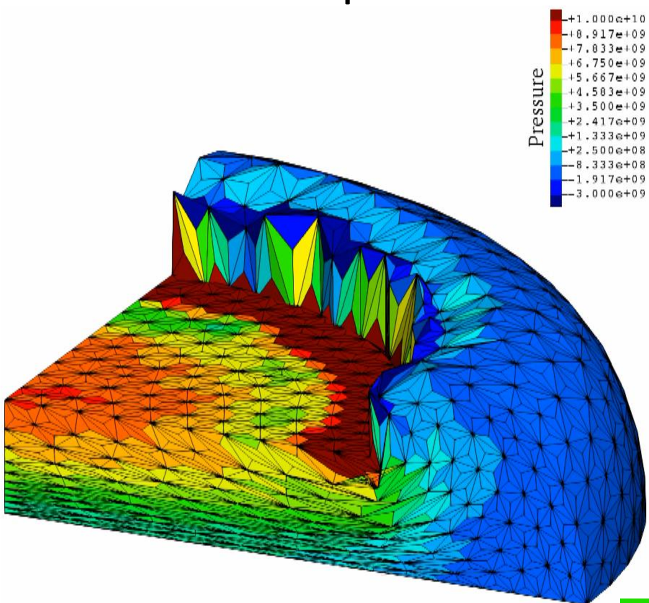
Explicit Dynamic Analysis using SelectiveCS-FEM-T10 with Radial Element Subdivision

Yuki ONISHI
(Tokyo Institute of Technology)

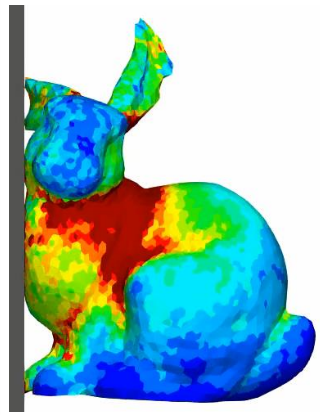
Applications of S-FEMs in Our Lab

- **Large deformation solid mechanics (still in academic)**

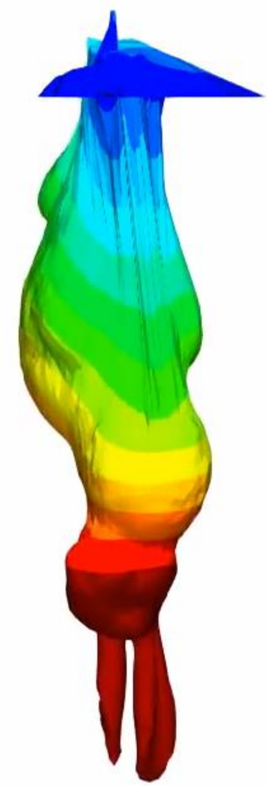
Static Implicit



Dynamic Explicit

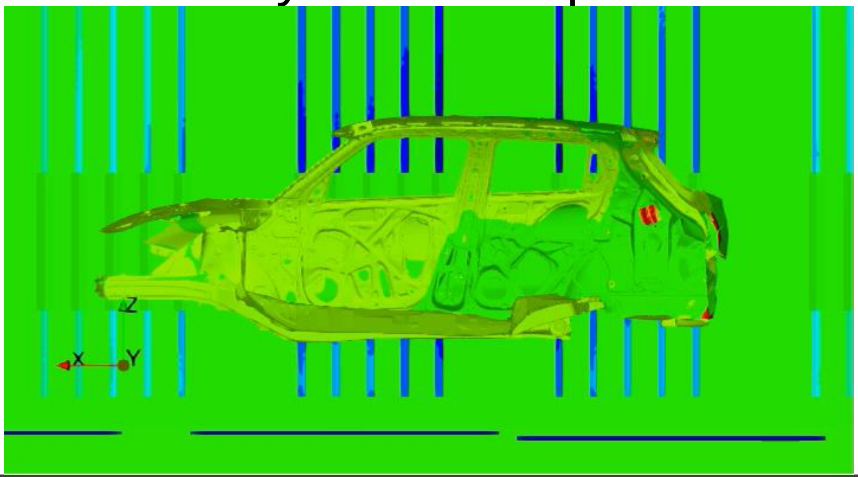


Viscous Implicit



- **Electrostatic (already in practice)**
  - 64 CPUs (512 cores)
  - MPI/OpenMP hybrid
  - 140M elements
  - 2000 timesteps
  - 18.5 h wall time

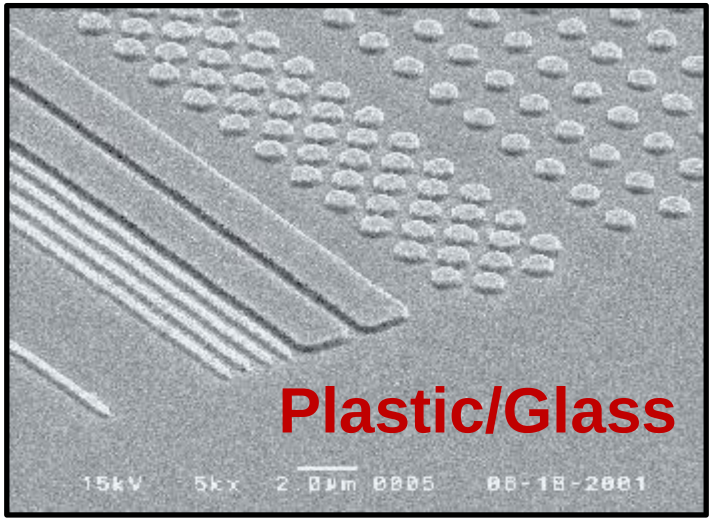
Carbody Electro Deposition



Motivation

What we want to do:

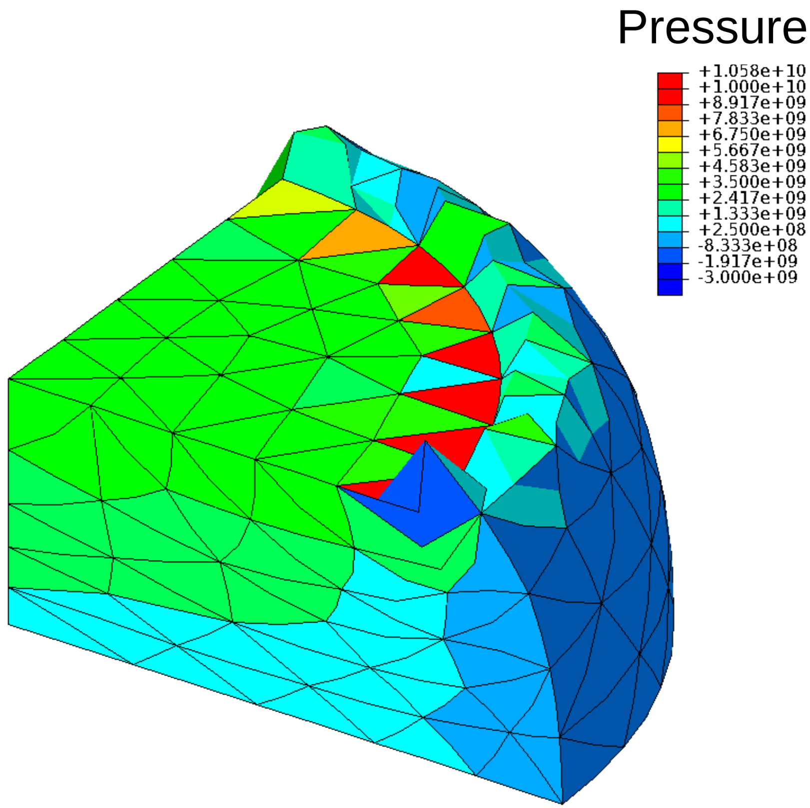
- Solve **severe large deformation** analyses accurately and robustly.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



Issues (e.g., barreling analysis of rubber cylinder)

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

With the best tetrahedral element in ABAQUS



T10:
10-node Tetrahedra

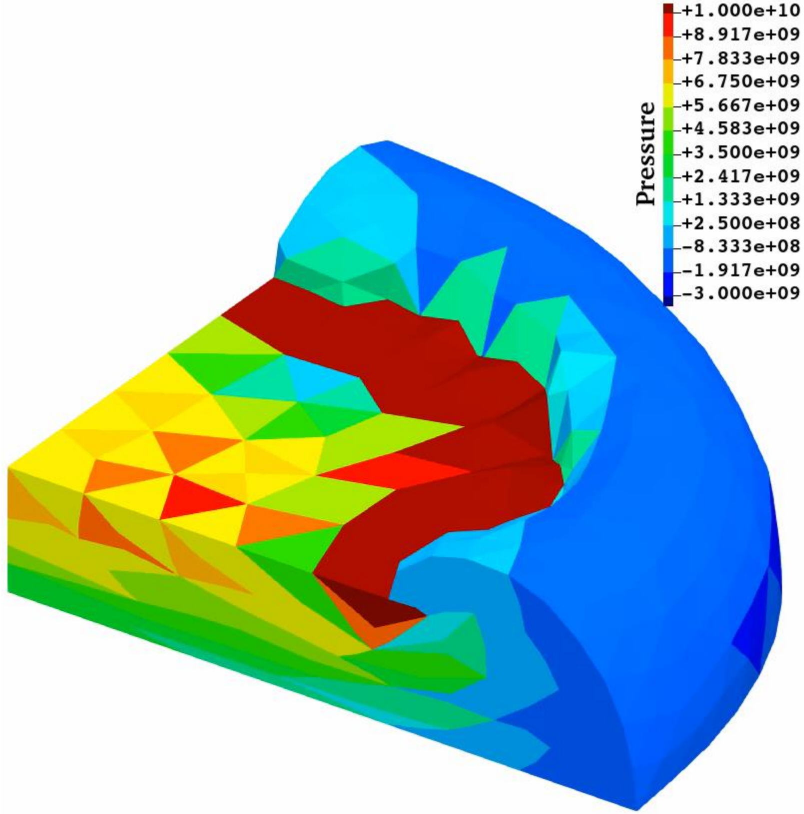
2nd order modified hybrid T10 (ABAQUS C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Short lasting (weak to severe deformation)
- ✗ Low interpolation accuracy

Our Approach (e.g., barreling analysis of rubber cylinder)

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

With the our latest S-FEM tetrahedral element



Same mesh & contour range as C3D10MH case.

Selective CS-FEM-T10 is much better than conventional tetrahedral elements in static analyses.

Y. Onishi, IJCM, (2021).

Further evaluation is necessary in dynamic analyses.

Latest S-FEM T10 (SelectiveCS-FEM-T10)

- ✓ No shear/volumetric locking
- ✓ Less pressure checkerboarding
- ✓ Long lasting (robust to severe deformation)
- ✓ Same CPU time as T10 elements.

Objective

1. Development of a dynamic version of SelectiveCS-FEM-T10
2. Evaluation of its accuracy and robustness in dynamic severe large deformation analyses.

