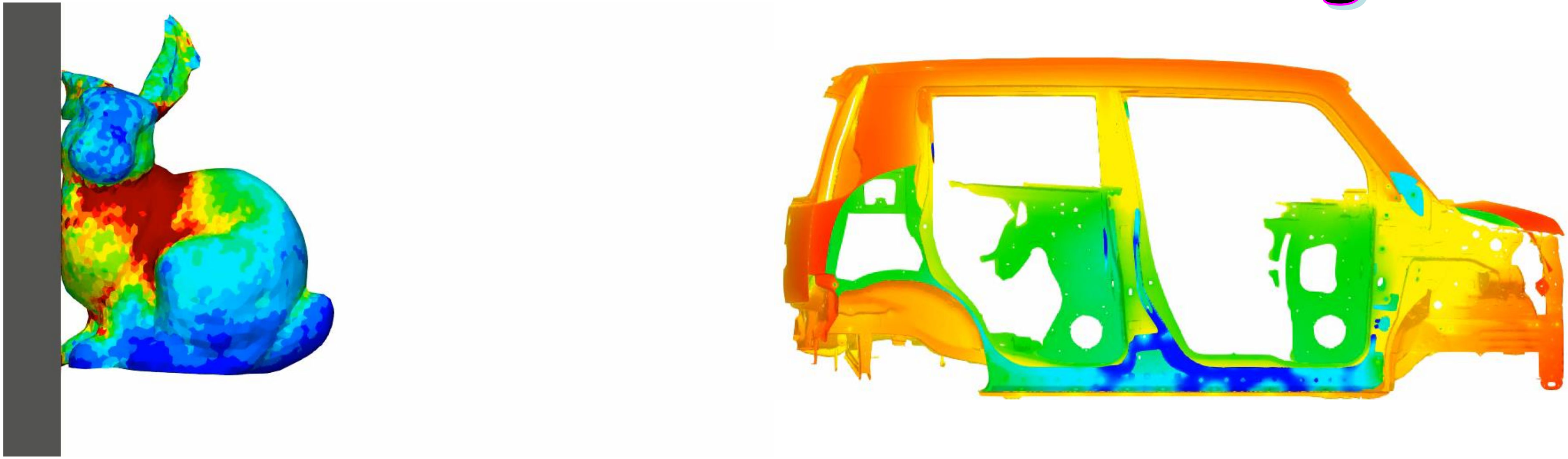
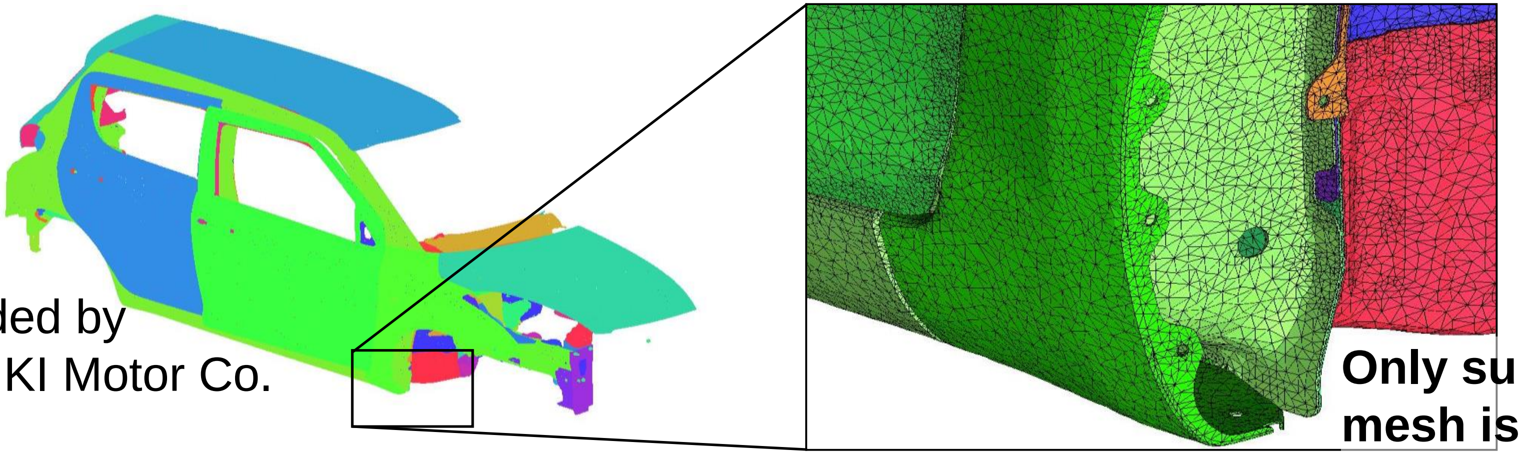
Smoothed Finite Element Methods: Recent Academic/Practical Progress



Yuki ONISHI
Tokyo Institute of Technology, Japan

Motivation of My S-FEM Researches

- Our group is mainly interested in practical research topics for **industrial applications**.
- Manufactured products usually have **complex body shapes**, which are difficult to be discretized with Hex meshes in FEA.
- Tet meshes are easy to generate, but the analyses with **conventional Tet elements are inaccurate**.



Provided by
SUZUKI Motor Co.

Only surface mesh is shown.

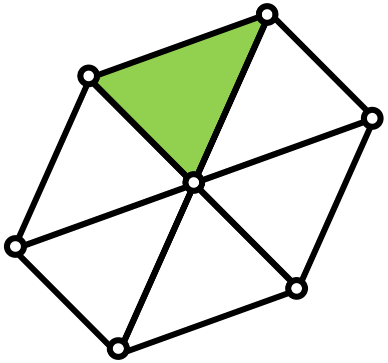
We have started researching on
smoothed FEM (S-FEM) with Tet meshes.

What is S-FEM?

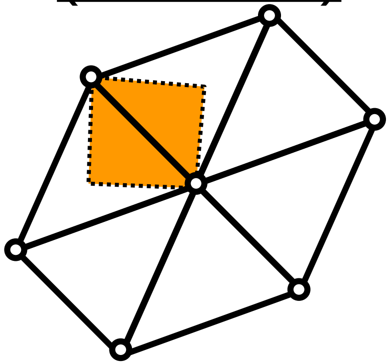
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example in 2D triangular mesh:

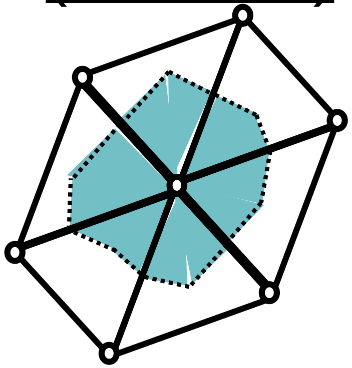
Standard FEM



Edge-based S-FEM (ES-FEM)

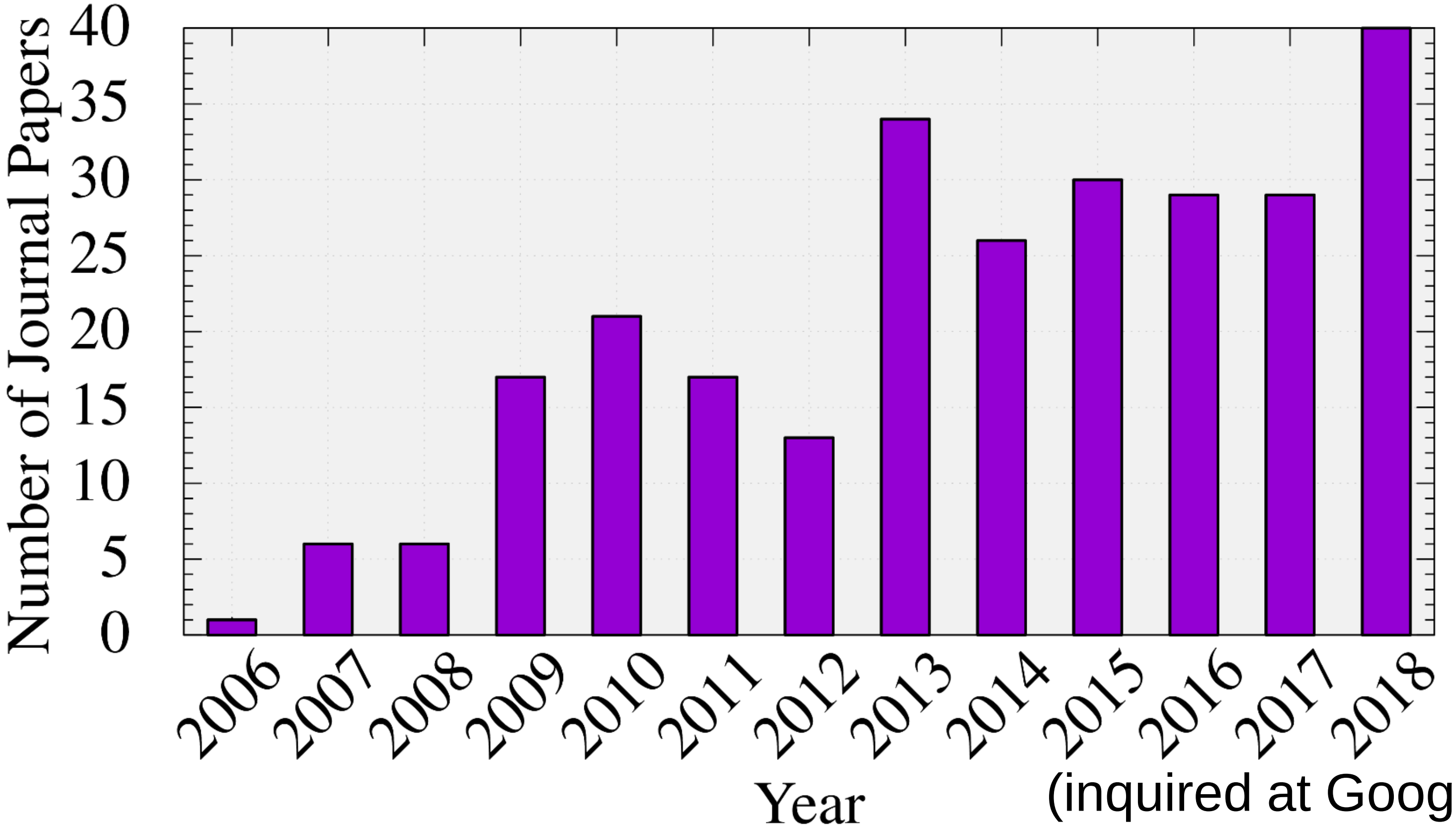


Node-based S-FEM (NS-FEM)



What is S-FEM?

- Number of journal papers written **in English** whose **title** contains "**smoothed finite element**":

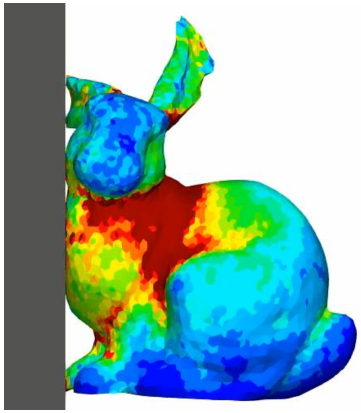


(inquired at Google Scholar)

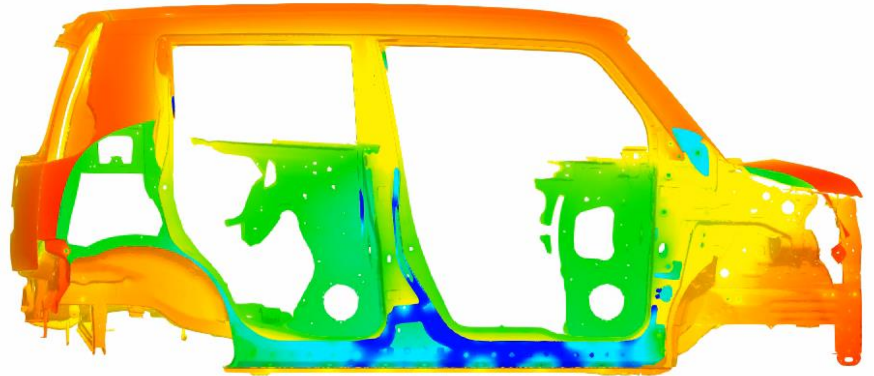
The attraction of S-FEM is expanding continuously.

Today's Agendas

Part 1: *Academic* Progress of S-FEM
in **Large Deformation Analysis** of Solids.



Part 2: *Practical* Progress of S-FEM
in **Electrodeposition Process Simulation**
of Auto Car Bodies.



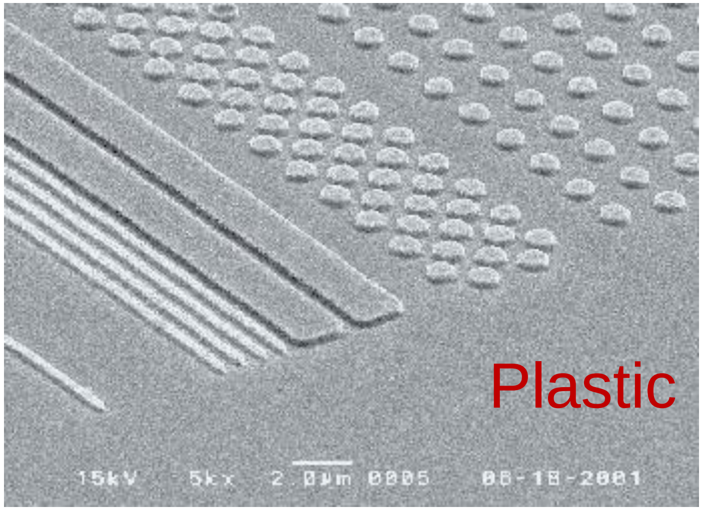
Part 1:
Academic Progress of S-FEM in Large Deformation Analysis of Solids

Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.

- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



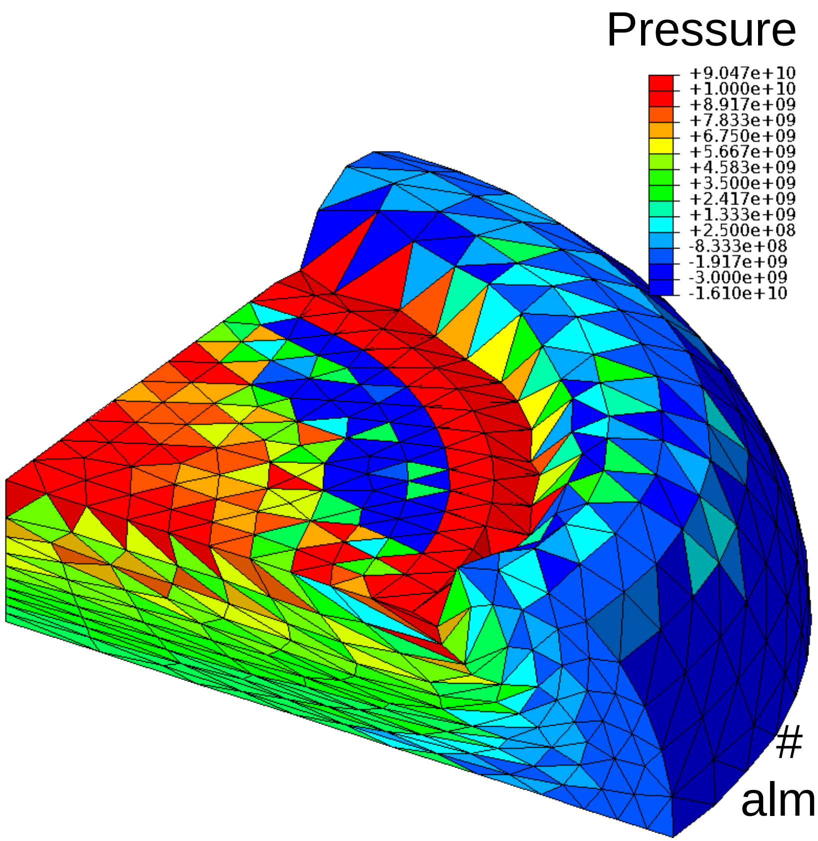
Issues

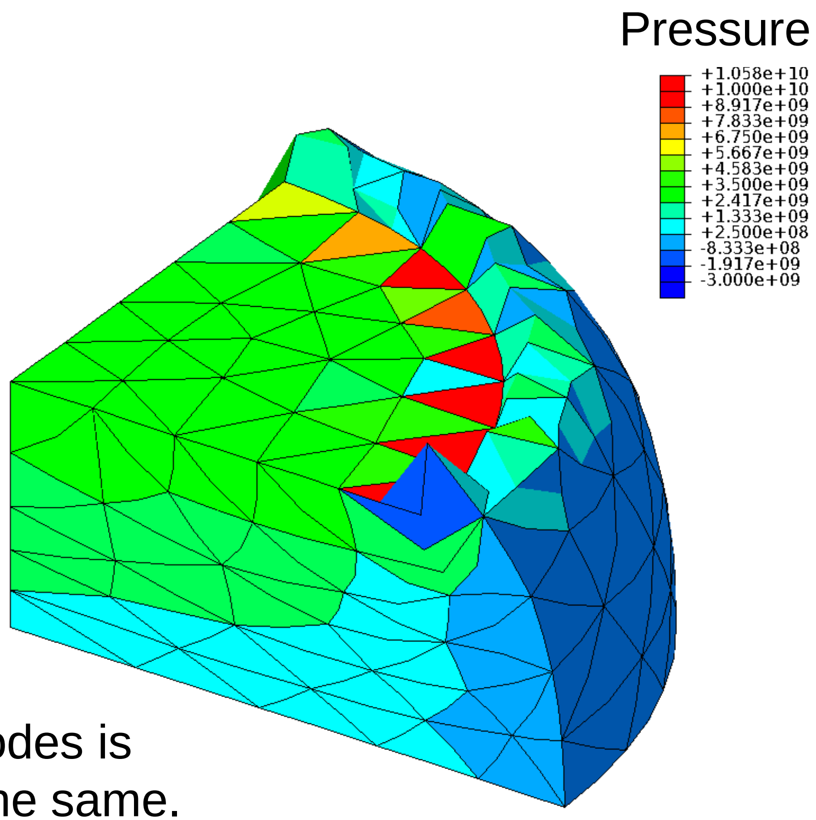
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
    Accuracy loss in large strain due to intermediate nodes.
- Enhanced assumed strain method (EAS):
  - ✗ Spurious low-energy modes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding etc..

Issues (cont.)

E.g.) Compression of neo-Hookean **hyperelastic** body with $\nu_{\text{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

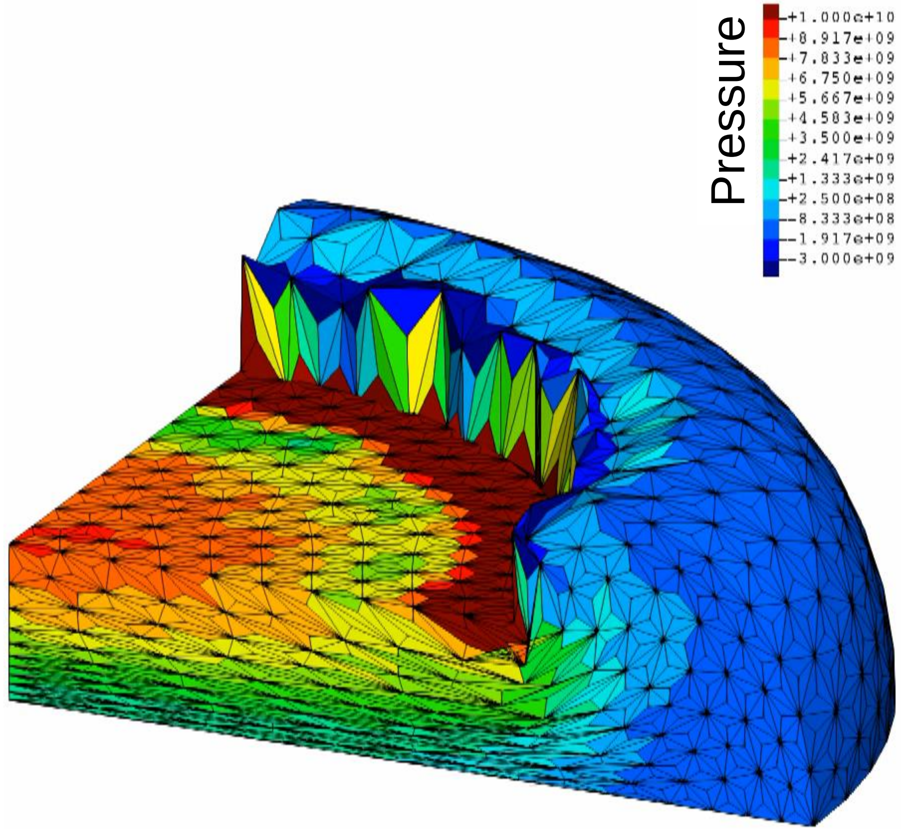
- ✓ No shear/volumetric locking
- ✗ Pressure checkerboarding
- ✗ Corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

A Recent Solution: S-FEM

E.g.) Compression of neo-Hookean **hyperelastic** body with $\nu_{\mathrm{ini}} = 0.49$



Same mesh
as C3D4H
case.

F-barES-FEM-T4 (One of the latest S-FEM)

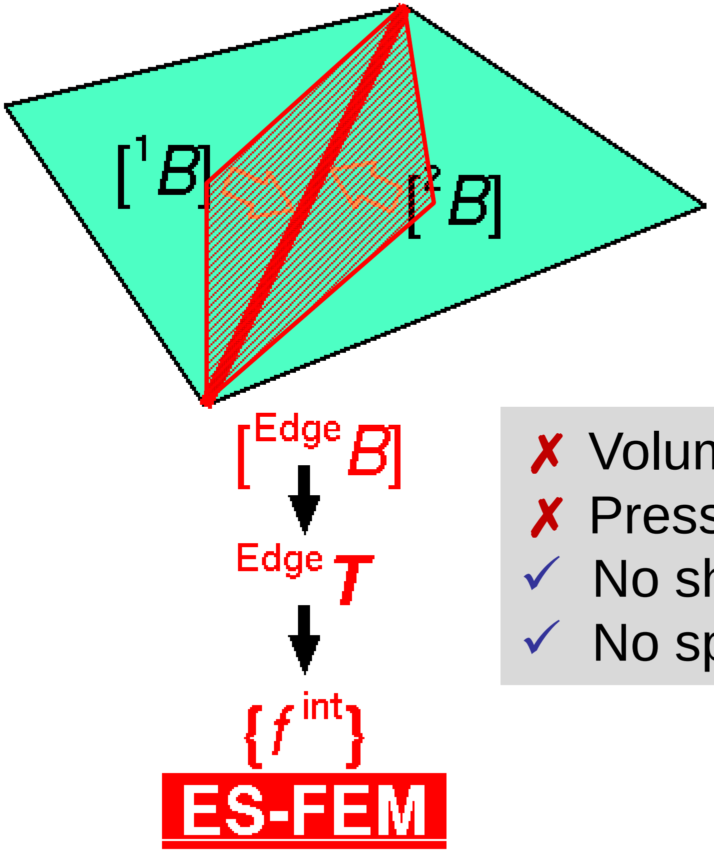
- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding
- ✓ No increase in DOF (i.e., No static condensation)

Formulation of
F-barES-FEM-T4

Quick Intro. of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[\,{}^{\mathrm{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each edge smoothing domain.

As if putting
an integration point
on each edge center

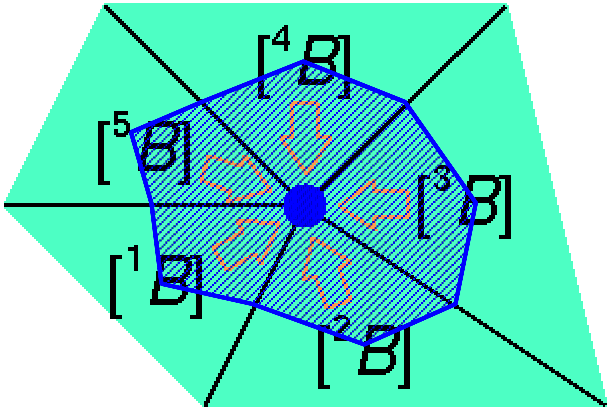


✗ Volumetric locking
✗ Pressure checkerboarding
✓ No shear locking
✓ No spurious modes

Quick Intro. of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[{}^{\mathrm{Node}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each node smoothing domain.

As if putting
an integration point
on each node



$[{}^{\mathrm{Node}}B]$

↓

${}^{\mathrm{Node}}\boldsymbol{T}$

↓

$\{f^{\mathrm{int}}\}$

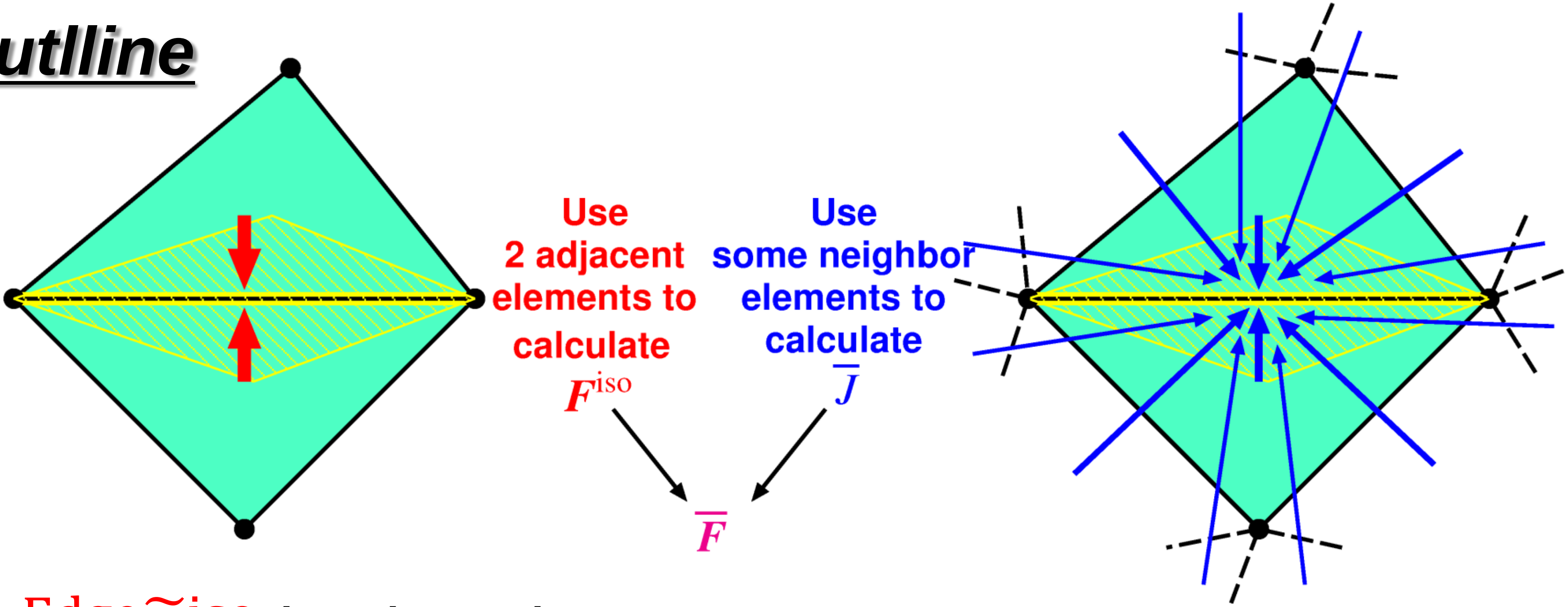
NS-FEM

- ✗ Spurious low-energy mode (or hour-glass mode)
- ✓ Less pressure checkerboarding
- ✓ No shear locking
- ✓ No volumetric locking

Concept of F-barES-FEM

Concept: combining ES-FEM and NS-FEM using F-bar method

Outlline



Use 2 adjacent elements to calculate $\boldsymbol{F}^{\mathrm{iso}}$

Use some neighbor elements to calculate $\bar{J}$

$\bar{\boldsymbol{F}}$

- $^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}$ is given by ES-FEM.
- $^{\mathrm{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- $^{\mathrm{Edge}}\bar{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

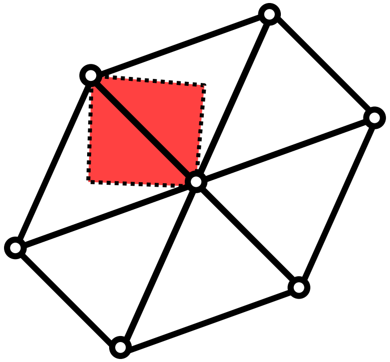
$$^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{\,1/3}\ {}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}.$$

Formulation of F-barES-FEM (1 of 2)

Deformation gradient of each edge ($\bar{\boldsymbol{F}}$) is derived as

$$\bar{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

in the manner of F-bar method.



