Material	Pressure Oscillation	Corner Locking	Severe Strain
Standard FEM-T4	✗	✓	✓	✗	✗	✓
ABAQUS C3D4H	✓	✓	✓	✗	✗	✓
Selective S-FEM-T4	✓	✓	✗	✗	✗	✓
bES-FEM-T4 hES-FEM-T4	✓	✓	✓	✗	✗	✗
F-bar ES-FEM-T4	✓	✓	✓	✓*	✓*	✓

*) when the num. of cyclic smoothings is sufficiently large.

#2: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

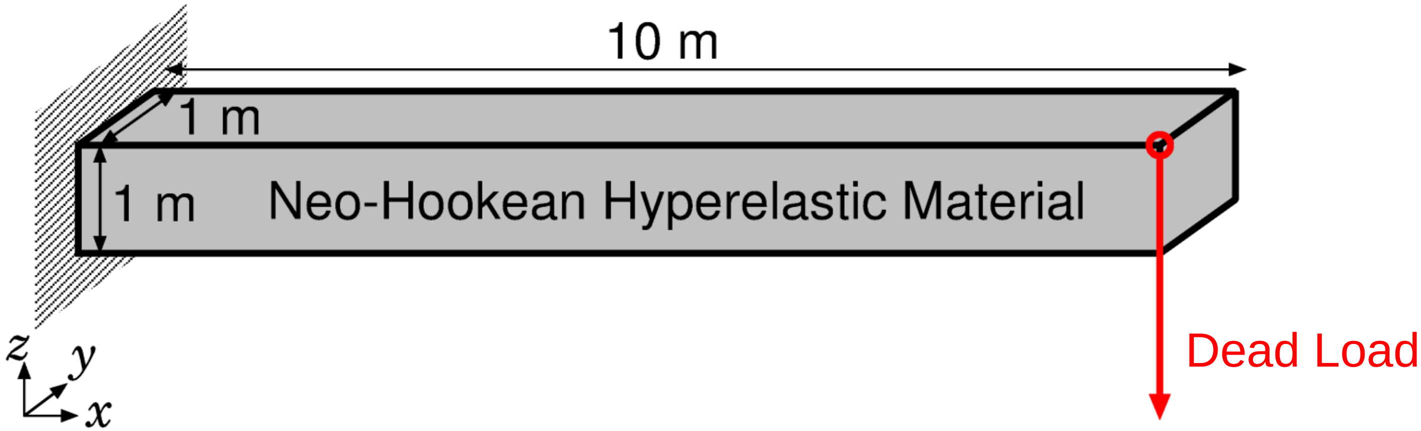
50% nominal compression



Smooth Mises stress distribution is obtained except just around the rim.

#0: Bending of a Cantilever

Outline



- Neo-Hookean hyperelastic material

$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

  with a constant C_{10}(=1 GPa) and various D_1s
  so that the initial Poisson's ratios are 0.49 and 0.499.
- Two types of tetra meshes: structured and unstructured.
- Compared to ABAQUS C3D4H (1st-order hybrid tetrahedral element).