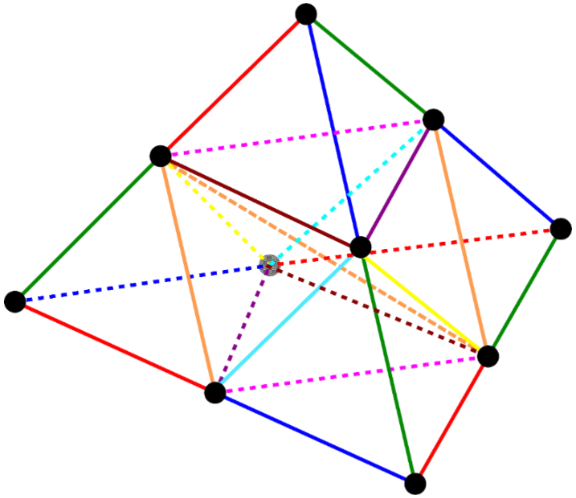
Table of Body Contents

- Methods: Formulation of SelectiveCS-FEM-T10
- Results: Demonstrations of SelectiveCS-FEM-T10
- Summary

Methods:
Formulation of
SelectiveCS-FEM-T10

Concepts of SelectiveCS-FEM-T10

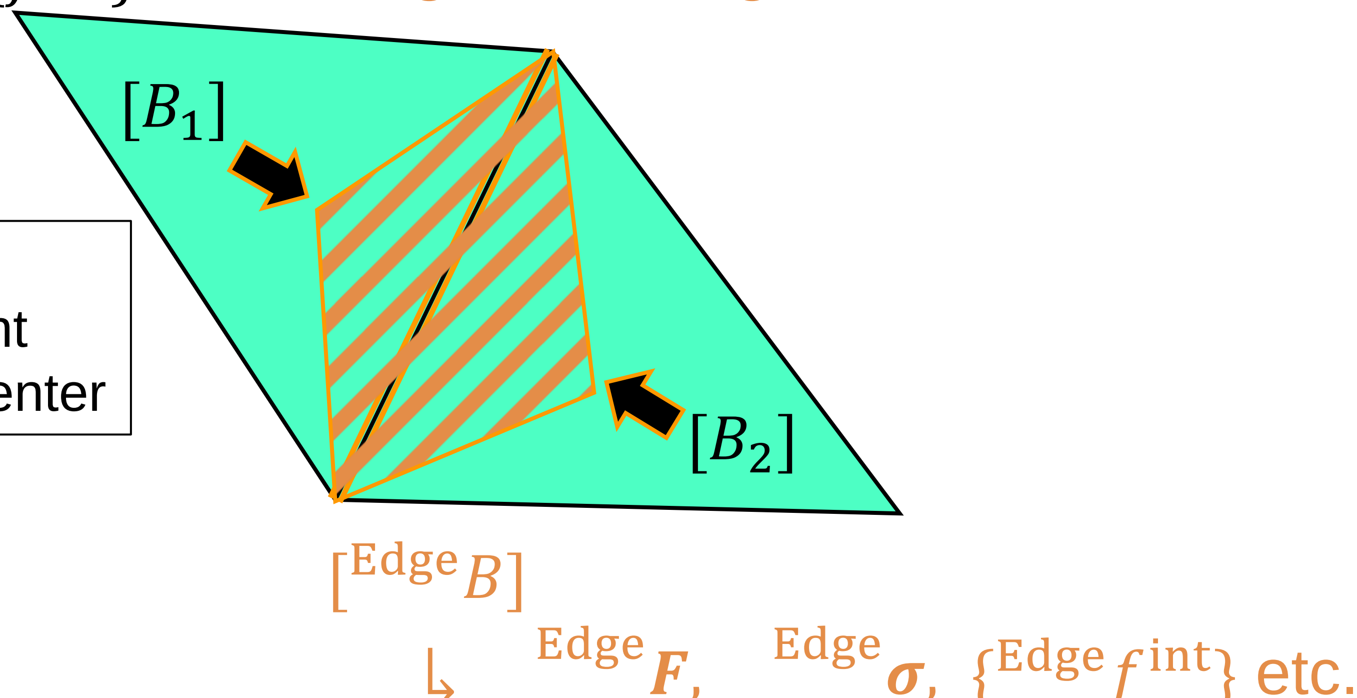
- Using T10 element and subdivide it into T4 sub-elements.
  ⟹ Overcomes the drawbacks of intermediate nodes.
- Adopting intra-element ES-FEM (a kind of CS-FEM) having no strain smoothing across multiple elements.
  ⟹ Becomes an independent element of existing FE codes.
- Applying selective reduced integration (SRI).
  ⟹ Overcomes volumetric locking.



Brief Formulation of ES-FEM

Let us consider two 3-node triangular elements in 2D for simplicity.

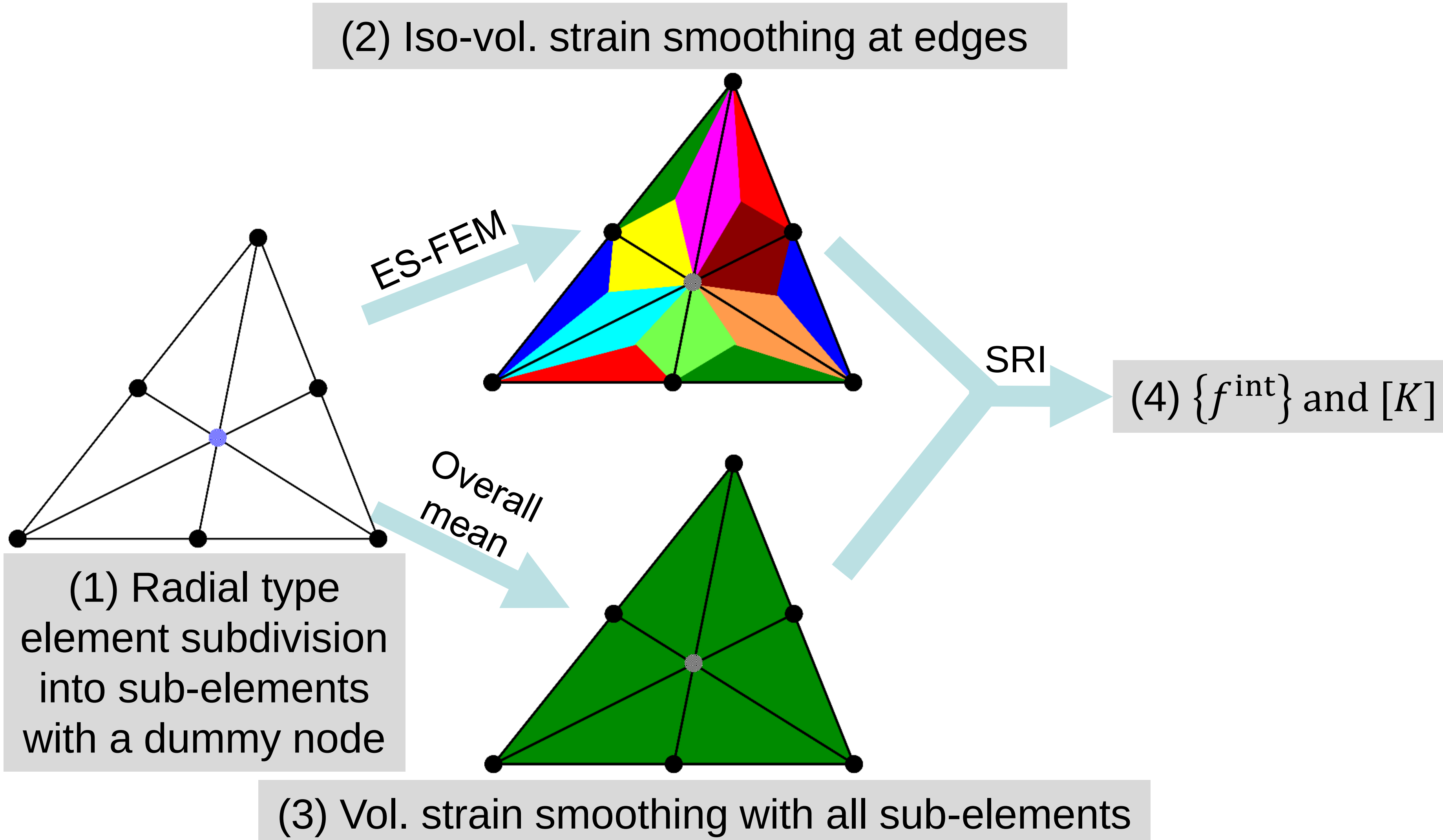
- Calculate $[B](= \mathrm{d}\boldsymbol{N}/\mathrm{d}\boldsymbol{x})$ at each element as usual.
- Distribute each $[B]$ to the connecting edge with an area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate deformation gradient $(\boldsymbol{F})$, Cauchy stress $(\boldsymbol{\sigma})$ and nodal internal force $\{f^{\mathrm{int}}\}$ in each edge smoothing domain.



As if putting
a Gauss point
on each edge center

Flowchart of SelectiveCS-FEM-T10

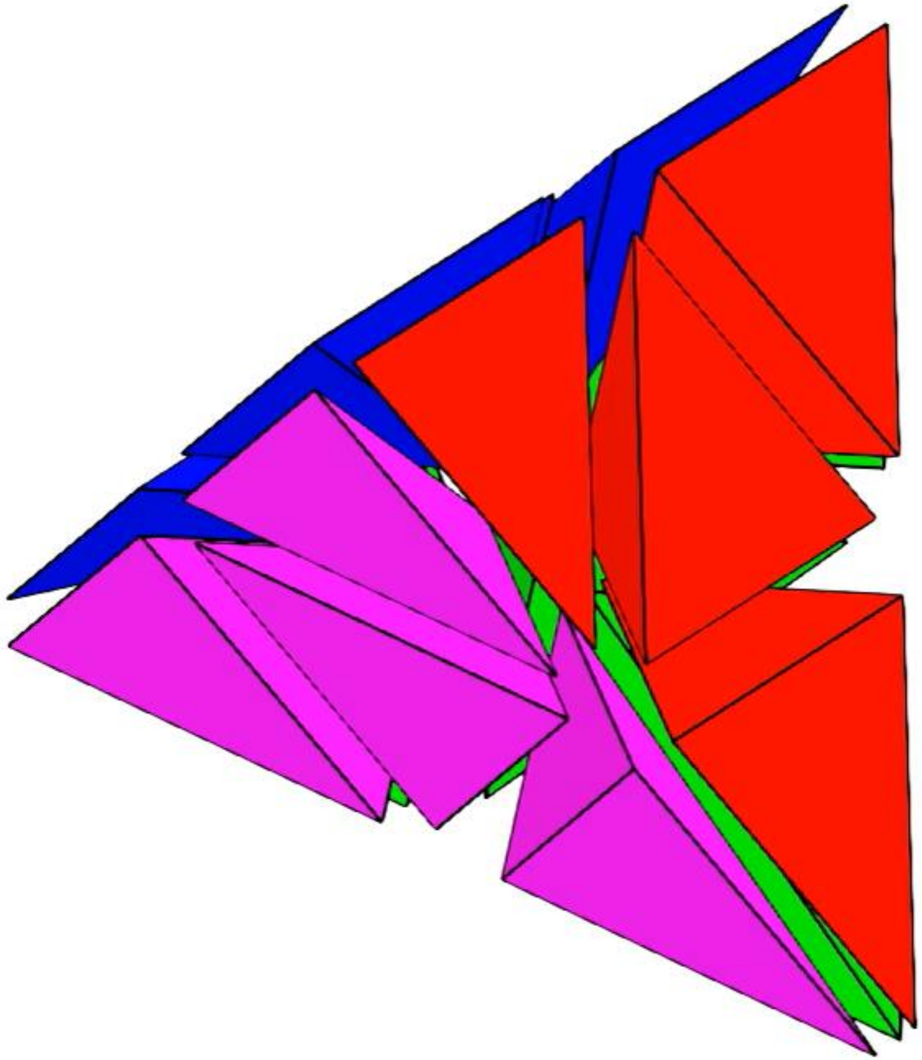
Explanation in 2D (6-node triangular element) for simplicity



T10 Element Subdivision in 3D

Radial subdivision (30% shrunk mesh)

There are 16 T4 sub-elements in total.