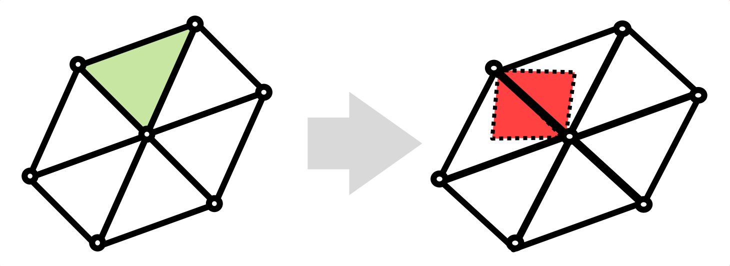
ES−FEM

Formulation of F-barES-FEM (2 of 2)

Each part of $\bar{\boldsymbol{F}}$ is calculated as

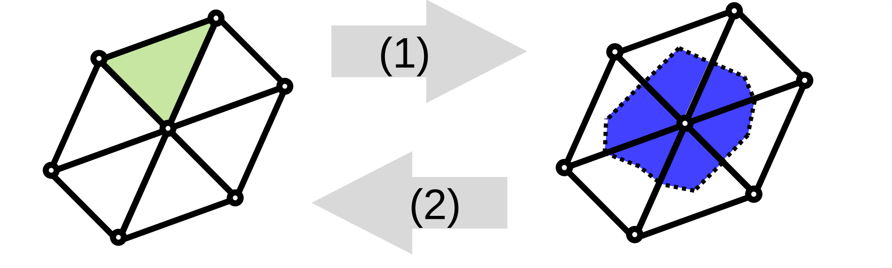
$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part



Smoothing the value of adjacent elements.
(same manner as ES-FEM)

Volumetric part



(1)Calculating node's value by smoothing the value of adjacent elements

(2)Calculating elements' value by smoothing the value of adjacent nodes

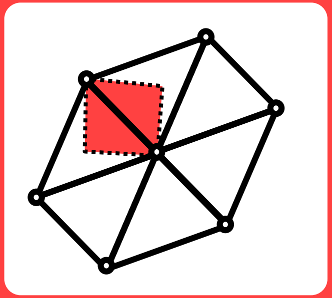
(3)Repeating (1) and (2) a few times

Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

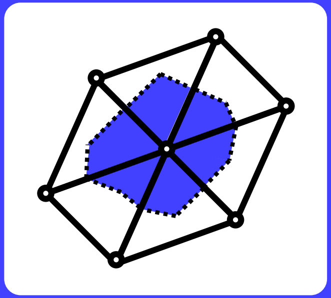
Isovolumetric part



Like a ES-FEM

1. Shear locking free

Volumetric part



Like a NS-FEM

2. Little pressure oscillation

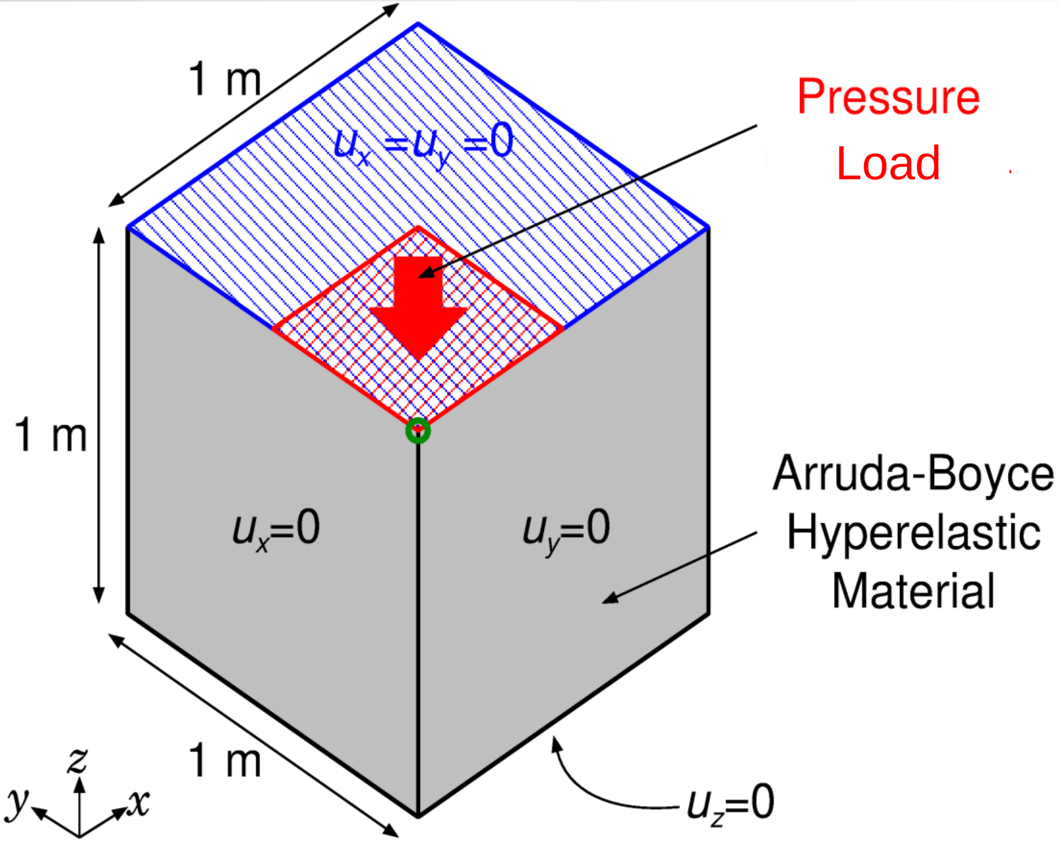
3. Volumetric locking free

with the aid of F-bar method

Demonstrations of
F-barES-FEM-T4

Compression of Rubber Block

Outline



- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static Implicit

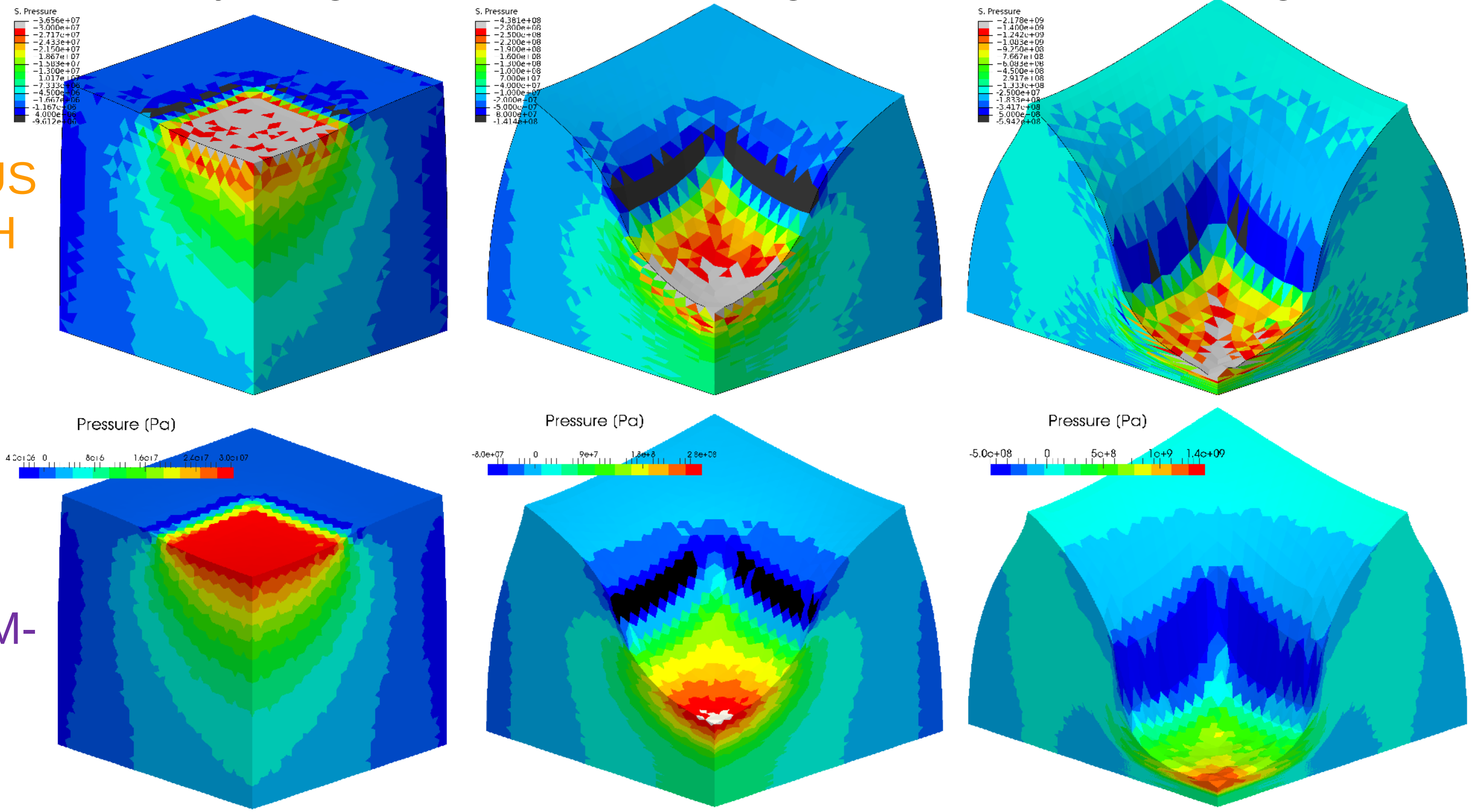
Compression of Rubber Block

Pressure dist.

Early stage | Middle stage | Later stage

ABAQUS C3D4H

F-bar ES-FEM-T4(3)



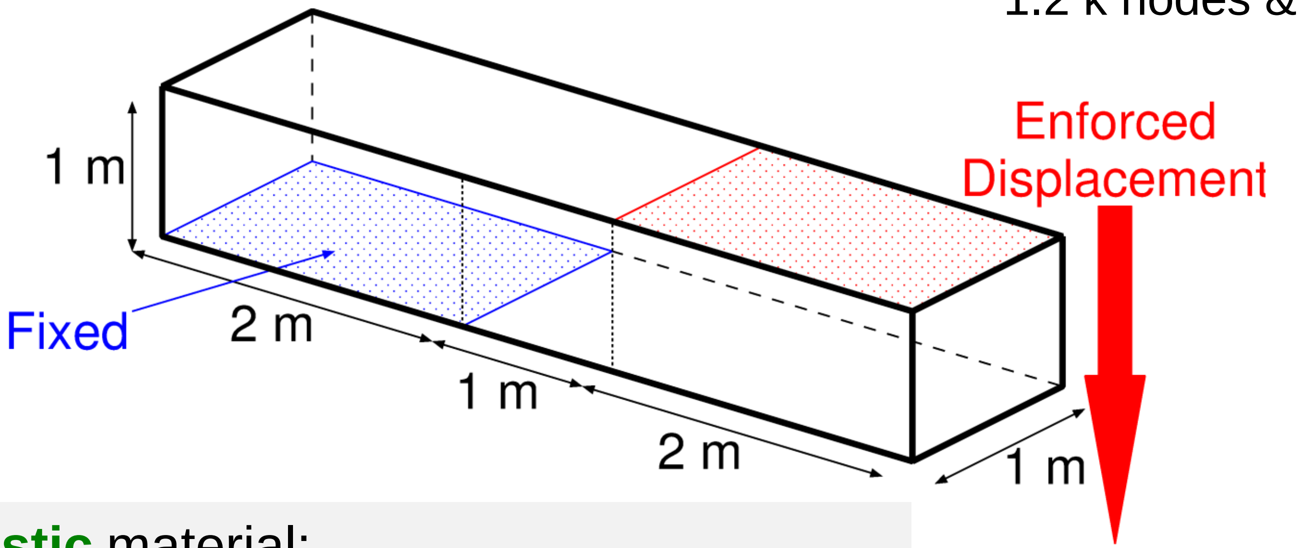
Smooth pressure distributions are obtained.