#0: Bending of a Cantilever

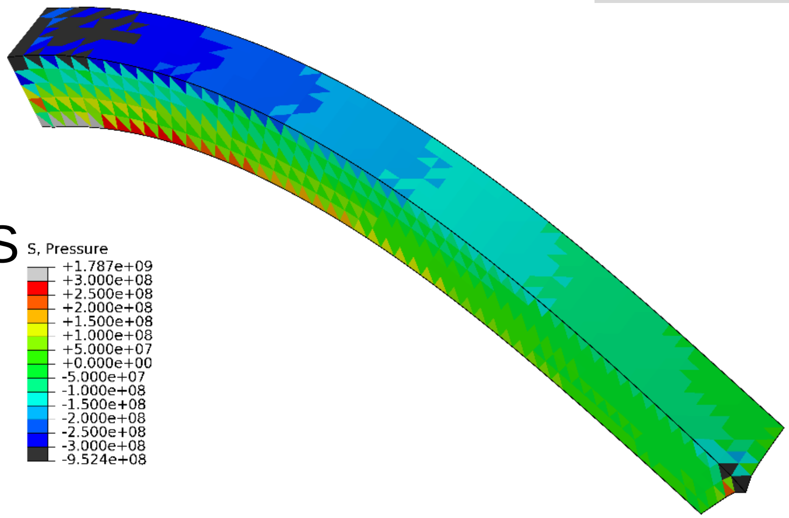
Pressure Distributions

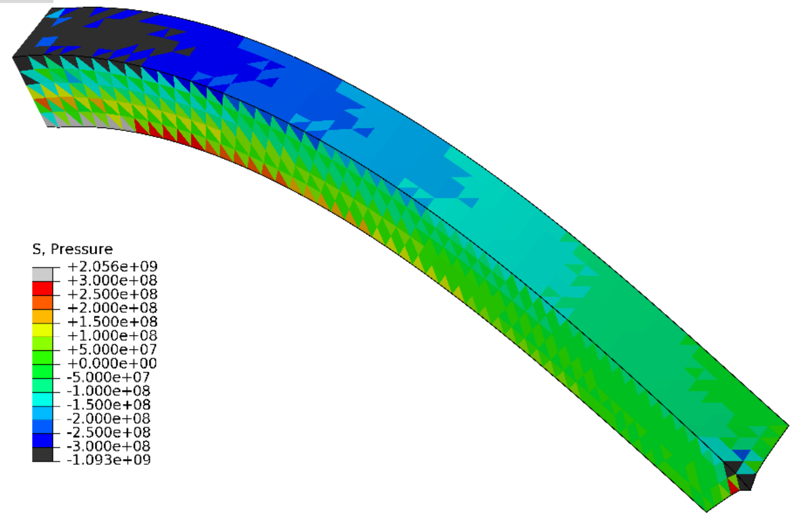
$\nu^{\mathrm{ini}} = 0.49$

Structured Mesh

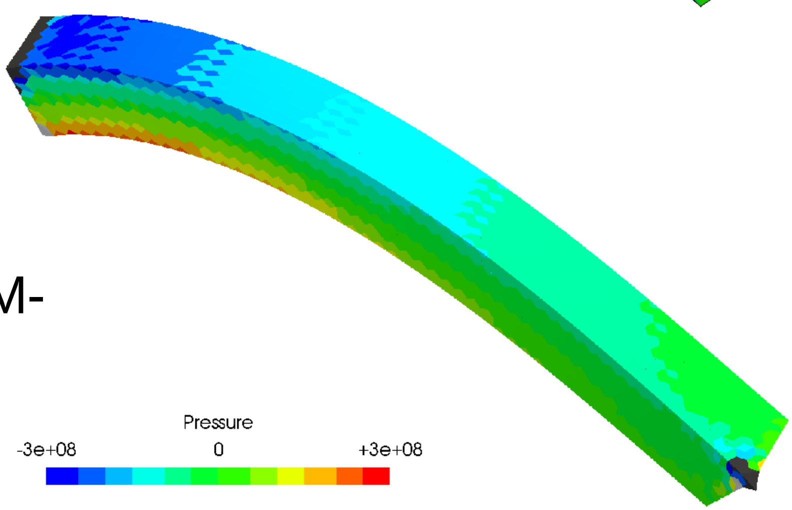
$\nu^{\mathrm{ini}} = 0.499$

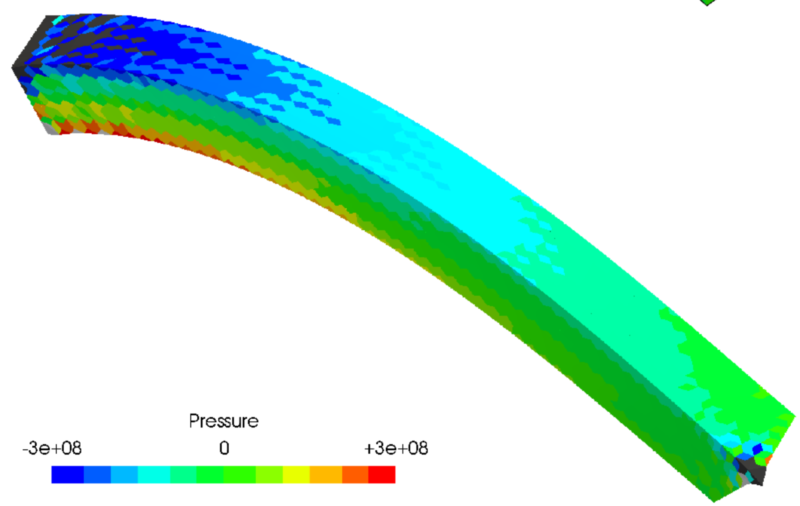
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