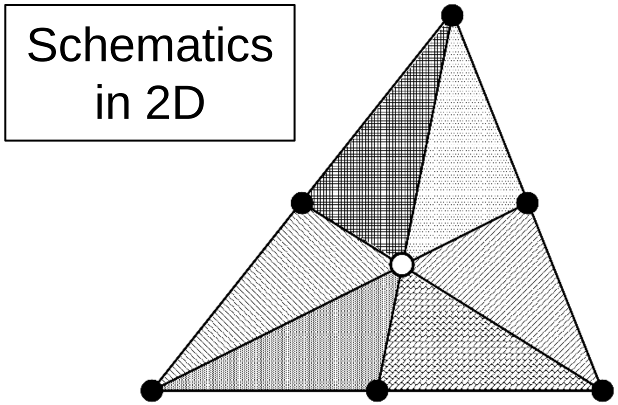
Strain on all 34 edges are smoothed by ES-FEM.

Sub-elements have a little larger skewness.

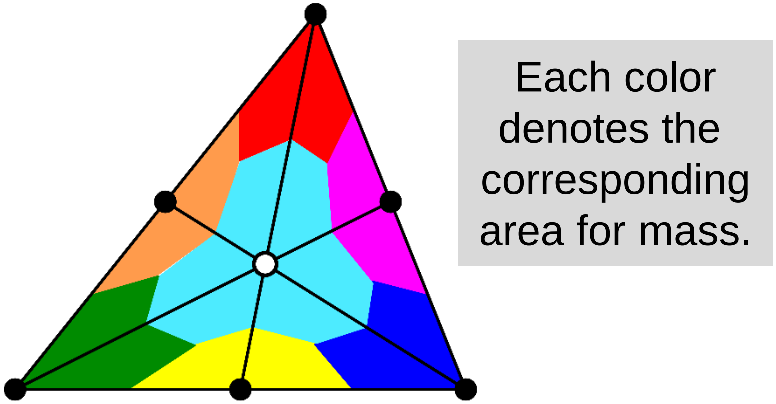
but skewness is not a big issue for ES-FEM.

Building Lumped Mass Matrix

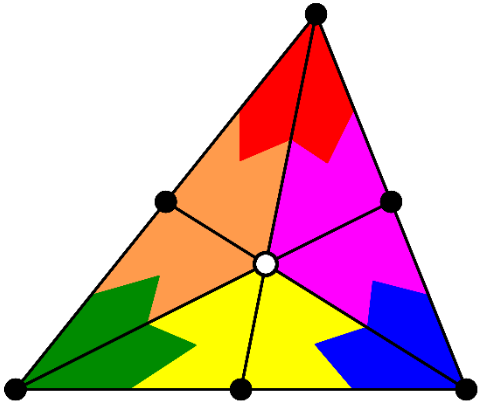
1. Calculate the mass of each sub-element.



2. Distribute it to composing 4 nodes.
   (3 nodes in 2D.)



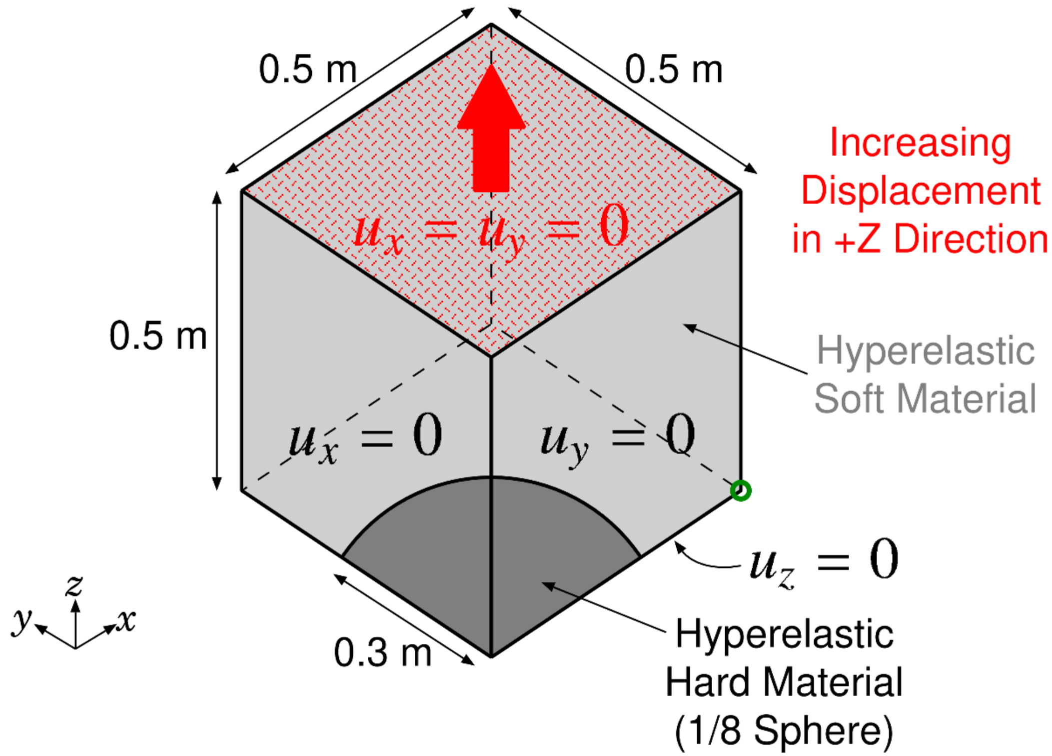
3. The mass of the dummy node is distributed to the connecting 6 mid-nodes.
   (3 mid-nodes in 2D.)



Results:
Demonstration of
SelectiveCS-FEM-T10

Tensioning of Rubber-Filler Composite

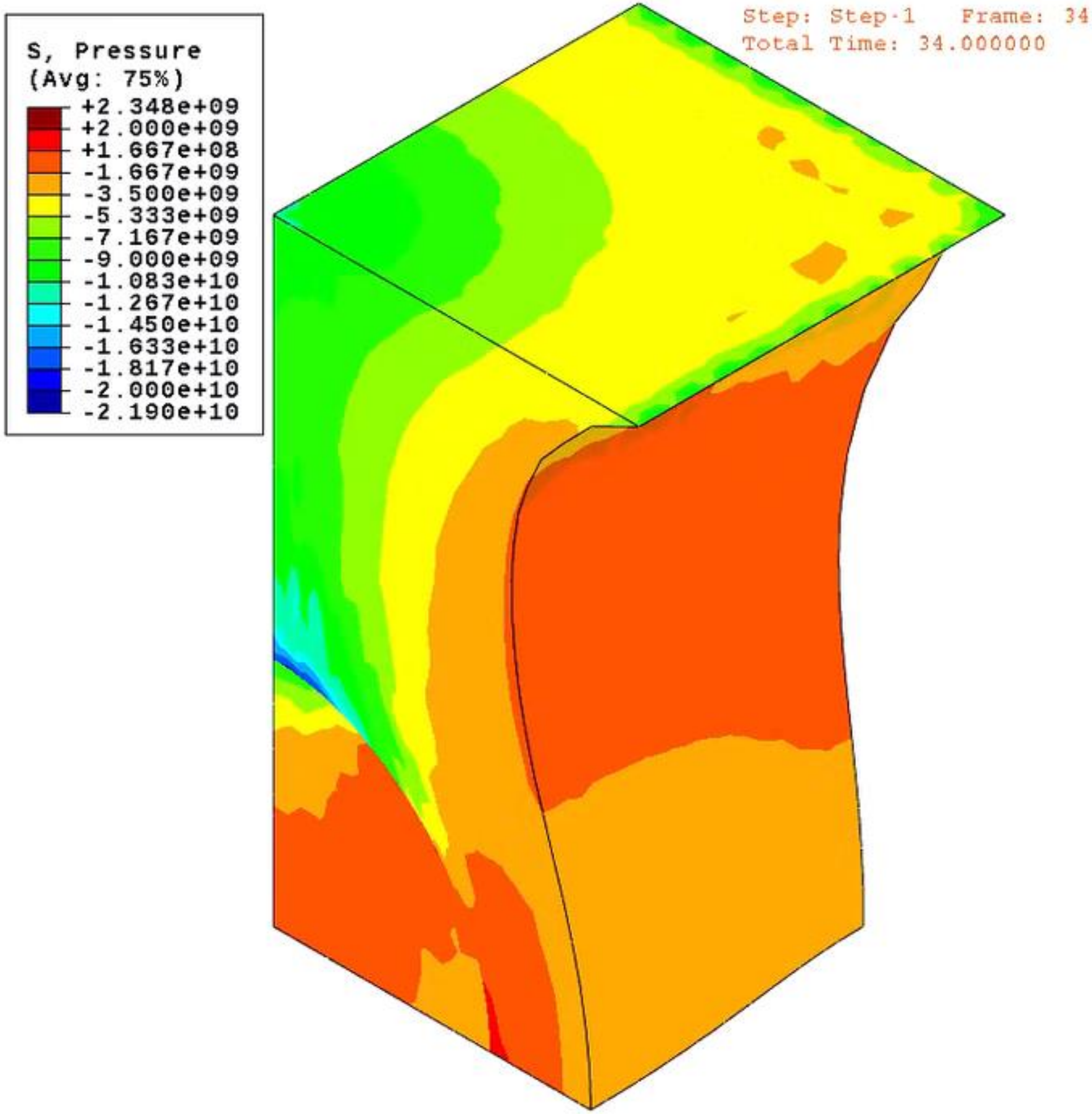
Outline



- Soft material: Neo-hookean, $E_{\mathrm{ini}} = 6\ \mathrm{GPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.49}$.
- Hard material: Neo-hookean, $E_{\mathrm{ini}} = 260\ \mathrm{GPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.3}$.
- Discretized into T10 mesh. (about 11,000 nodes and 7,000 elements)
- Compared to ABAQUS C3D10MH, the best T10 element of ABAQUS, with the same mesh.

Tensioning of Rubber-Filler Composite

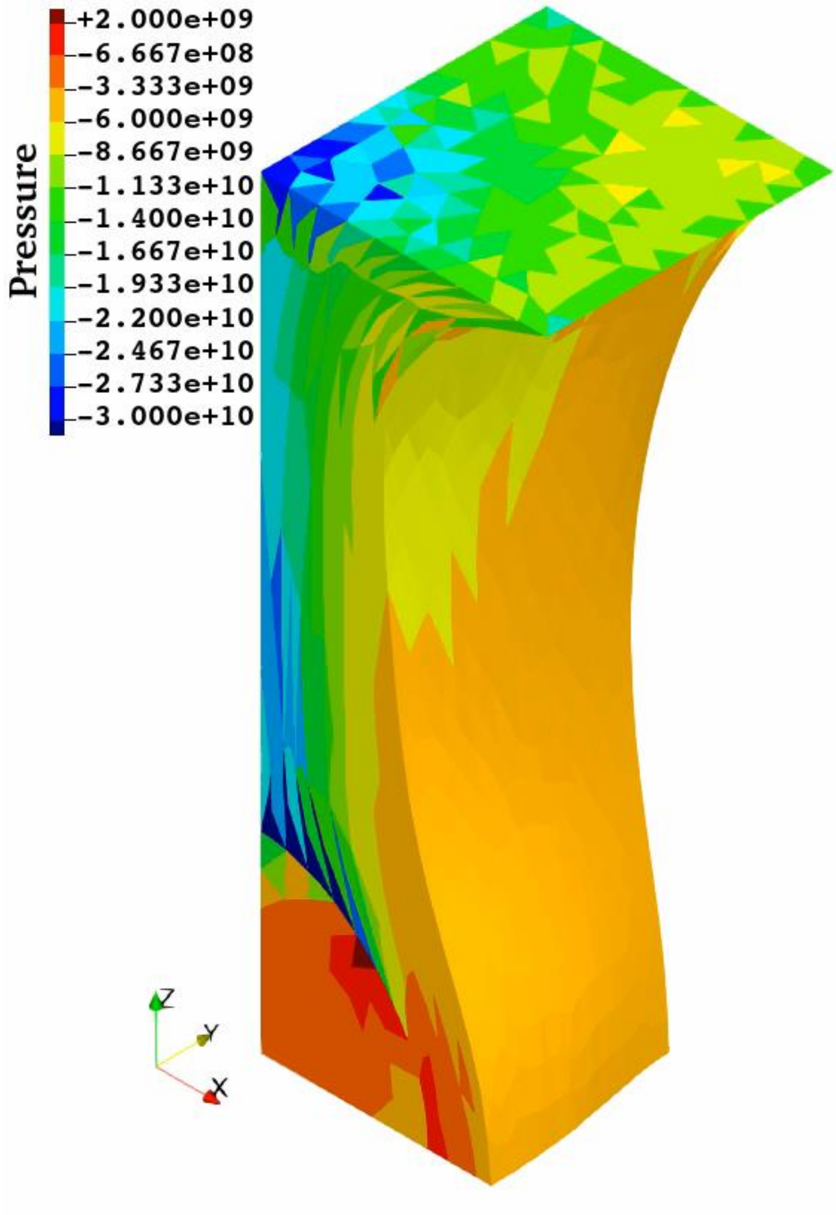
Result of ABAQUS C3D10MH with pressure contour