Static Implicit

Shear-tensioning of Elasto-plastic Bar

Outline

1.2 k nodes & 4.8 k elems.



Elasto-plastic material:

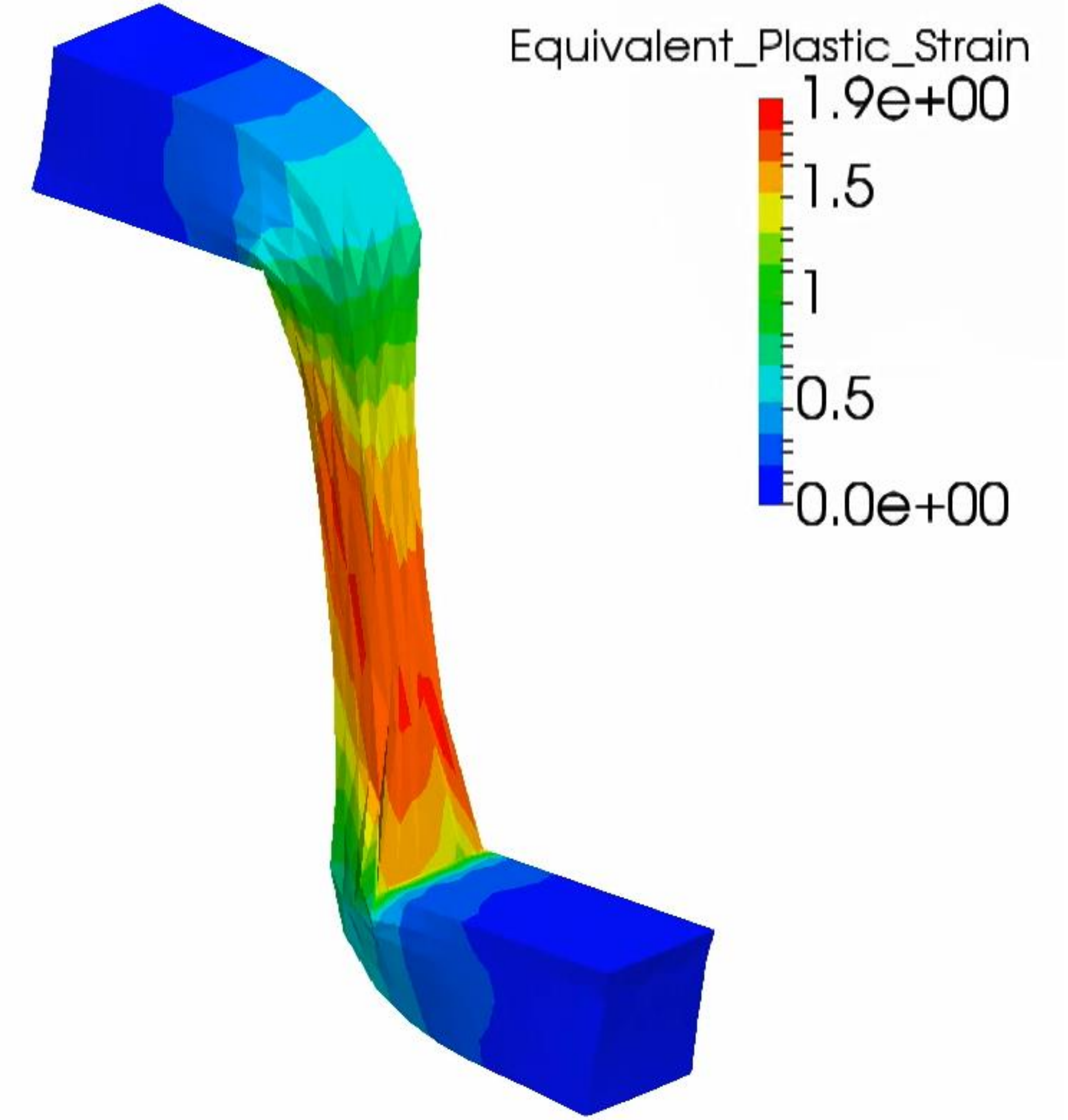
- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static
Implicit

Shear-tensioning of *Elasto-plastic* Bar

Result of
F-bar
ES-FEM(2)
(Equiv. plastic strain)

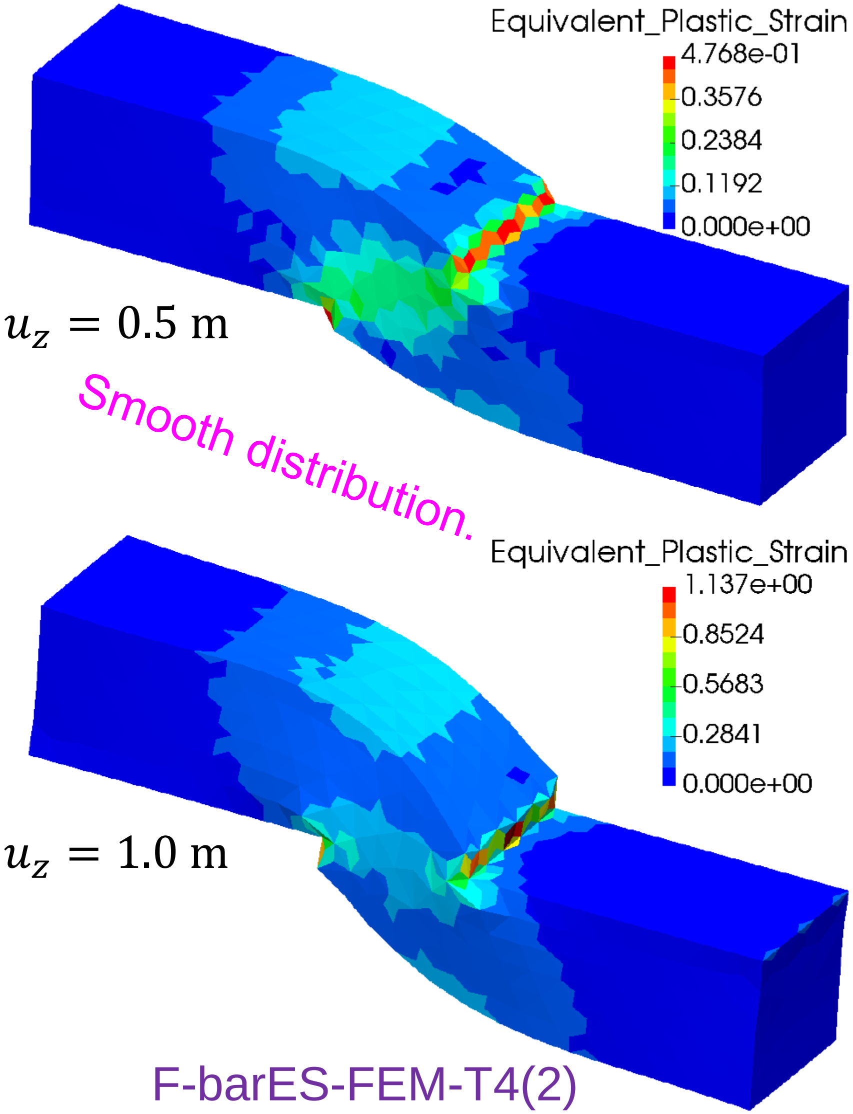


Extreme large deformation with smooth strain dist. is successfully achieved.

Static
Implicit

Shear-tensioning of *Elasto-plastic* Bar

Equivalent plastic strain dist. in middle states

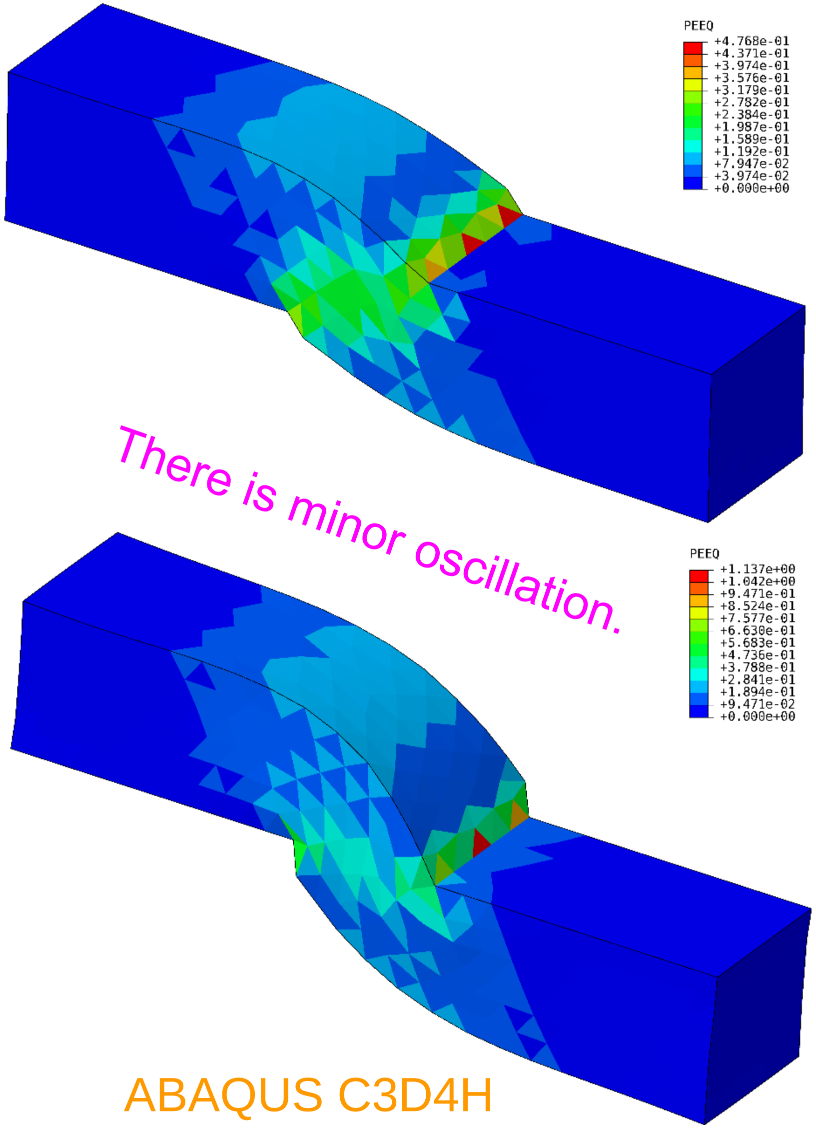


$u_z = 0.5$ m

Smooth distribution.

$u_z = 1.0$ m

F-barES-FEM-T4(2)



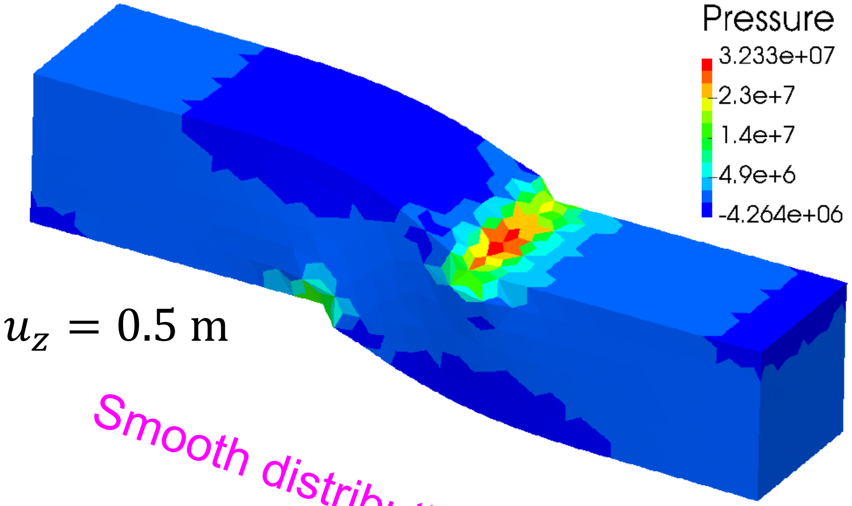
There is minor oscillation.