#0: Bending of a Cantilever

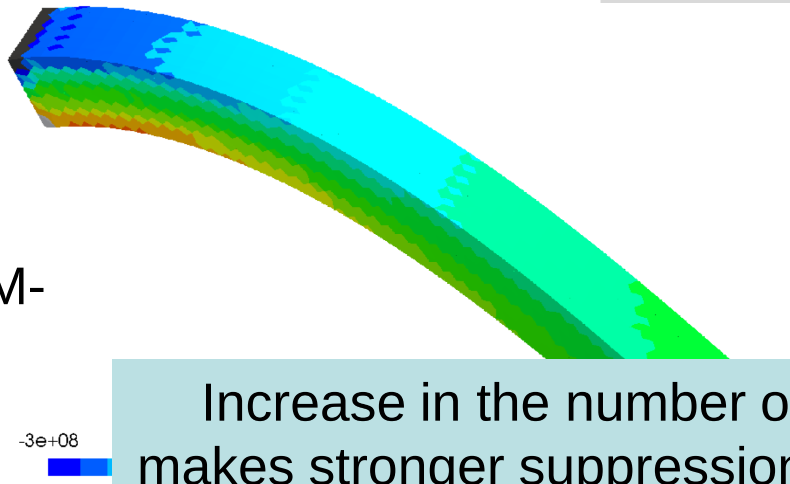
Pressure Distributions

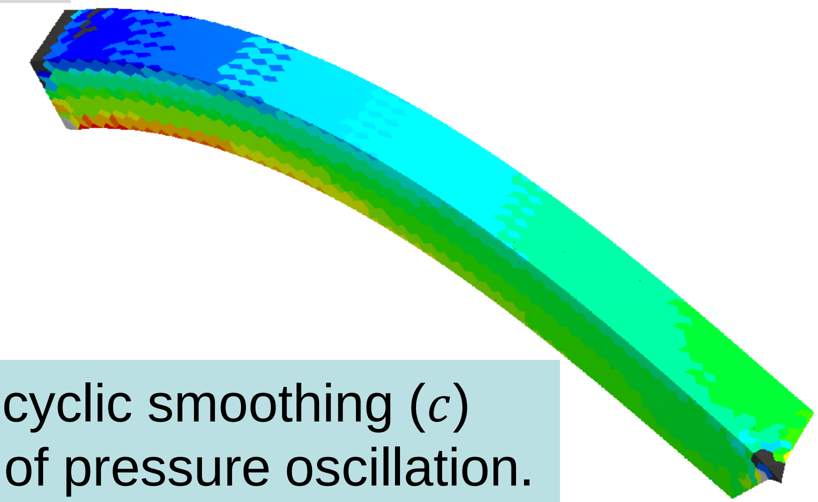
$\nu^{\text{ini}} = 0.49$

Structured Mesh

$\nu^{\text{ini}} = 0.499$