Convergence failure at 69% nominal stretch (short lasting)

Tensioning of Rubber-Filler Composite

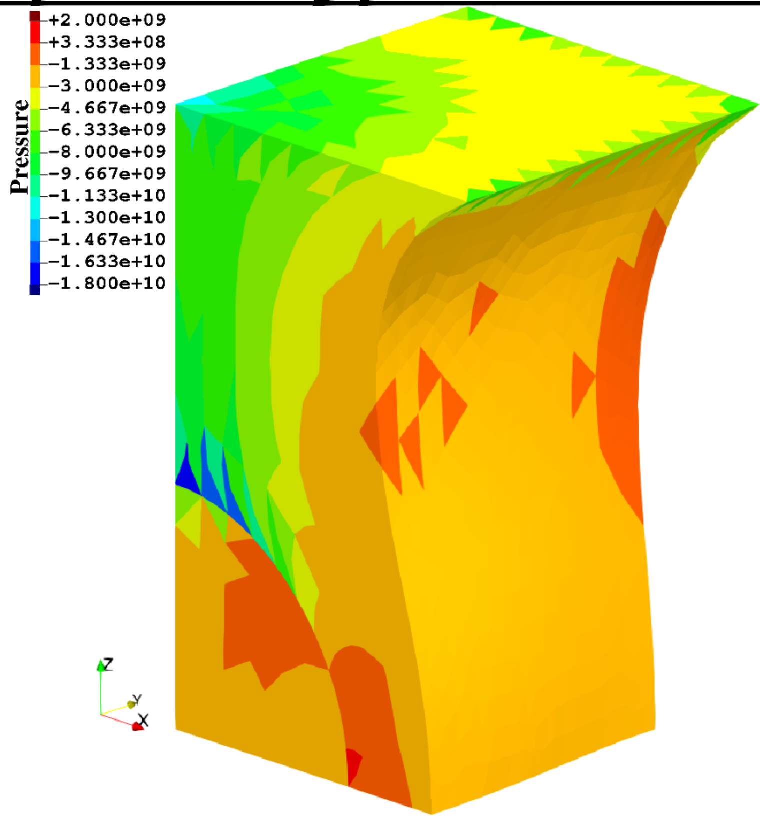
Result of
Selective CS-FEM-T10
with pressure contour



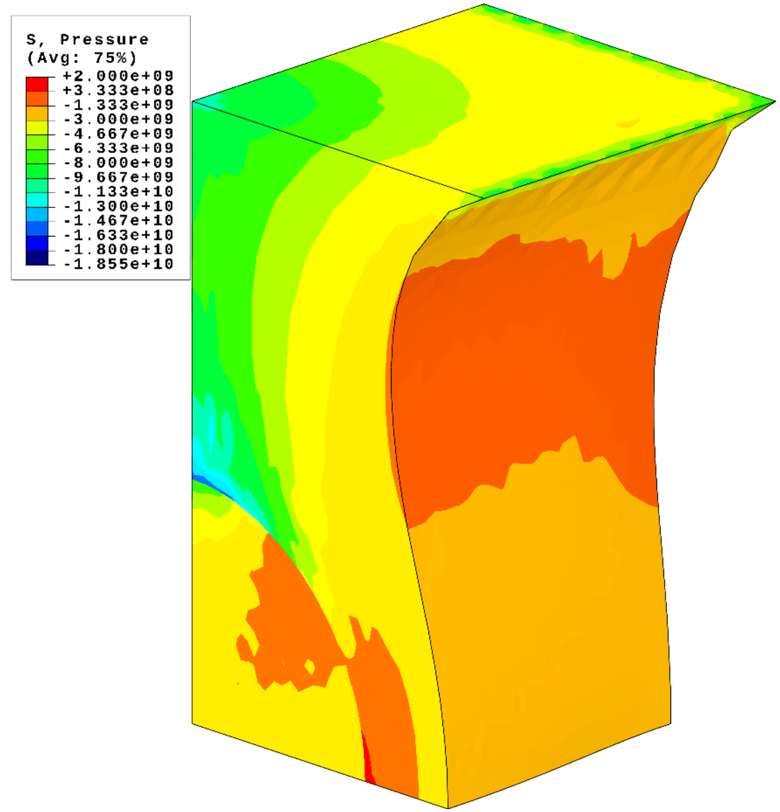
Convergence failure at 166% nominal stretch (long lasting)

Tensioning of Rubber-Filler Composite

Comparison of pressure dist. at 60% nominal stretch



SelectiveCS-FEM-T10

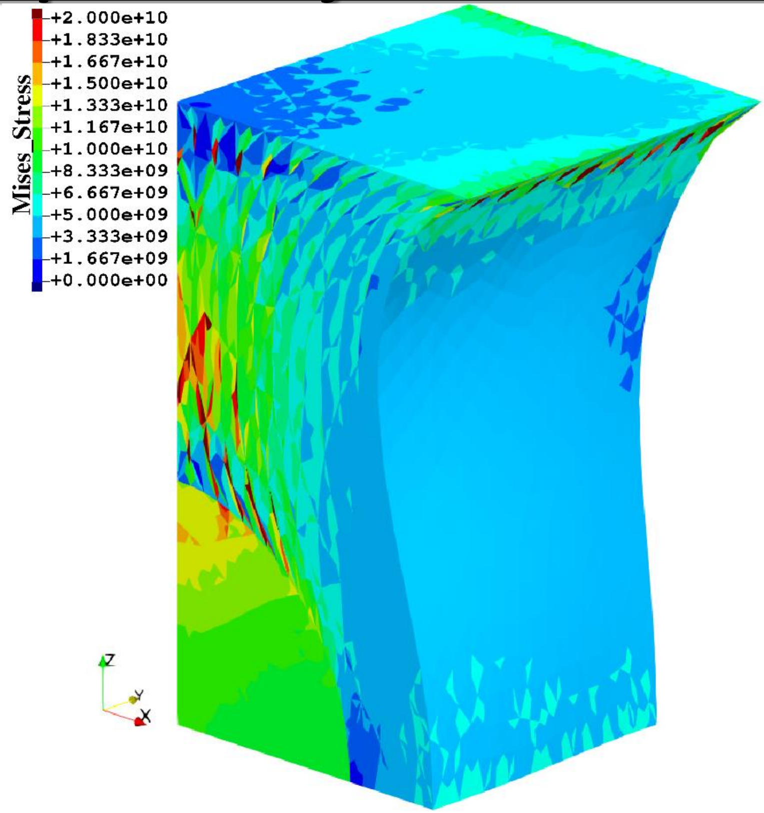


ABAQUS C3D10MH

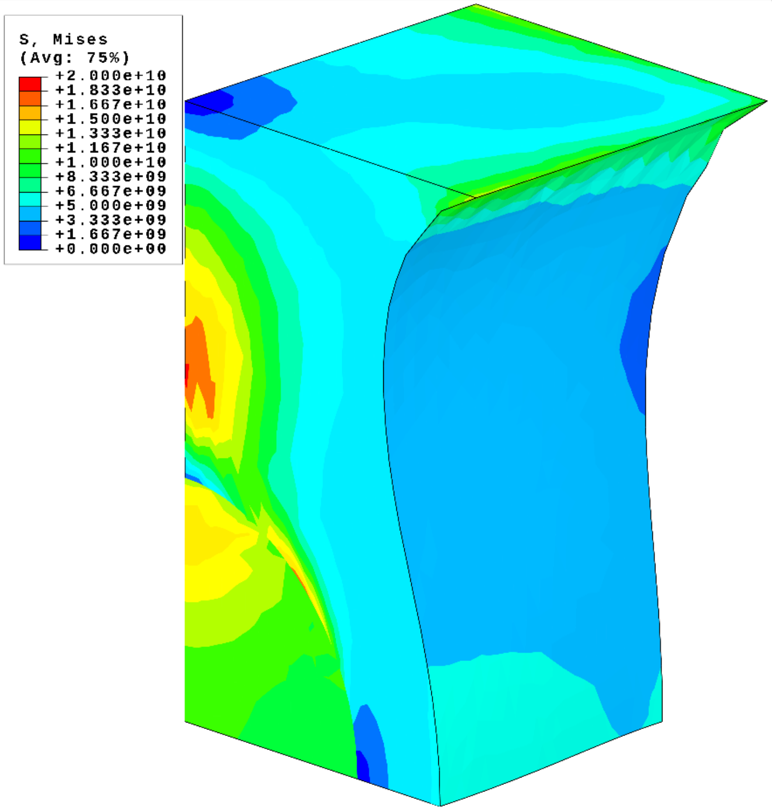
SelectiveCS-FEM-T10 has good pressure accuracy.