ABAQUS C3D4H

Static
Implicit

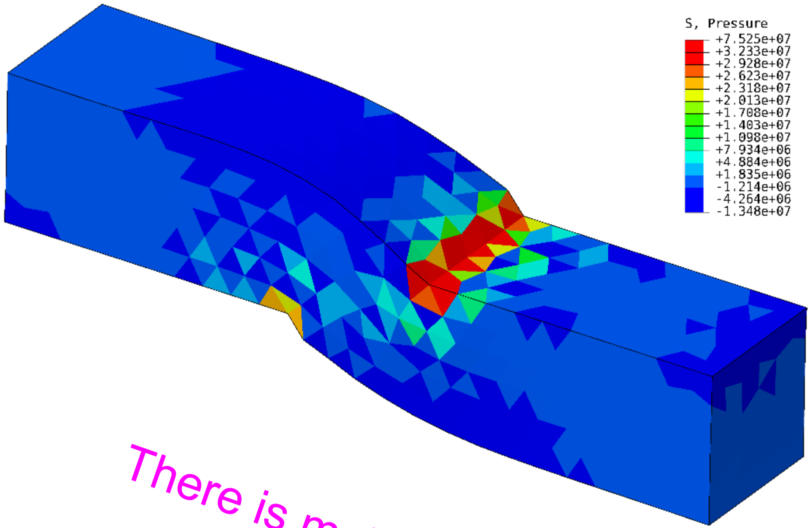
Shear-tensioning of Elasto-plastic Bar

Pressure dist. in middle states

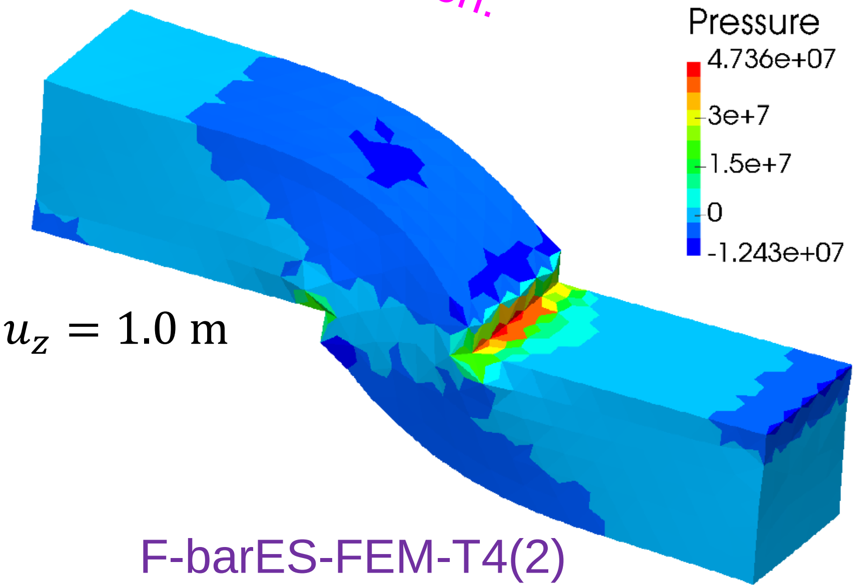


$u_z = 0.5$ m

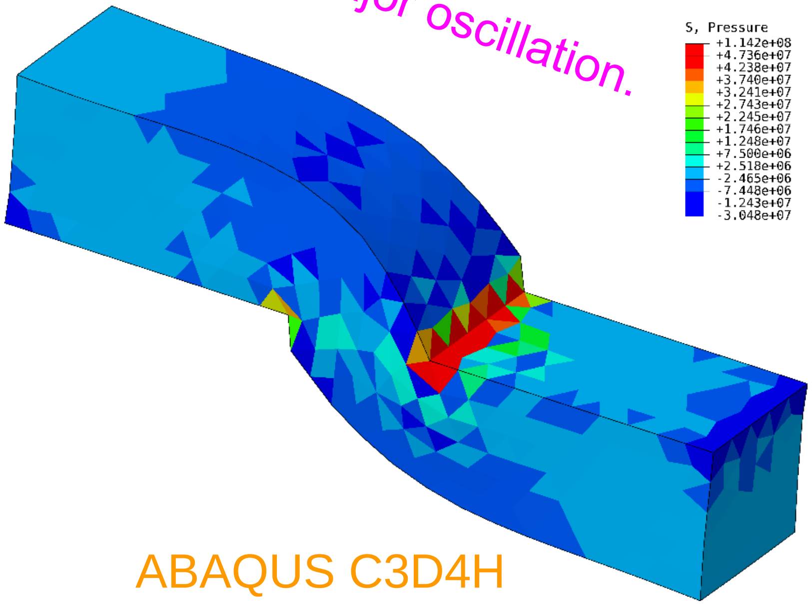
Smooth distribution.



There is major oscillation.



$u_z = 1.0$ m



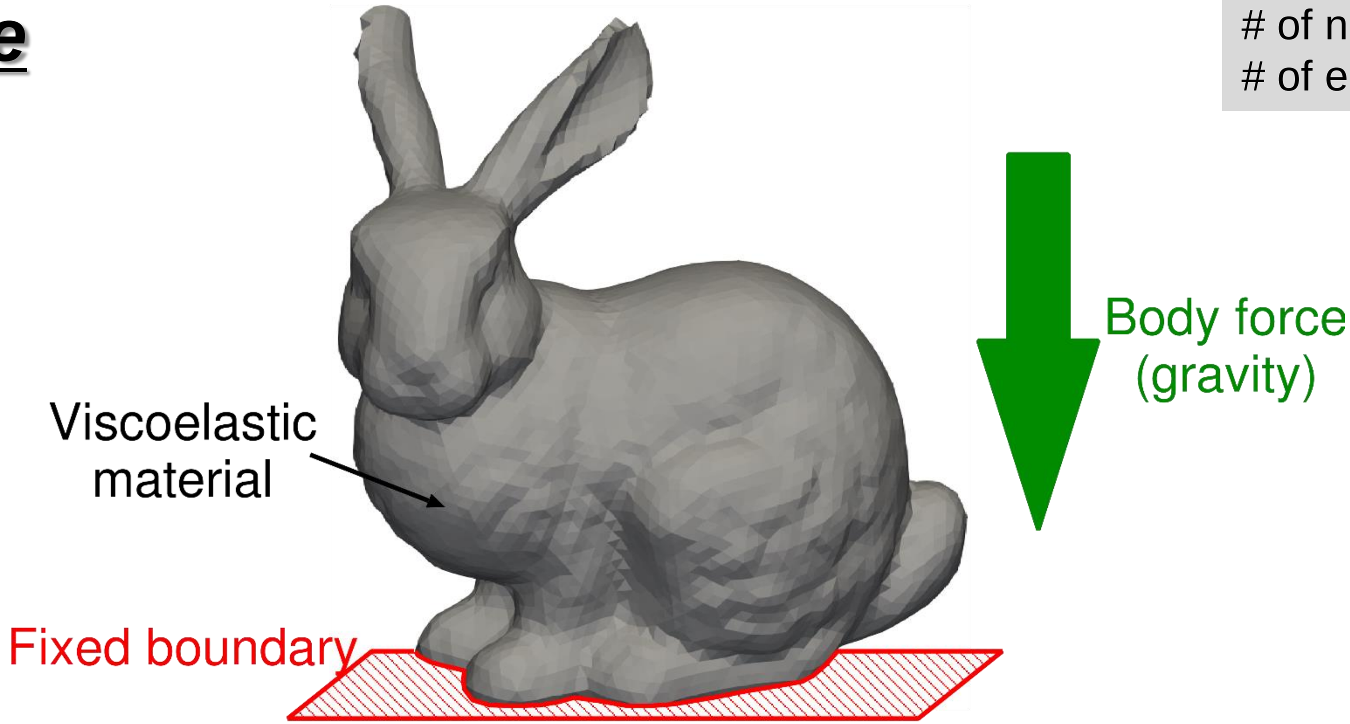
F-barES-FEM-T4(2)

ABAQUS C3D4H

Collapse Analysis of *Viscoelastic* Bunny

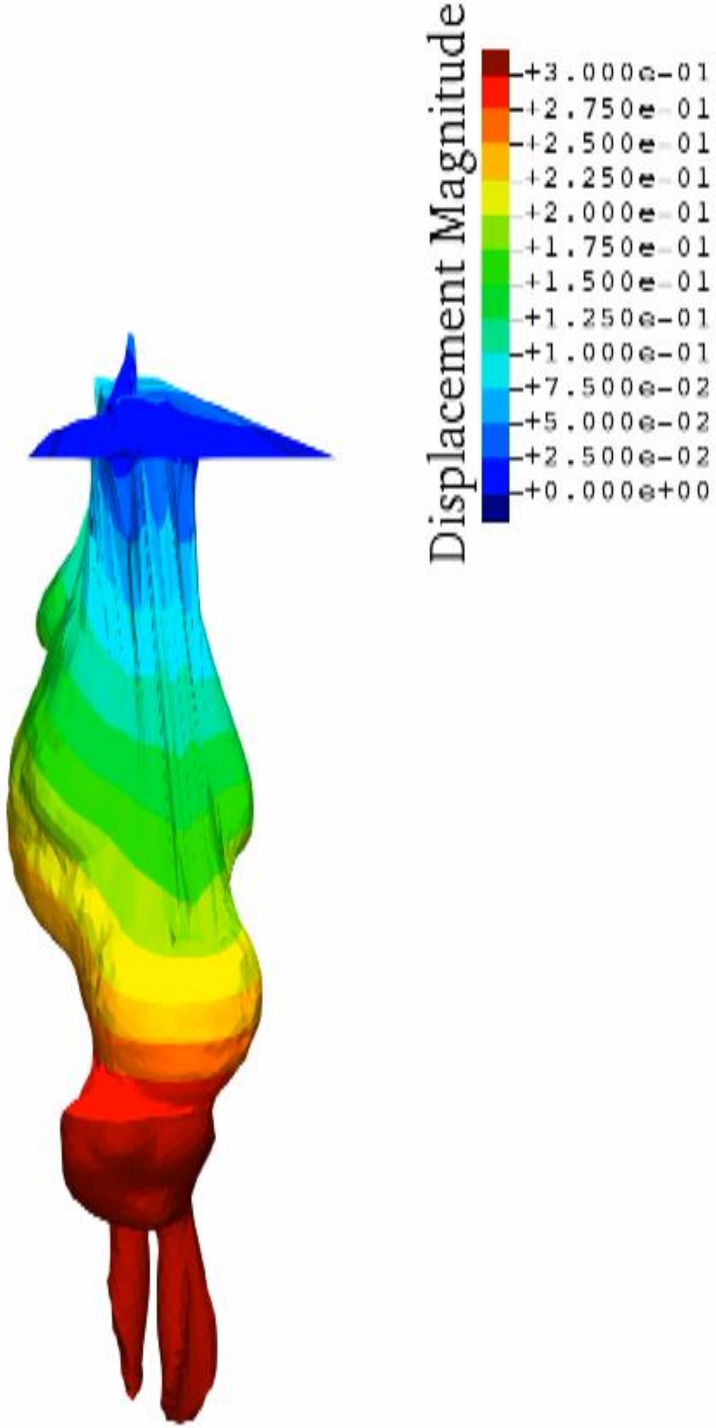
Outline

of nodes: 24136
# of elems: 126231



- Applying gravity to the Stanford Bunny and let it collapsed by its self-weight.
- Soft viscoelastic material $(\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10\ s)$.
- Contact is NOT considered.
- Comparing F-barES-FEM-T4(2) and ABAQUS C3D4H.

Animation of Deformation

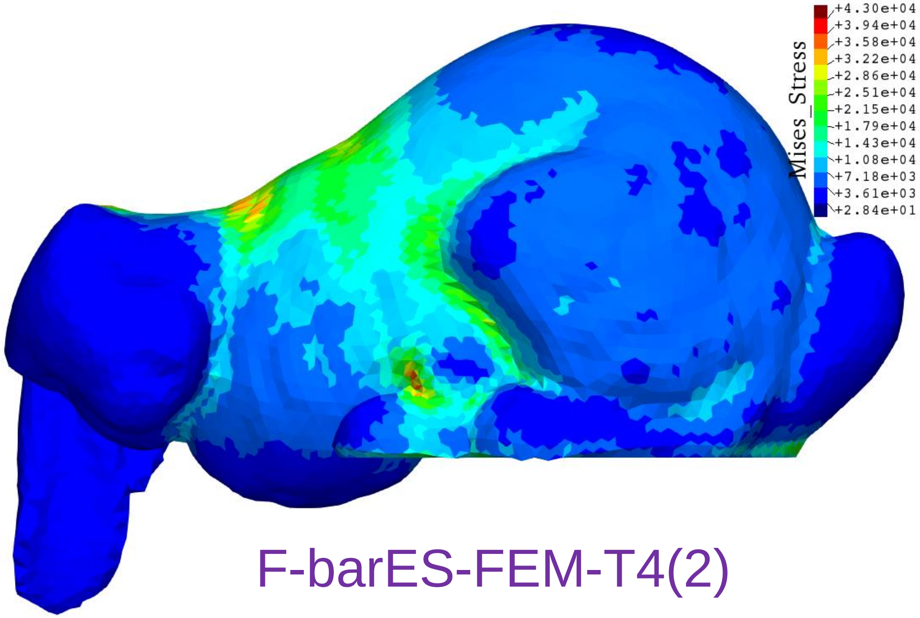


Because contact is not considered, the body penetrates the feet and finally becomes upside downside. The analysis lasts till the necking.

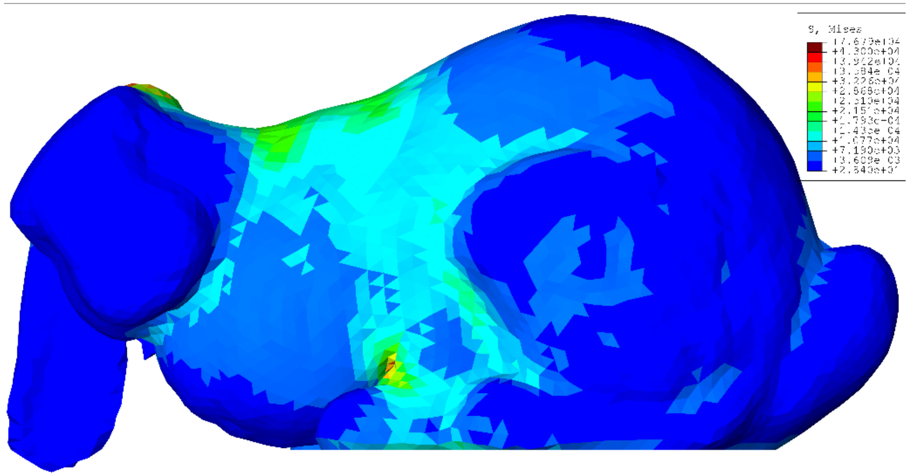
Viscous
Implicit

Collapse Analysis of Viscoelastic Bunny

Mises stress dist. when C3D4H get a convergence failure



F-barES-FEM-T4(2)

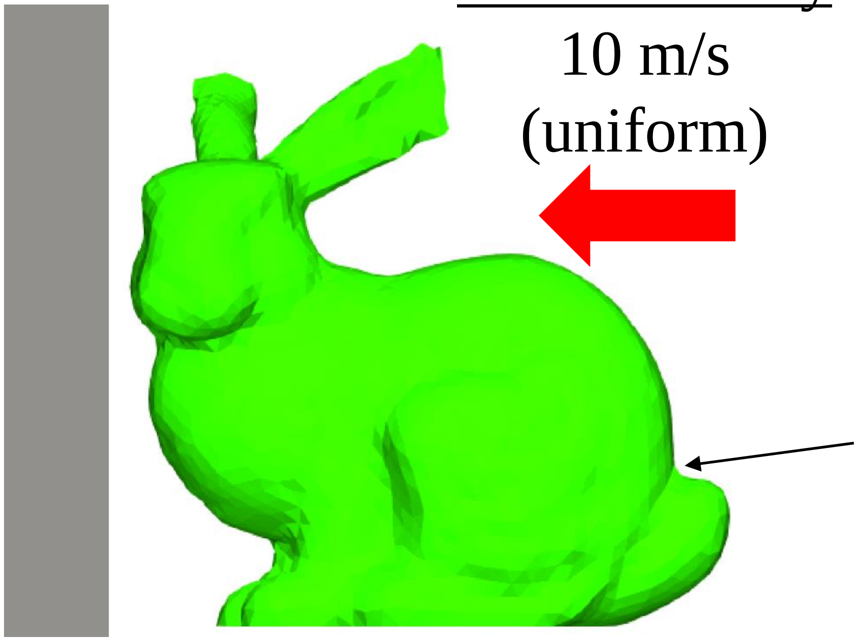


ABAQUS C3D4H

- ABAQUS C3D4H shows a stiffer result due to shear locking.
- The result of F-barES-FEM-T4 would be better.