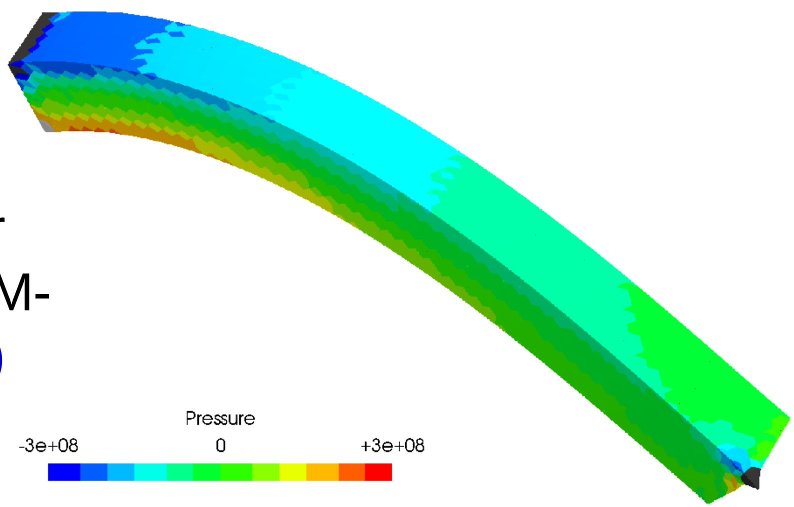
F-bar ES-FEM-T4(2)

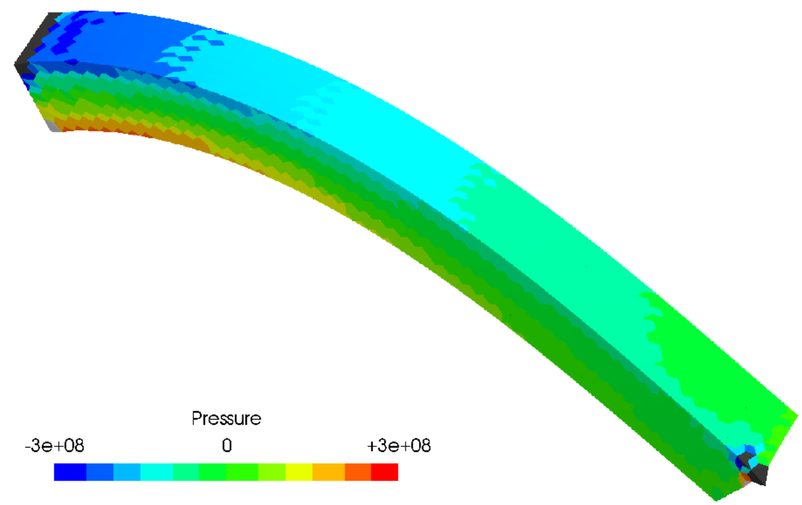




Increase in the number of cyclic smoothing (c) makes stronger suppression of pressure oscillation.

F-bar ES-FEM-T4(3)