Tensioning of Rubber-Filler Composite

Comparison of Mises stress dist. at 60% nominal stretch



SelectiveCS-FEM-T10

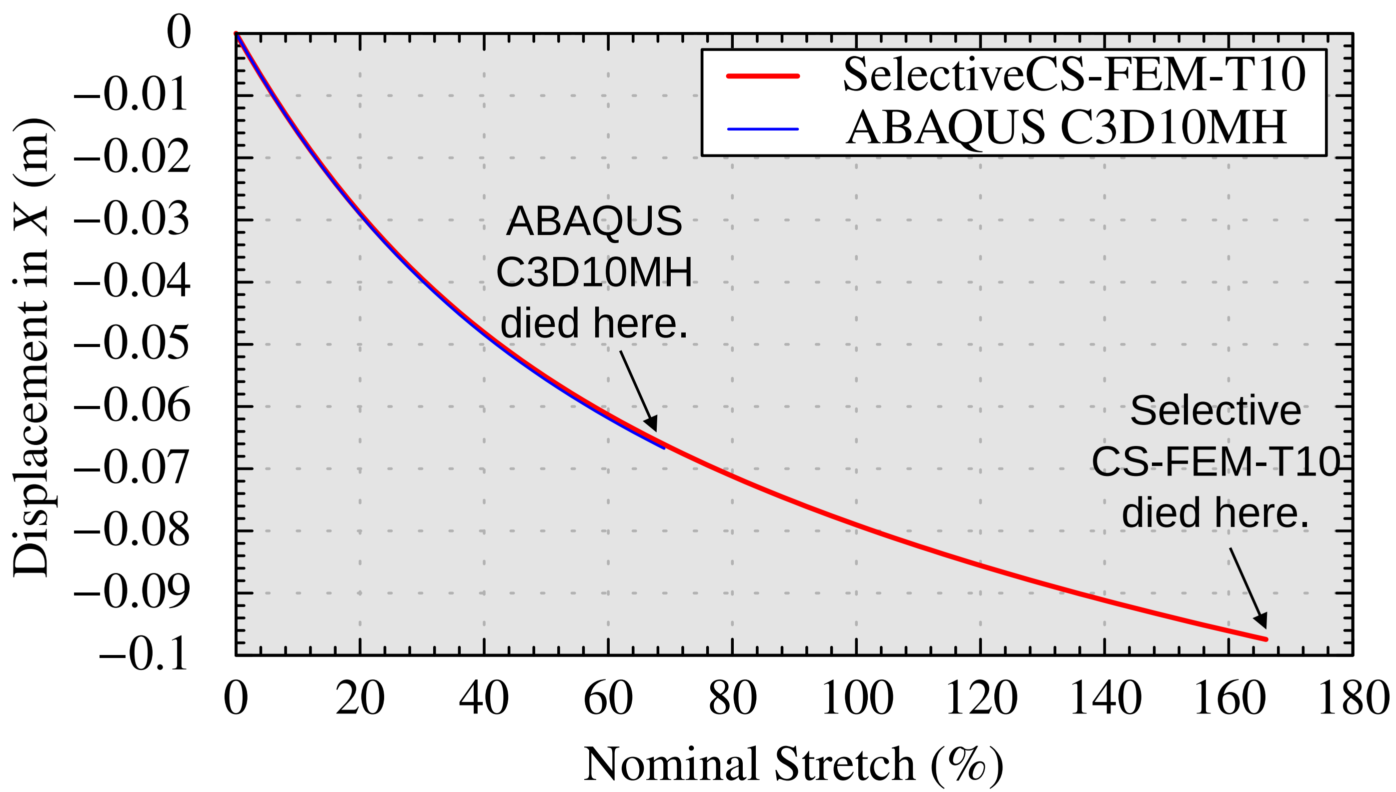


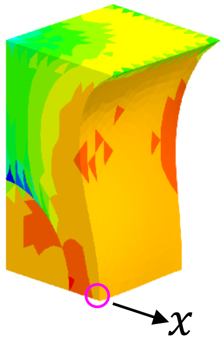
ABAQUS C3D10MH

SelectiveCS-FEM-T10 has an issue of Mises stress oscillation, which should be resolved in the future.

Tensioning of Rubber-Filler Composite

Comparison of history of u_x at the bottom corner





SelectiveCS-FEM-T10 has enough accuracy in displacement (and force, also) in addition to large deformation robustness.

Deformation Modes of Armadillo

Outline

- Rubber body.
  (Young's modulus: 5MPa,
  Poisson's ratio: 0.49)
- Discretized in T10 mesh.
  (about 80,000 nodes
  and 52,000 elements)
- Both soles of the feet are perfectly constrained.
- Modal analysis up to 40 eigen modes.
  (This is not a large deformation analysis.)
- Compared to ABAQUS C3D10MH with the same mesh.

Eigen Mode

Deformation Modes of Armadillo

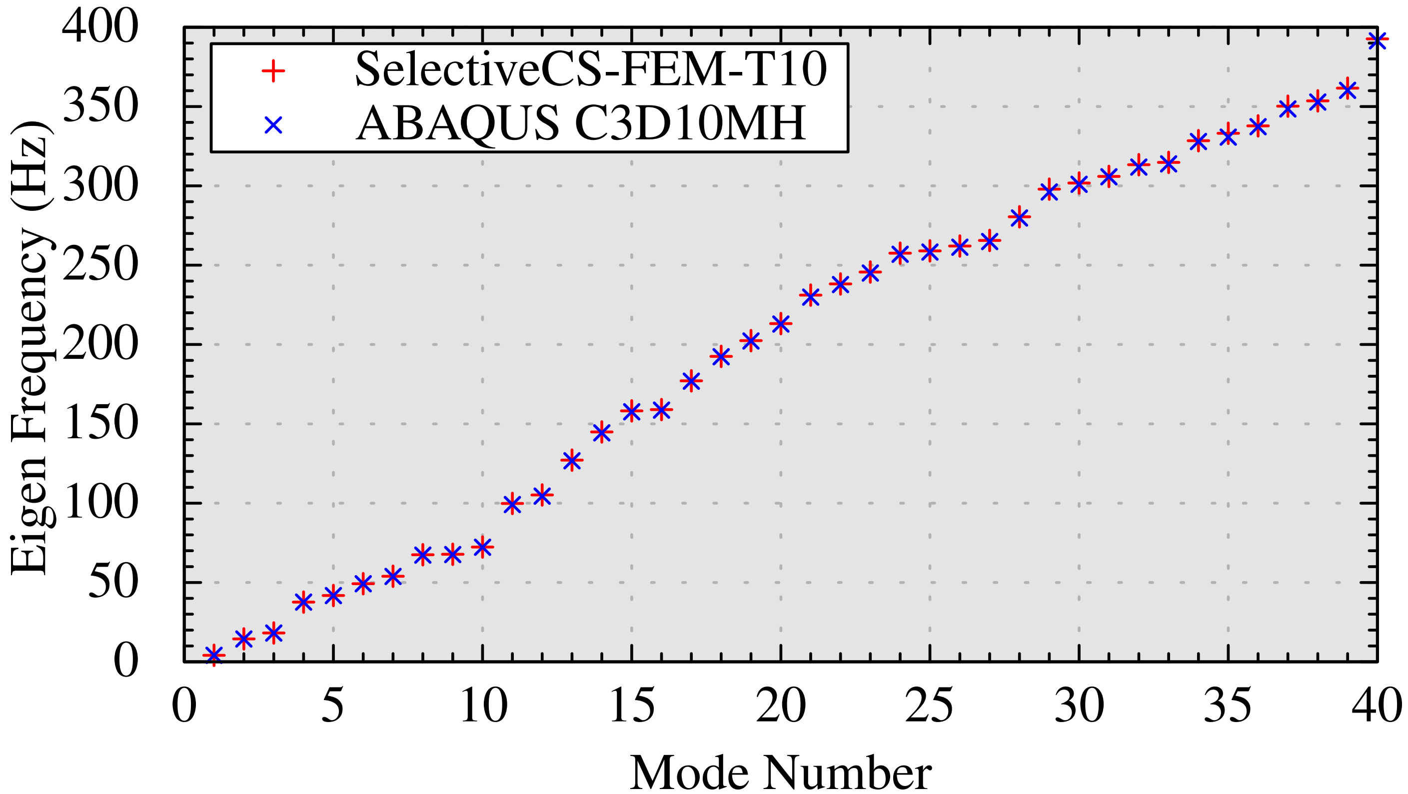
Eigen modes up to Mode 40 with SelectiveCS-FEM-T10

There are no unnatural modes.

SelectiveCS-FEM-T10 has no spurious low-energy modes like hour-glass modes.

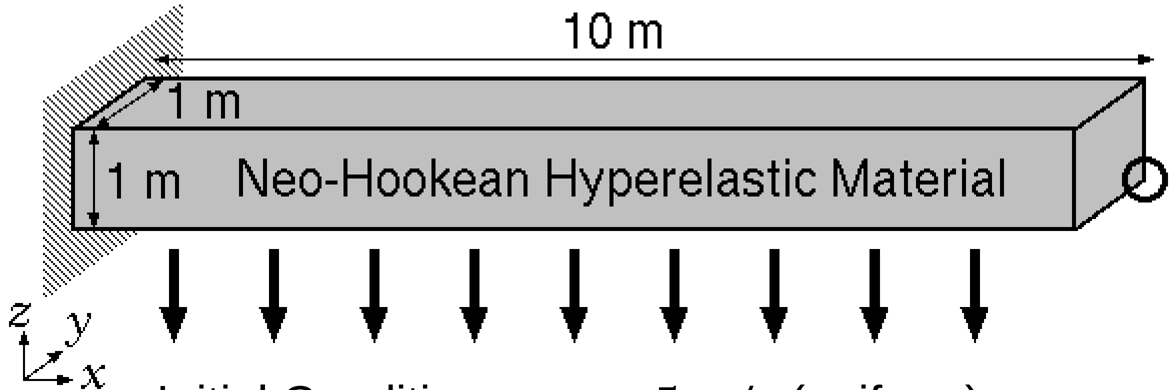
Deformation Modes of Armadillo

Comparison of eigen frequencies



SelectiveCS-FEM-T10 has practical accuracy in modal analyses as ABAQUS C3D10MH; therefore, SelectiveCS-FEM-T10 would be stable in dynamic analyses.

Dynamic Bending of Cantilever

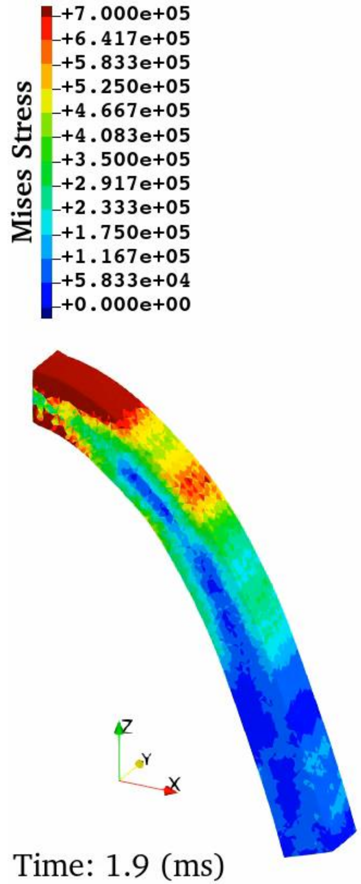


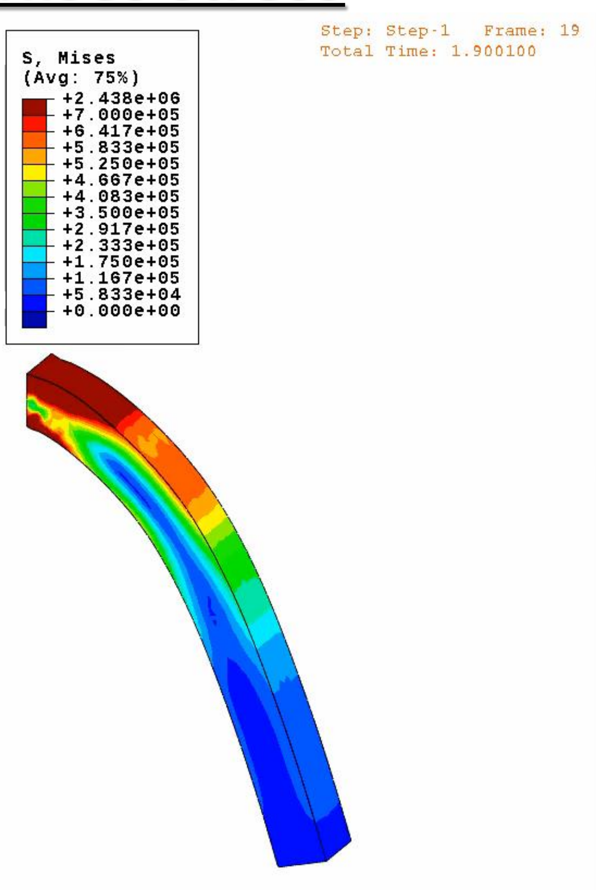
Initial Condition: $v_z = -5$ m/s (uniform)

- Neo-Hookean, $E_{\mathrm{ini}} = 6.0\ \mathrm{MPa}$, $\nu_{\mathrm{ini}} = 0.49$, $\rho = 920\ \mathrm{kg/m^3}$
- Initial velocity: $v_z = -5\ \mathrm{m/s}$ for all nodes of cantilever
- Discretized into T10 mesh. (about 4,000 nodes and 2,000 elements)
- Compared to ABAQUS/Explicit C3D10M (NOT C3D10MH) with the same mesh and $\Delta t\ (= 0.1\ \mathrm{ms})$.