Impact of **Rubber** Bunny

Outline



Rigid Wall

Contact condition

free-slip, free-separation

Initial velocity

10 m/s

(uniform)

Rubber body

$E = 6.0\ \mathrm{MPa}$

$\nu = \mathbf{0.49}$

$\rho = 920\ \mathrm{kg/m^3}$

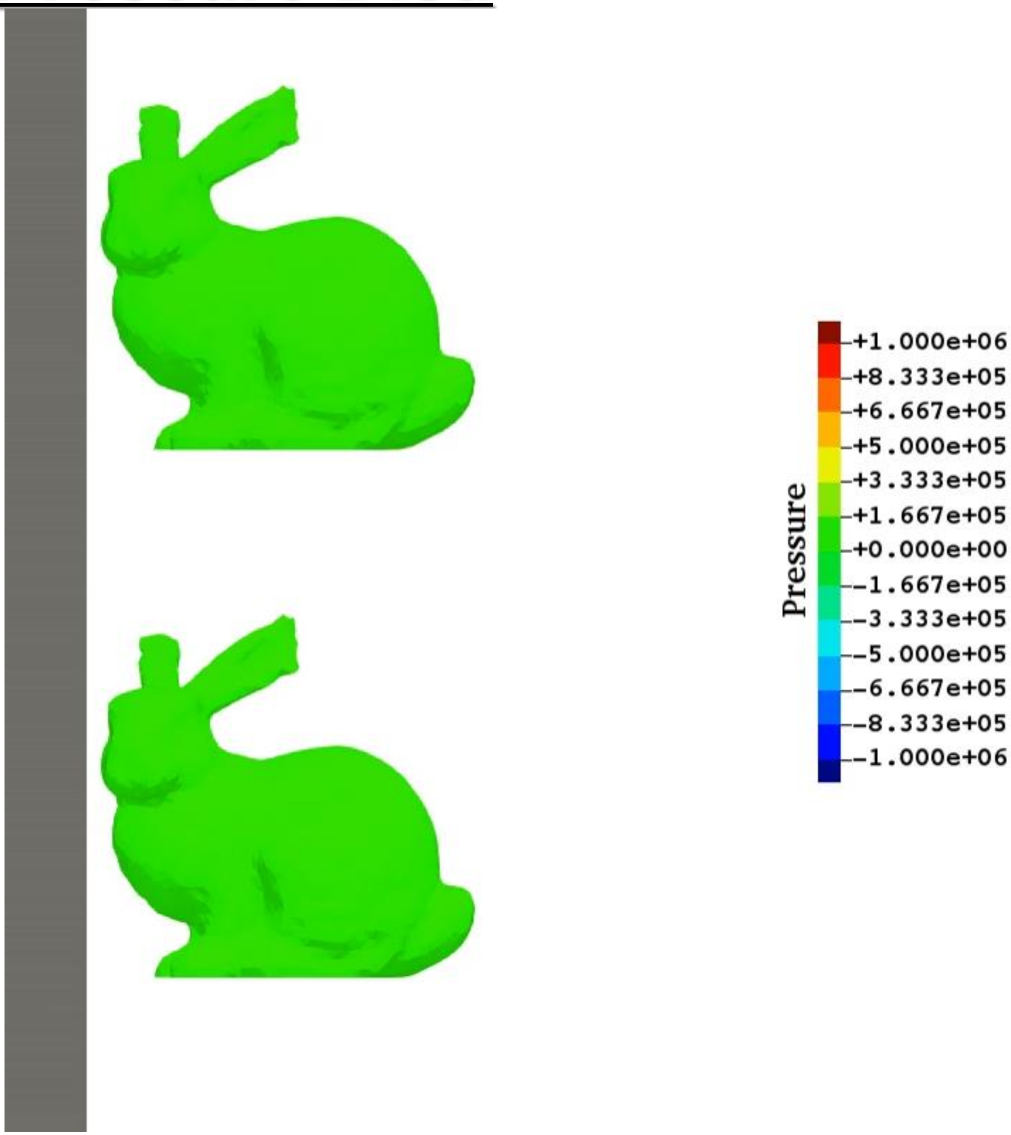
- A bunny made of rubber (Neo-Hookean) is crushed to a rigid wall.
- Compared with ABAQUS/Explicit C3D4 using a same T4 mesh.
- Note that neither Hex mesh nor hybrid elements is not available in this problem.

Impact of Rubber Bunny

Animation of Pressure Dist.

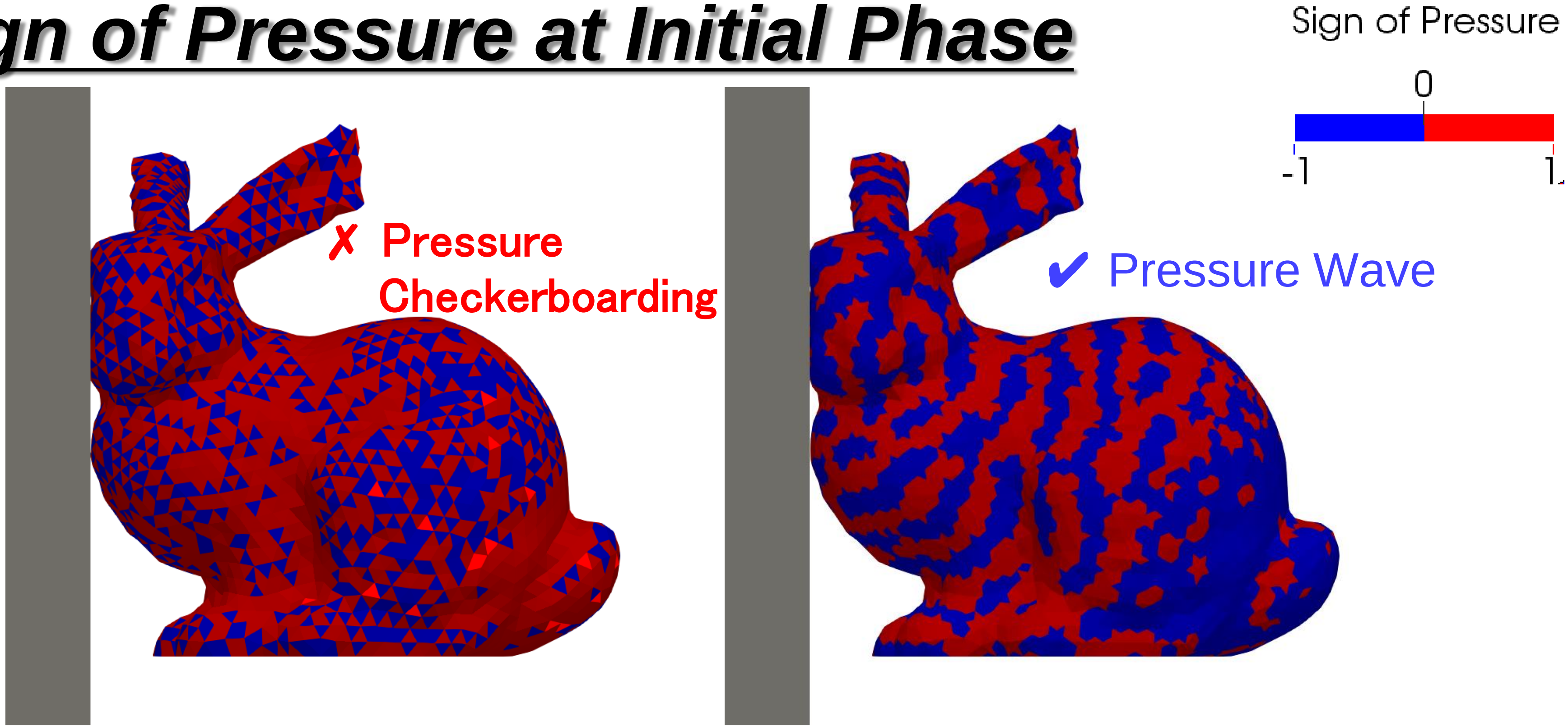
ABAQUS/Explicit
C3D4
- ✗ Pressure Checkerboarding
- ✗ Shear Locking

SymF-barES-FEM-T4(1)
- ✓ Smooth pressure
- ✓ No Locking



Impact of Rubber Bunny

Sign of Pressure at Initial Phase



✗ Pressure Checkerboarding

✔ Pressure Wave

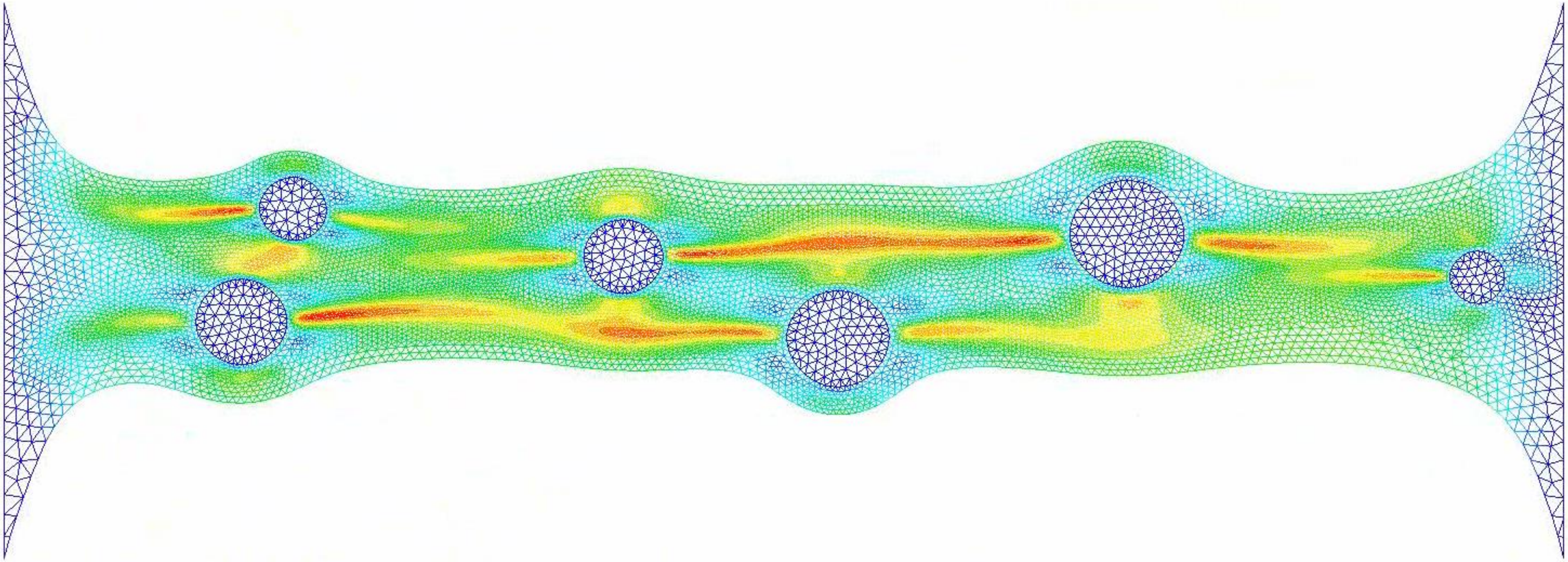
ABAQUS/Explicit C3D4
(Standard T4 element)

SymF-barES-FEM-T4(1)

The proposed S-FEM captures the pressure wave in a complex body successfully!!

Stretch of Filler-containing Rubber with Remesing

- Several hard circular fillers are distributed in a square soft matrix rubber (neo-Hookean hyperelastic with $\nu_{\mathrm{ini}} = 0.49$).
- E_{ini} of the filler is 100 times larger than E_{ini} of the matrix.
- Left side is constrained and right side is displaced.