#0: Bending of a Cantilever

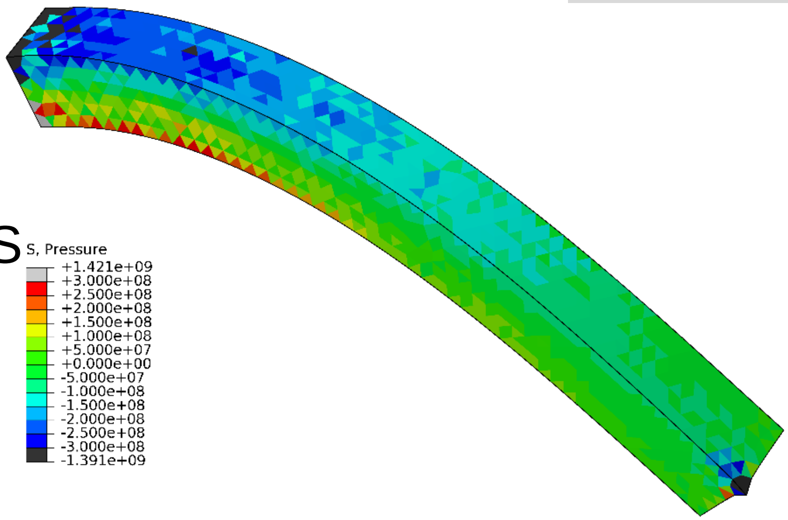
Pressure Distributions

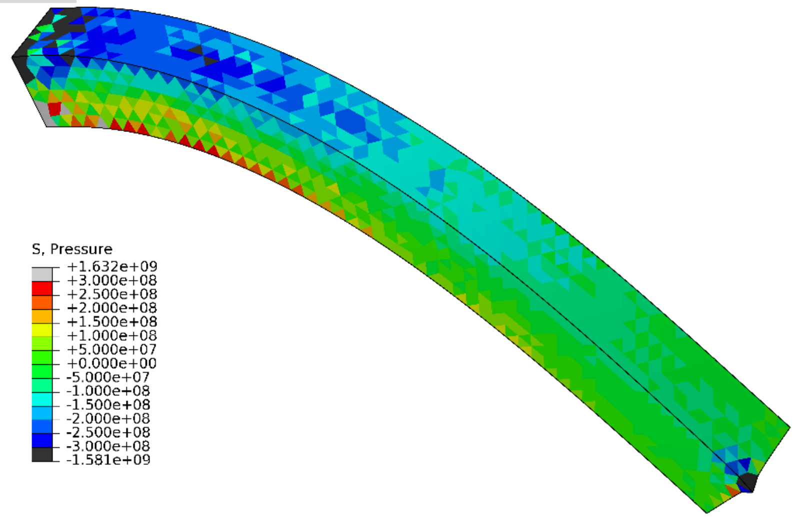
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

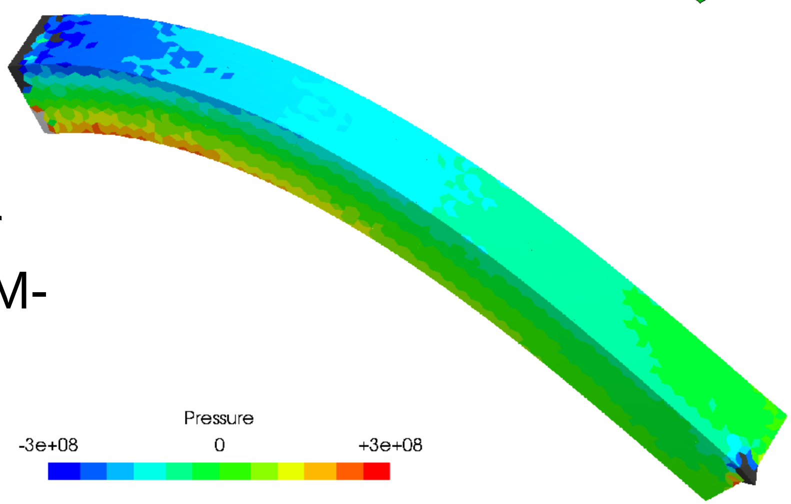
$\nu^{\text{ini}} = 0.499$

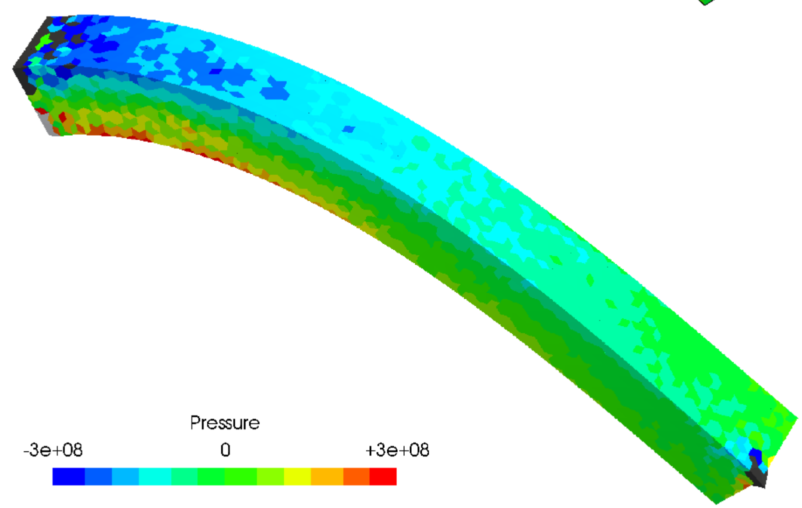
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