Dynamic Bending of Cantilever

Comparison of animation of Mises stress



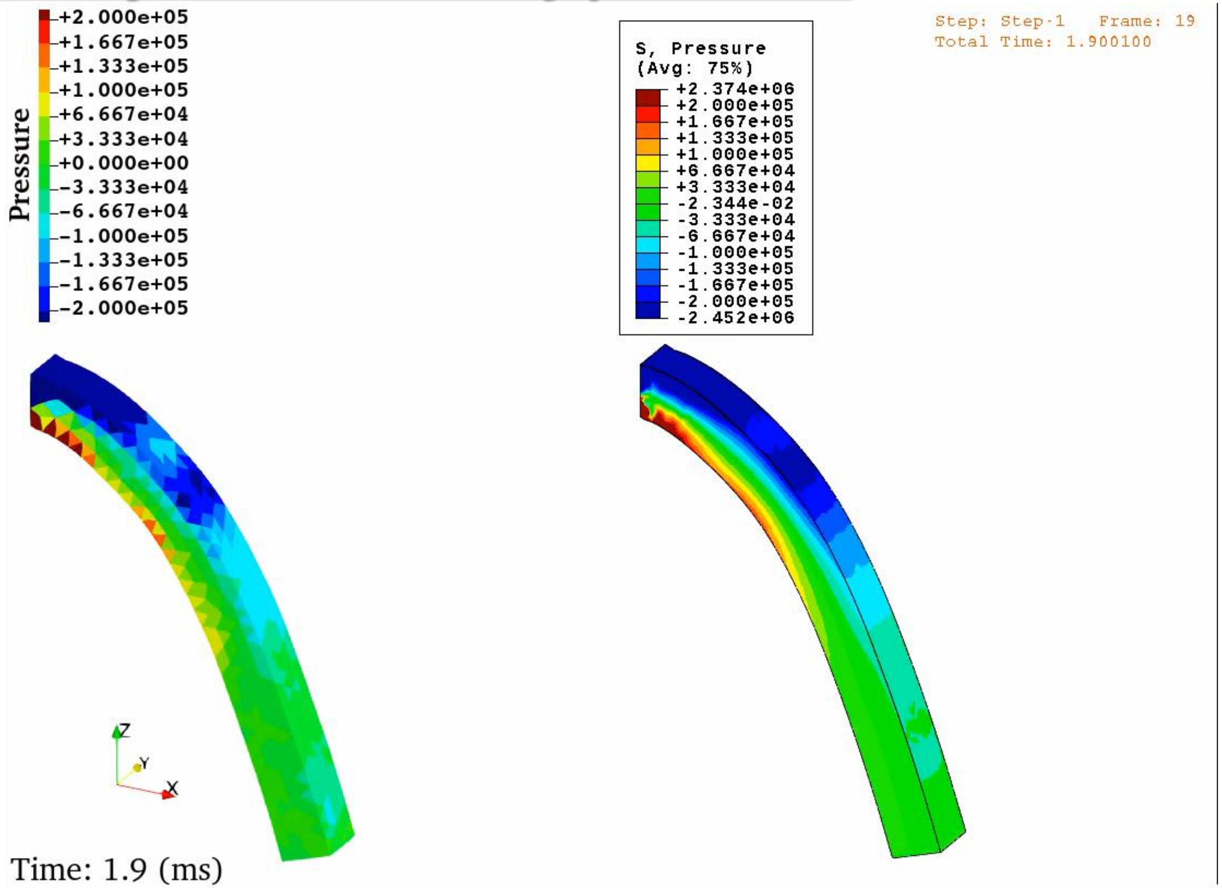


SelectiveCS-FEM-T10/Explicit ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 has similar accuracy
in displacement and Mises stress to ABAQUS C3D10M.

Dynamic Bending of Cantilever

Comparison of animation of pressure

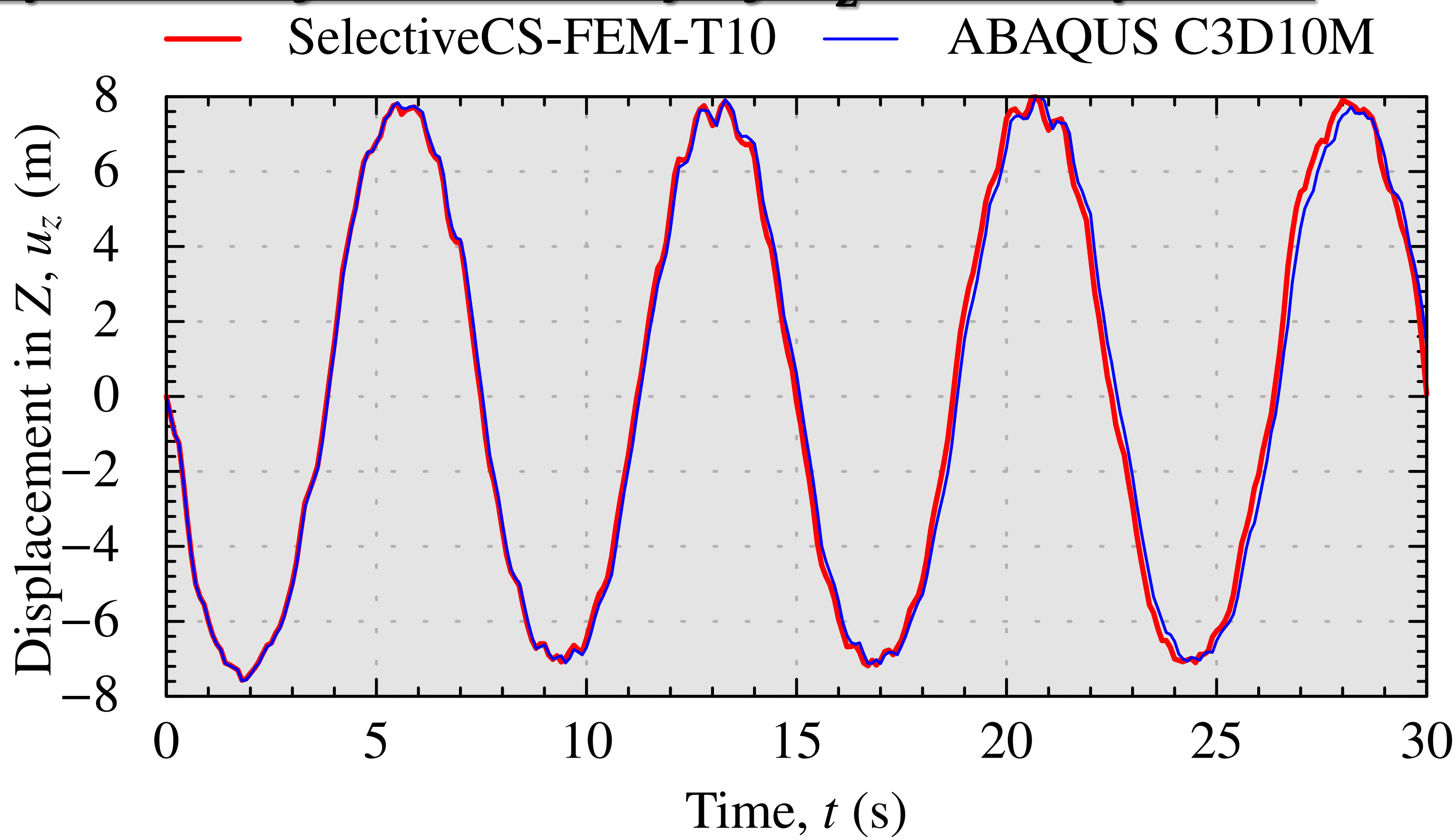


SelectiveCS-FEM-T10/Explicit ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 has similar accuracy
in displacement and pressure to ABAQUS C3D10M.

Dynamic Bending of Cantilever

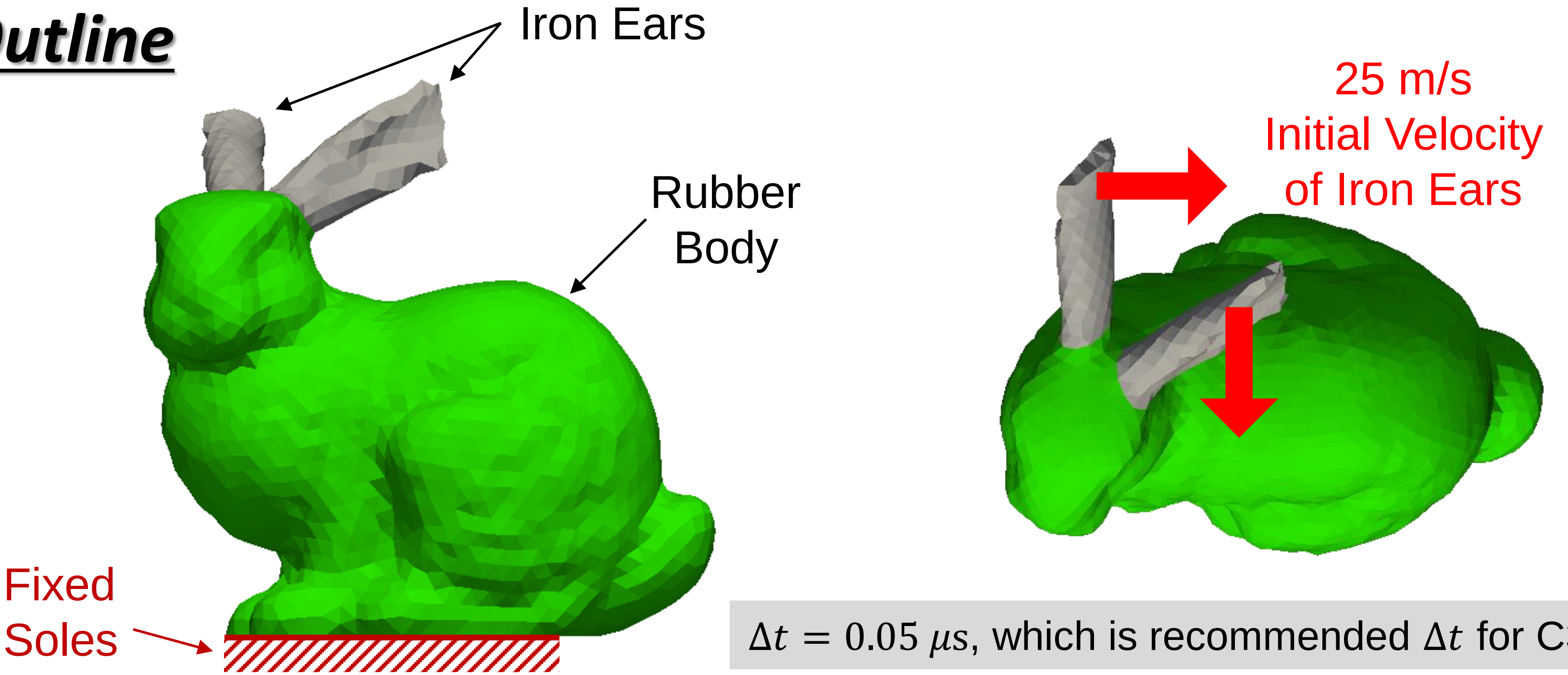
Comparison of time-history of u_z at the tip node



SelectiveCS-FEM-T10 has similar accuracy in displacement to ABAQUS C3D10M.