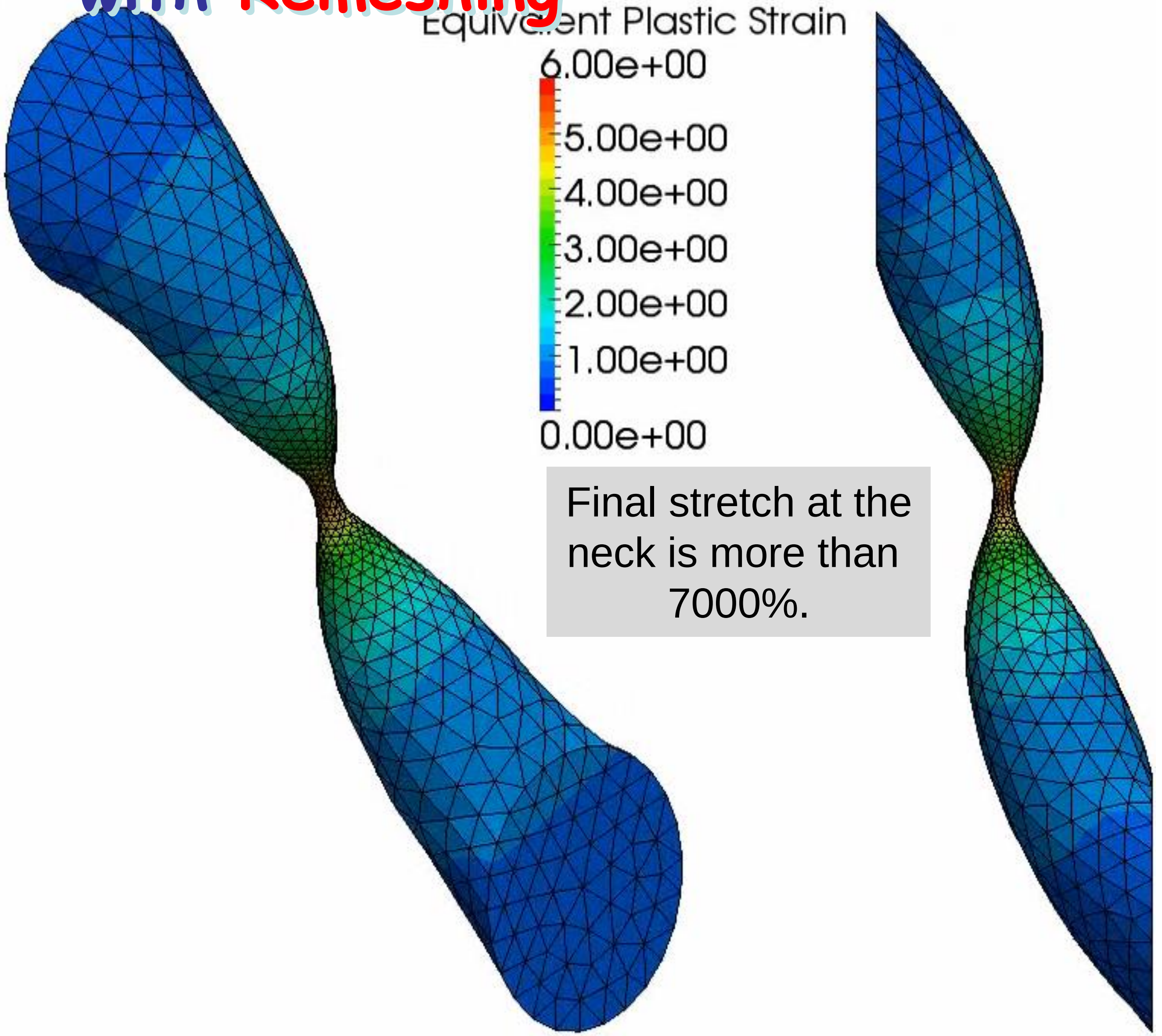
Valid Mises stress dist. is obtained after many time remeshings.

Static Implicit

Shear-tensioning of Elasto-plastic cylinder with Remeshing

- Aluminium cylinder subjected to enforced disp..
- Pure shear at the initial stage, but stretch dominates at the later stage.
- Necking occurs in the end.

Valid plastic strain dist. is obtained after many time remeshings.



Final stretch at the neck is more than 7000%.

Characteristics of F-barES-FEM-T4

✓ No increase in DOF.
(No Lagrange multiplier. No static condensation.)

✓ Locking- & checkerboarding-free with T4 mesh.

✗ Higher costs in memory and CPU time
due to wider bandwidth of $[K]$.

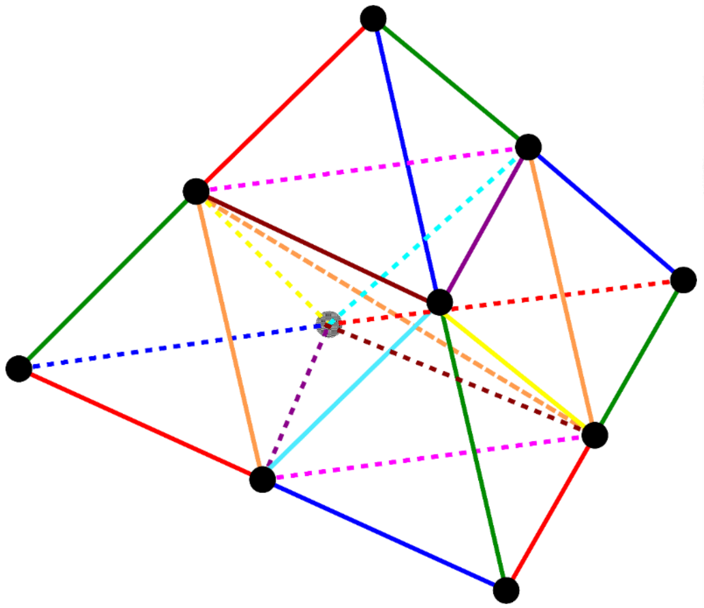
In case of standard unstructured T4 meshes:

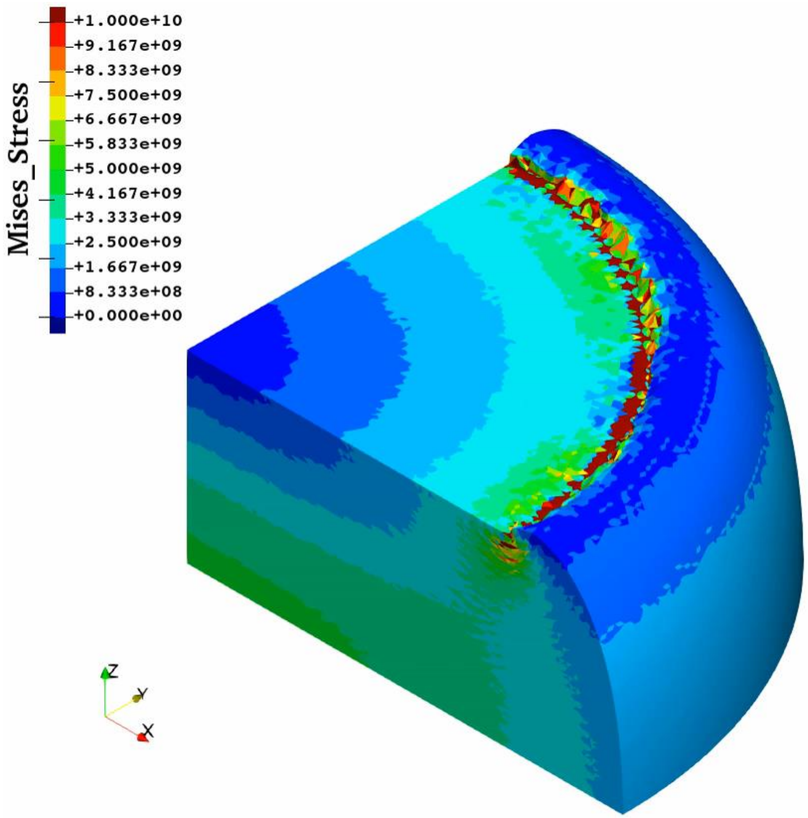
Method	Approx. Bandwidth	Approx. Ratio
Standard FEM-T4	40	1
F-barES-FEM-T4(1)	390	x10
F-barES-FEM-T4(2)	860	x20

Another approach should be addressed
for full industrial applications.

Concept of SelectiveCS-FEM-T10

- Our new another approach using T10 mesh.
- Same memory & CPU costs as the T10 elements.
- Details of SelectiveCS-FEM-T10 will be presented soon (10:40~) in the S-FEM MS at Room D today!!

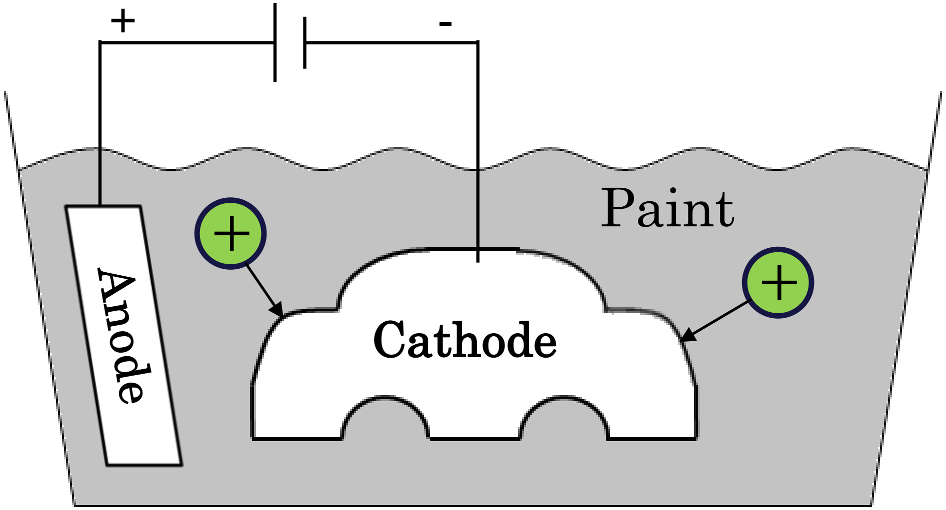




Part 2:
Practical Progress of S-FEM in Electrodeposition Process Simulation of Auto Car Bodies

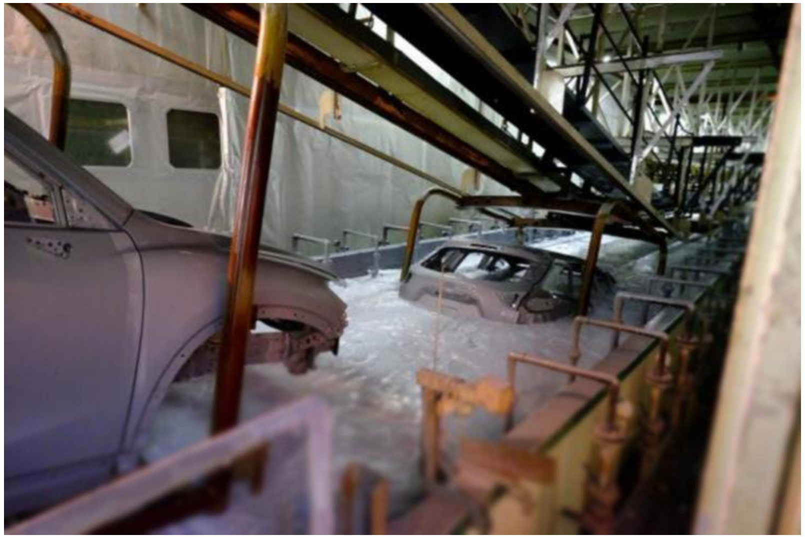
What is Electrodeposition (ED) ?


http://n-link.nissan.co.jp/NOM/PLANT/



- Most widely-used basecoat methods for car bodies.
- Making coated film by applying direct electric current (up to 300 V) in a paint pool.
- Relatively good at making uniform film thickness but not satisfactory uniform in actual production lines.
- ED simulator is necessary for the optimization of carbody design and coating conditions in actual lines.

Photos of ED Process Line



1. dipping and deposition process

2. water rinse process

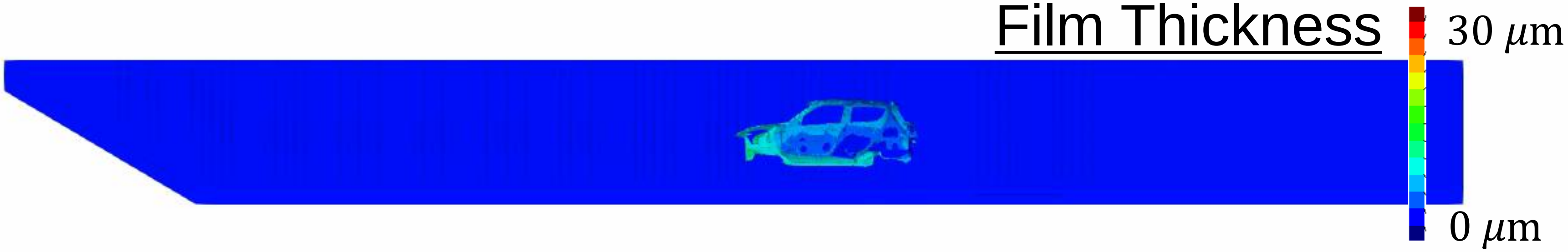
We focus on this process.

3. baking process

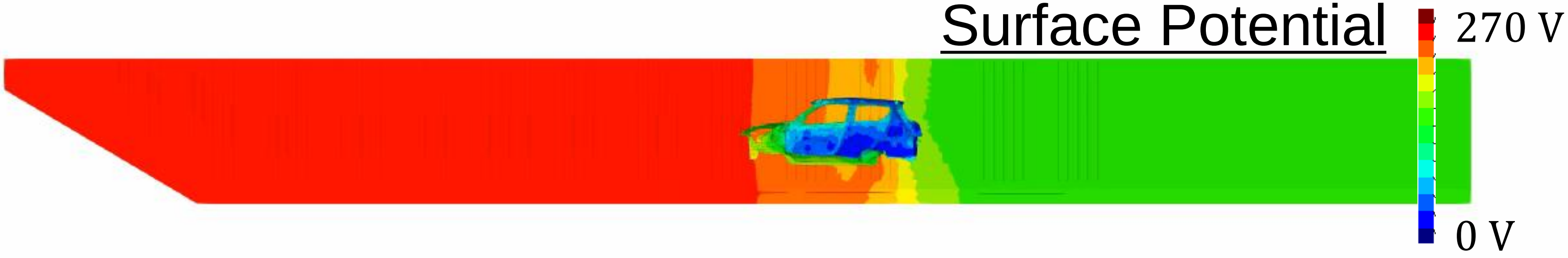
What is ED Simulation ?