#0: Bending of a Cantilever

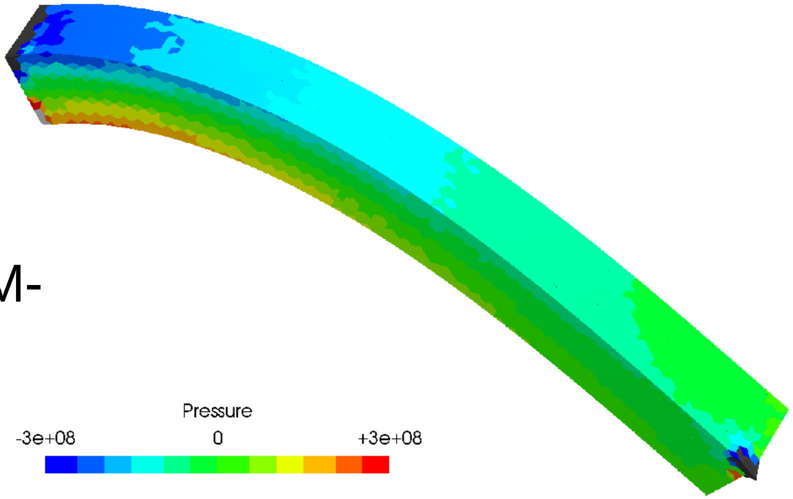
Pressure Distributions

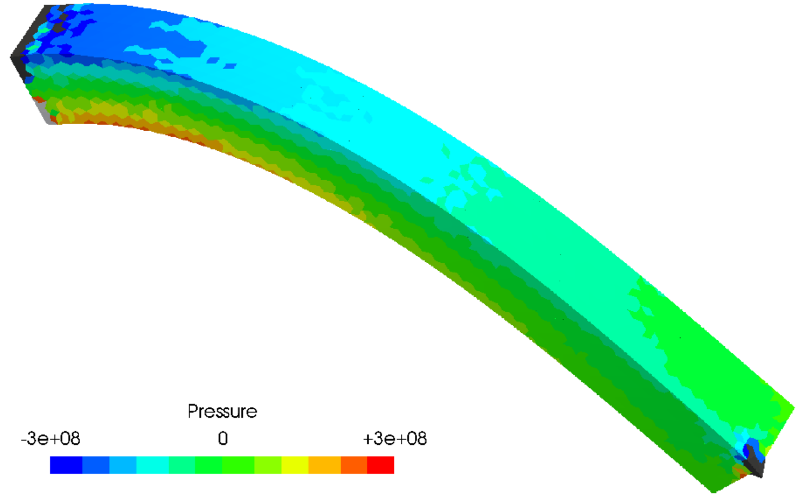
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

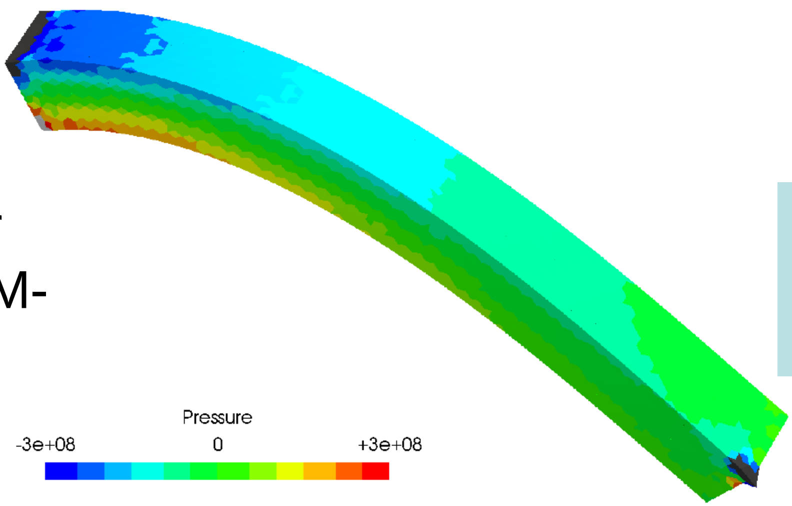
$\nu^{\text{ini}} = 0.499$

F-bar ES-FEM-T4(2)





F-bar ES-FEM-T4(3)



No mesh dependency is observed.