Swing of Bunny Ears

Outline

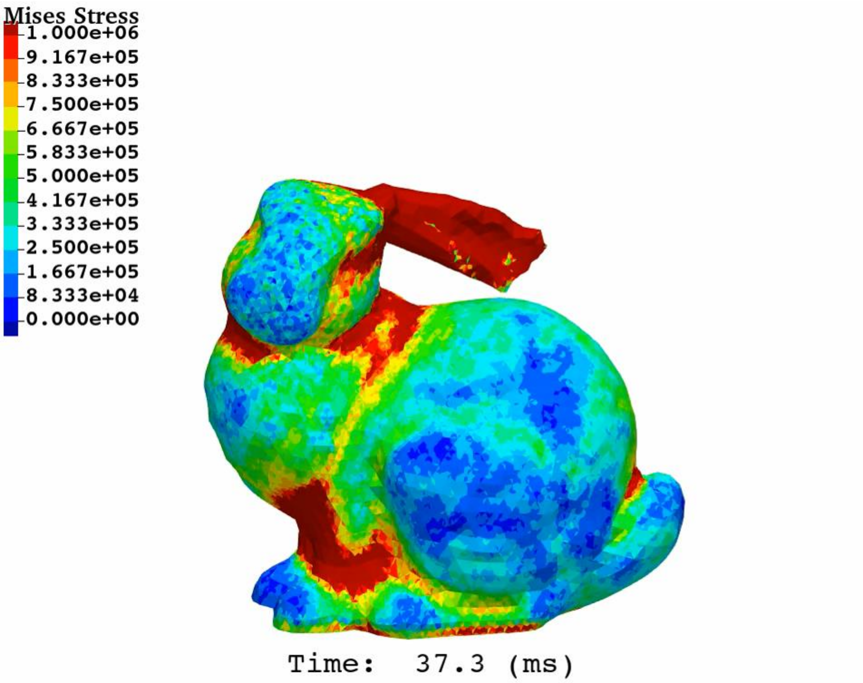


$\Delta t = 0.05\ \mu s$, which is recommended Δt for C3D10M

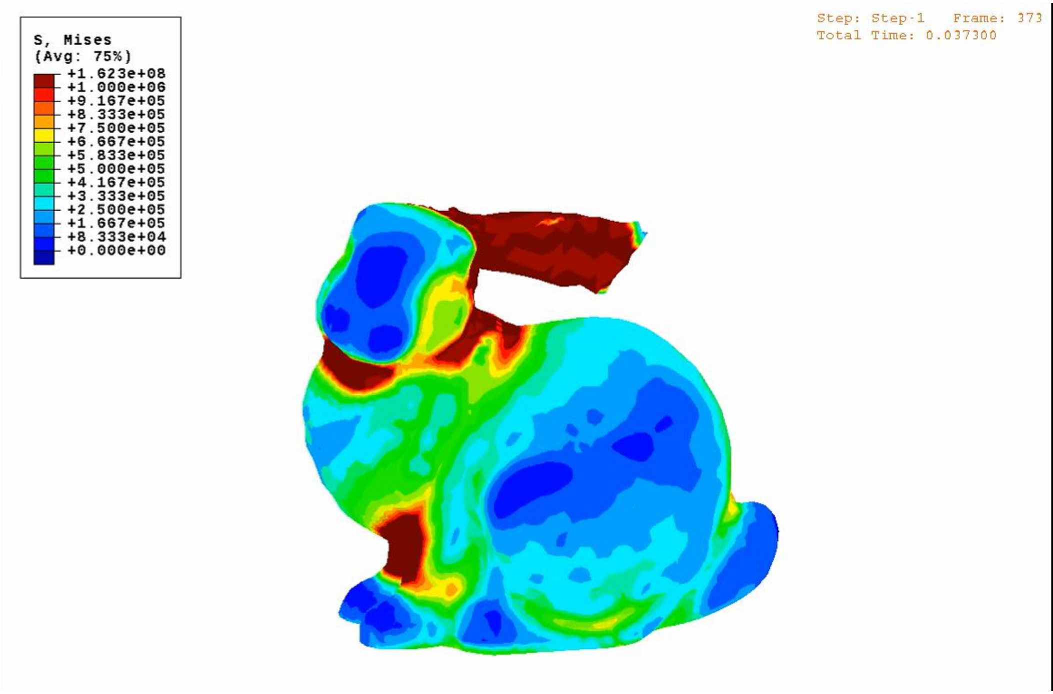
- Iron ears: Neo-Hookean, $E_{\mathrm{ini}} = 200\ \mathrm{GPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.3}$, $\rho = 7800\ \mathrm{kg/m^3}$.
- Rubber body: Neo-Hookean, $E_{\mathrm{ini}} = 6\ \mathrm{MPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.49}$, $\rho = 920\ \mathrm{kg/m^3}$.
- Discretized into T10 mesh. (about 61,000 nodes and 41,000 elements)
- Compared to ABAQUS/Explicit C3D10M (NOT C3D10MH) with the same mesh and Δt.
- Contact is not considered.

Swing of Bunny Ears

Comparison of Mises stress animation



SelectiveCS-FEM-T10/Explicit



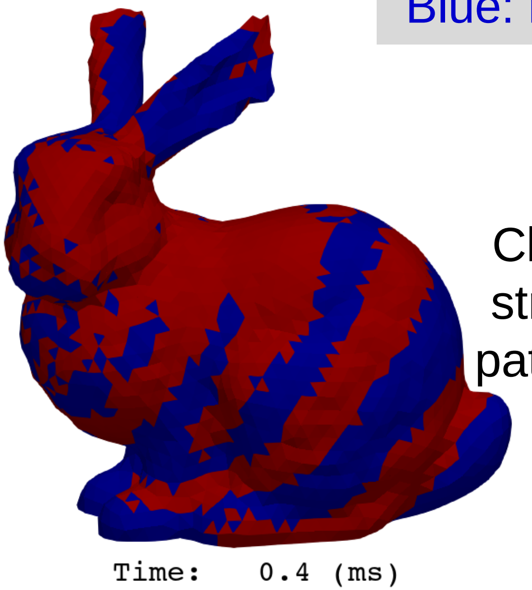
ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 has similar accuracy in displacement and Mises stress to ABAQUS C3D10M.

Swing of Bunny Ears

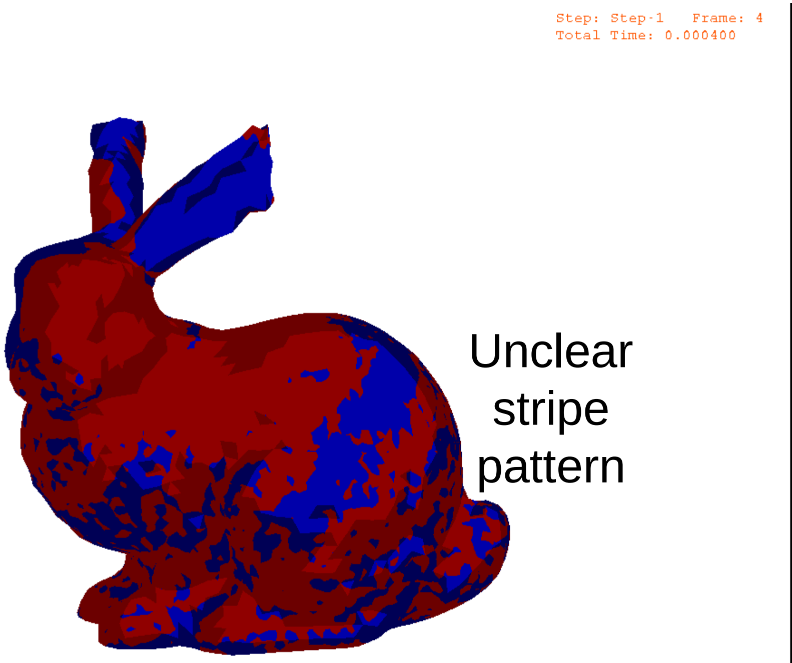
Comparison of pressure sign at $t = 0.4$ ms (right after the stat)

Red: positive pressure
Blue: negative pressure



Clear stripe pattern

SelectiveCS-FEM-T10/Explicit



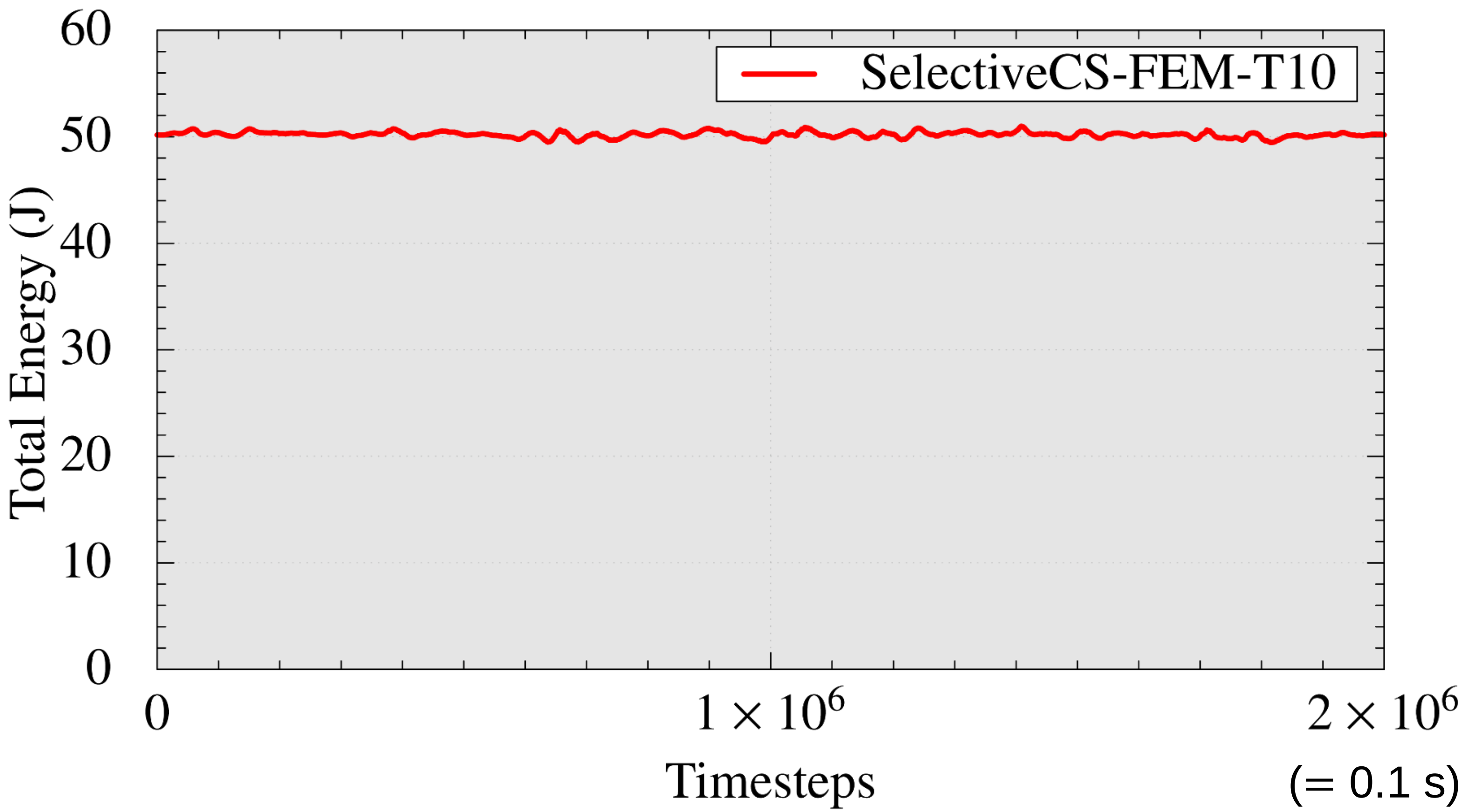
Unclear stripe pattern

ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 seems to calculate the initial pressure wave propagation more correctly than ABAQUS C3D10M.

Swing of Bunny Ears

Timestep-history of total energy (= kinetic + strain)



SelectiveCS-FEM-T10 has enough energetic stability in dynamic analysis.

Summary

Summary

Summary

- A new **SelectiveCS-FEM-T10** was proposed, which is:
  - **More robust to severe large deformation** than the conventional T10s.
  - **Enough accuracy for practical use** as compared to ABAQUS's best T10.
  - Slower than conventional T10s only in dynamic explicit analysis.
- More severe large deformation dynamic analyses should be performed for evaluation.

Take-home message

If you are interested in large deformation analysis,

please consider implementing SelectiveCS-FEM-T10 to your FE code.

It's supremely useful & easy to code!!

Thank you for your kind attention!

Appendix

Characteristics of SelectiveCS-FEM-T10

Benefits

- ✓ Accurate (no locking, no checkerboarding, no force oscillation).
- ✓ Robust (long-lasting in large deformation).
- ✓ No increase in DOF (No static condensation).
- ✓ Same CPU costs as the other T10 elements (except explicit analyses).
- ✓ Implementable to commercial FE codes (e.g., ABAQUS UEL).

Drawbacks

- ✗ Mises stress oscillation in some extreme analyses.
- ✗ Several times larger memory size than other T10 elements.
- ✗ No longer a T4 formulation.

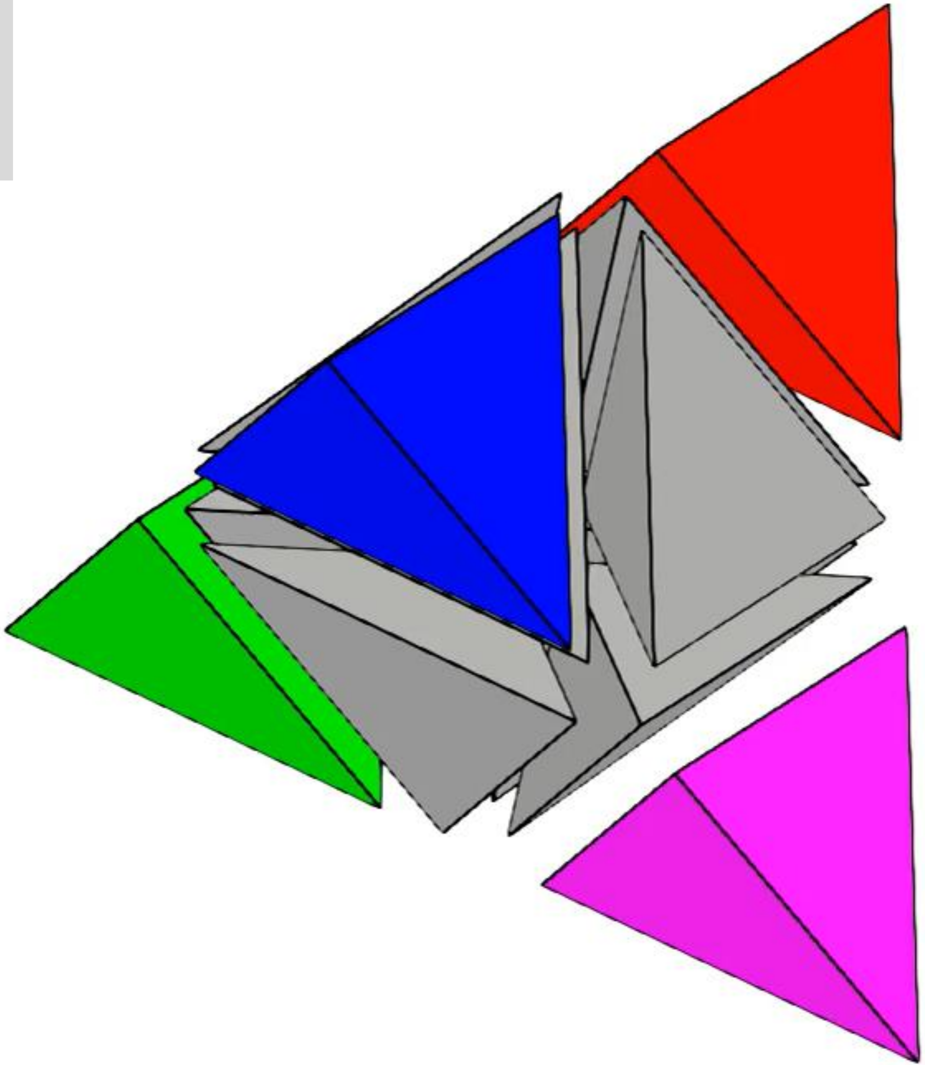
SelectiveCS-FEM-T10 is competitive with the best ABAQUS T10 element, C3D10MH.

T10 Element Subdivision in 3D

Natural subdivision (30% shrunk mesh)

Each frame edge is owned by only one sub-element.

There are 12 sub-elements in total.

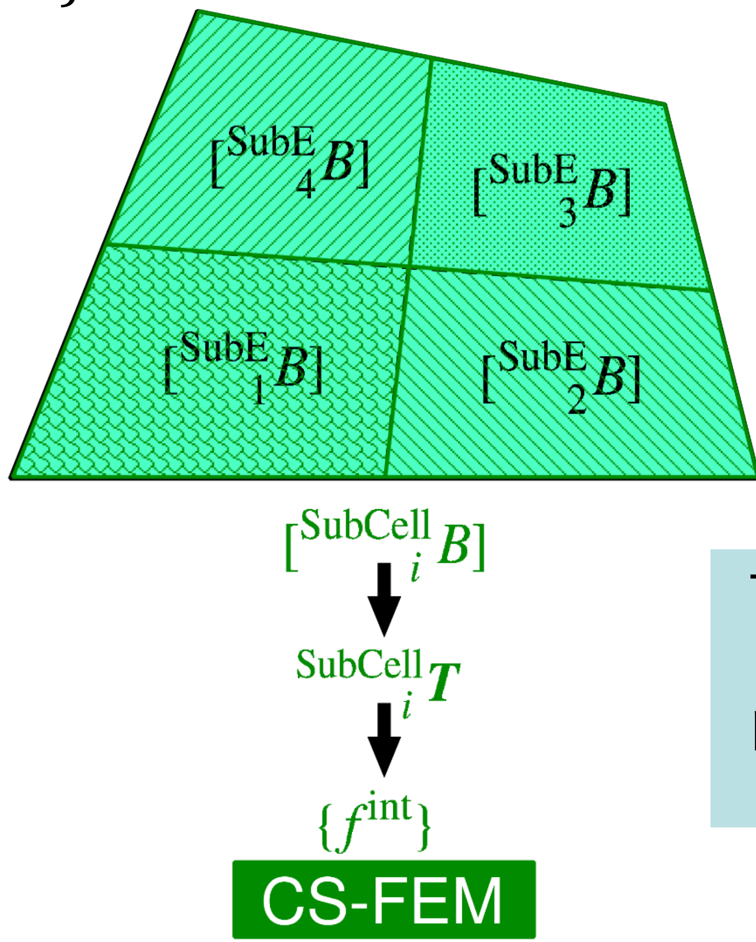


Strain on frame edges are NOT smoothed by ES-FEM.

Brief of Cell-based S-FEM (CS-FEM)

- Subdivide each element into some sub-element.
- Calculate $[{}^{\mathrm{SubE}}B]$ at each sub-element.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each sub-element.

As if putting an integration point on each sub-element



The biggest advantage of CS-FEM is its **portability to existing FE codes.**

Issues

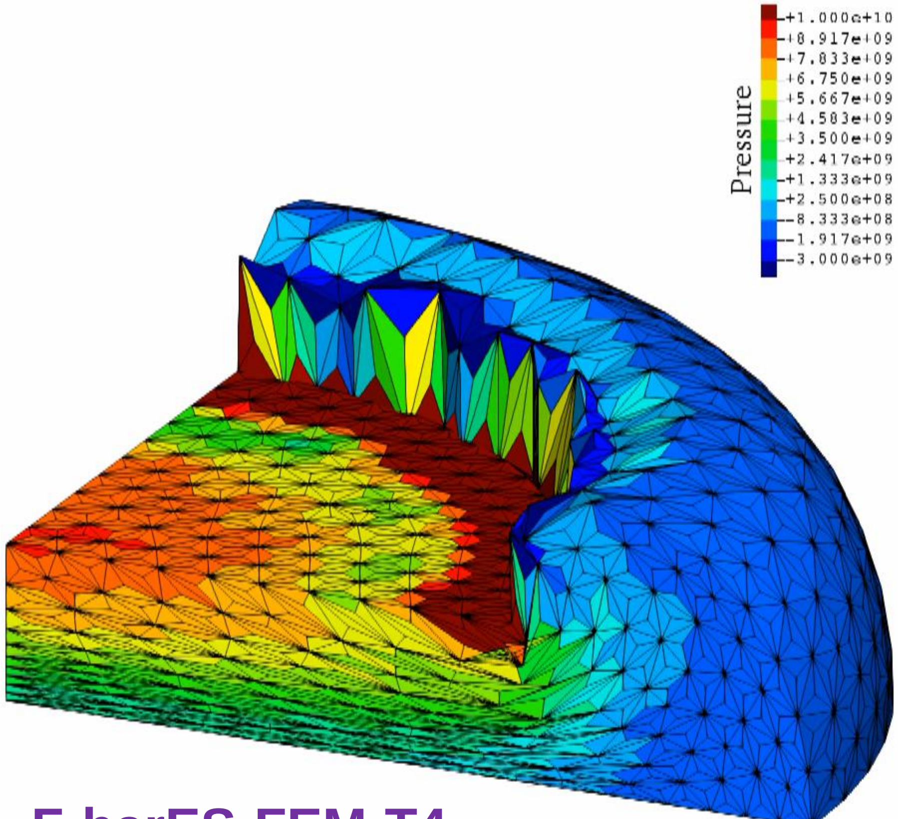
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy and/or robustness especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking. Accuracy loss in large strain.
- B-bar/F-bar method, Selective reduced integration (SRI):
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method** (ABAQUS **C3D10MH** etc.):
  - ✗ Early convergence failure. Accuracy loss in large strain.
- **F-bar aided ES-FEM-T4** [Y.Onishi, IJNME, 109 (2017)] :
  - ✓ Accurate & robust ✗ Hard to implement in FEM codes.
- **SelectiveCS-FEM-T10** [Y.Onishi, IJCM, 17 (2020)] :
  - ✓ Accurate, robust & easy to implement. ✗ Not yet optimal.

Issues in Barreling Analysis of Rubber Cylinder

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

Same mesh as C3D4H case.



F-barES-FEM-T4

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding
- ✓ No increase in DOF

Although F-barES-FEM-T4 is accurate and robust,

- ✘ it cosumes larger memory & CPU costs.
- ✘ it cannot be implemented in general-purpose FE software due to the adoption of ES-FEM.

↓

Another approach adopting **CS-FEM** with **T10** element would be effective.

Y. Onishi, IJMNE, Vol. 109 (2017).