ED simulation calculates film thickness derived from surface potential and current density with moving boundaries.

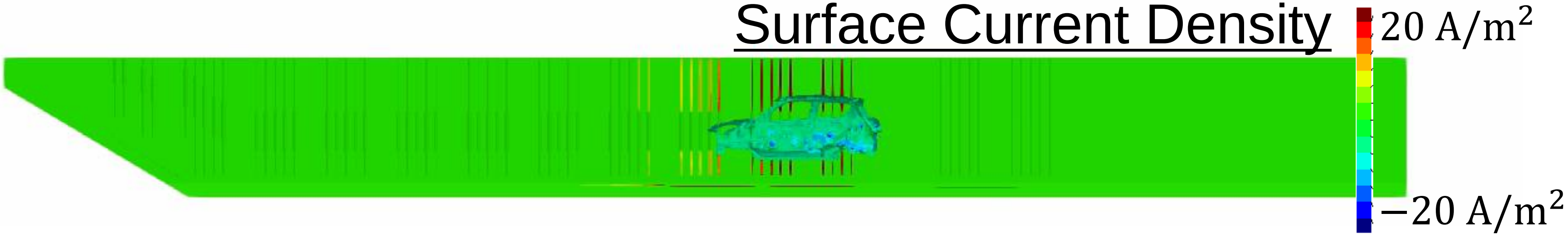
Film Thickness



Surface Potential

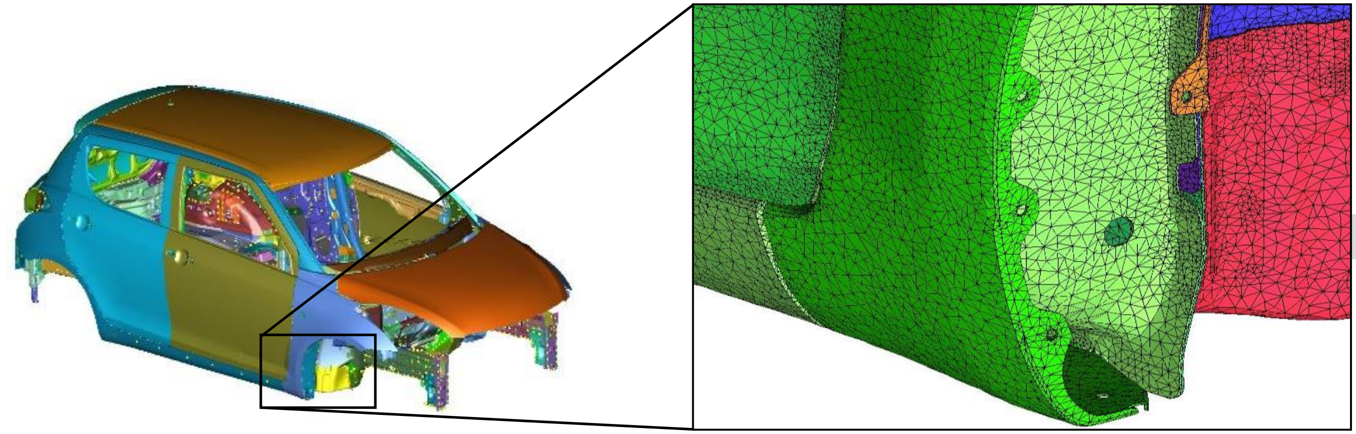


Surface Current Density

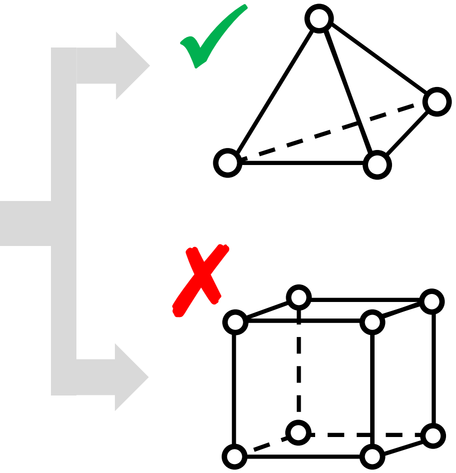


Issues in Meshing (1)

✗ It is difficult to discretize complex shapes such as car bodies with **hexahedral meshes**.



Only surface mesh is shown.
Many holes exist on body plates



→ We have to use **tetrahedral meshes** in ED simulation.

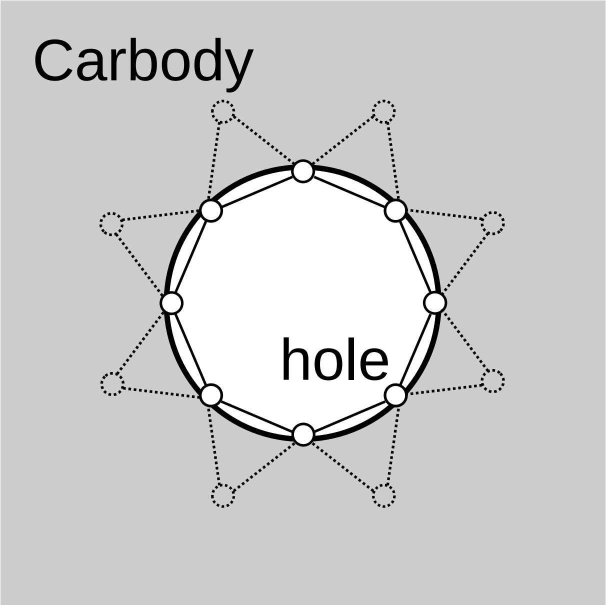
However…

Accuracy of the standard FEM-T4 is insufficient in complex shapes.

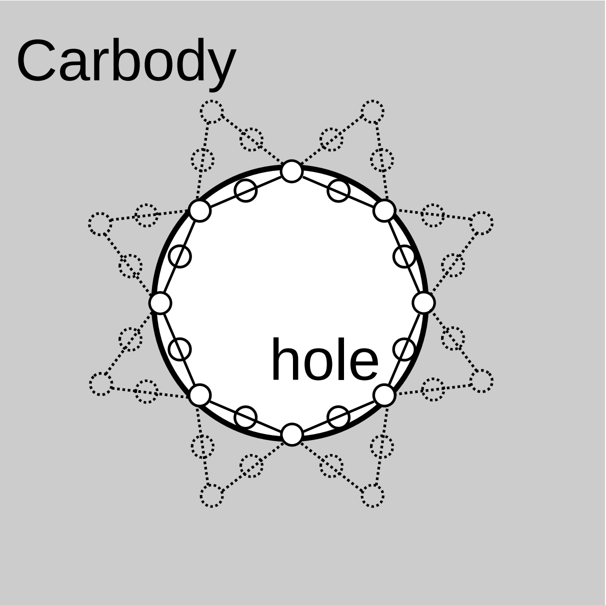
Issues in Meshing (2)

✗ 10-node tetrahedral (T10) mesh without kink generally requires more large number of nodes than T4 mesh.

T4



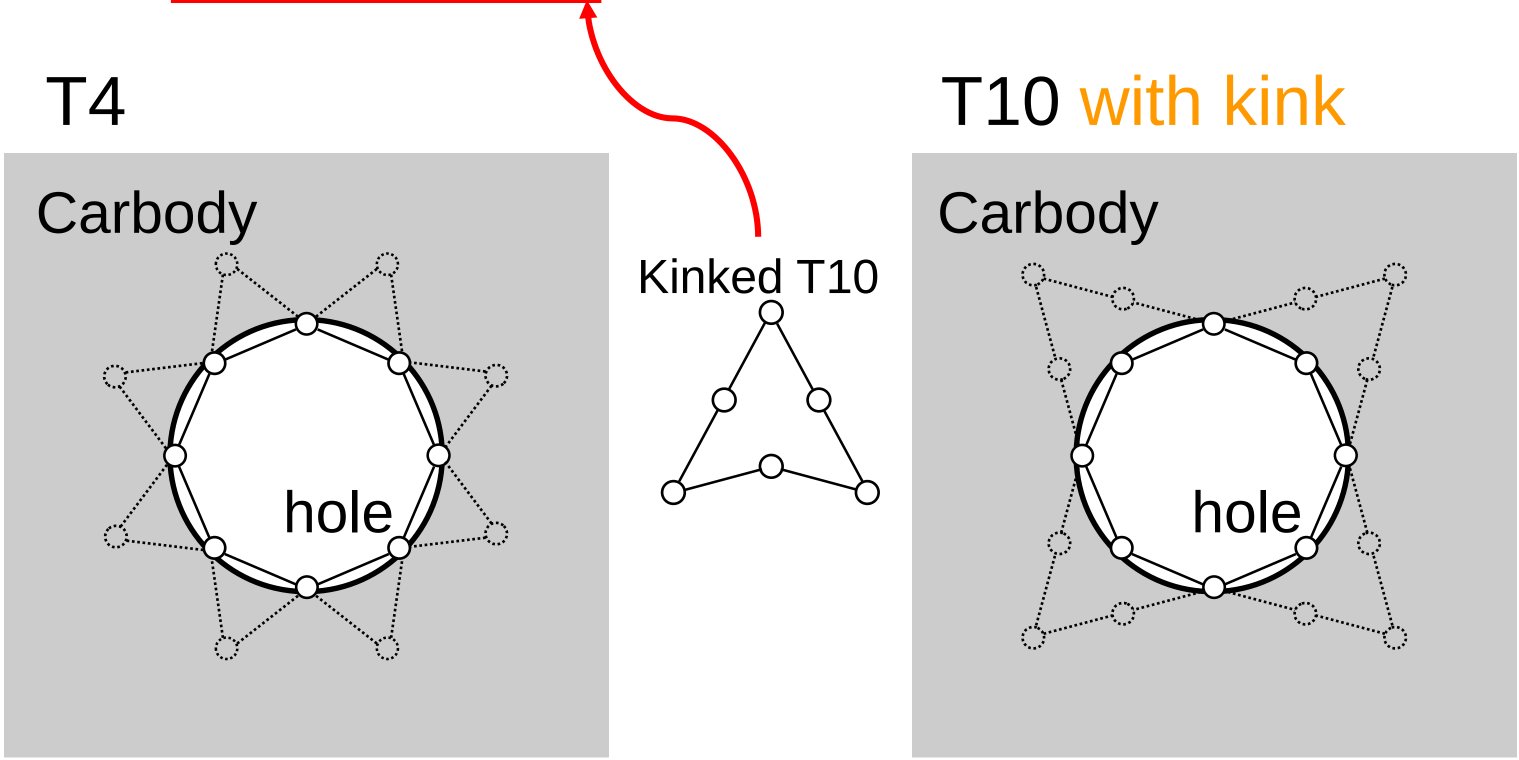
T10 without kink



For the same shape representation, T10 mesh without kink leads to massive increase in DOF.

Issues in Meshing (2 Cont.)

✗ 10-node tetrahedral (T10) mesh with kink causes severe accuracy loss.



T10 mesh with kink does not increase DOF but induces severe accuracy loss.

Motivation

- ■ Hexahedral elements:
  - ✗ It is difficult to discretize complex shapes.
- ■ T10 elements without kink:
  - ✗ It leads to massive increase in DOF.
- ■ T10 elements with kink:
  - ✗ It causes severe accuracy loss.

→ We want to realize high accuracy analysis with T4 mesh.

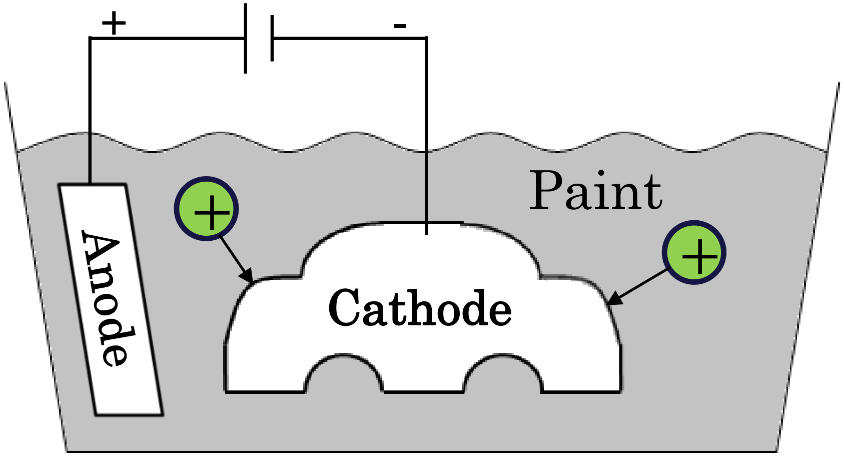
ES-FEM-T4 could be a solution to these issues.

Formulation of ES-FEM for ED Simulation

Fundamental Equations

Governing equation

The electrostatic Laplace equation, $\nabla^2 \phi = 0$, in the paint pool domain.



Boundary conditions (BCs)

1. Insulation BC
2. Anodic (Electrode surface) BC
3. Cathodic (Carbody surface) BC

ED boundary models are identified with lab experiments.

The role of ES-FEM is to solve the Laplace equation for each timestep with the iterative solver (MINRES).

Outline of ES-FEM

What is ES-FEM-T4?

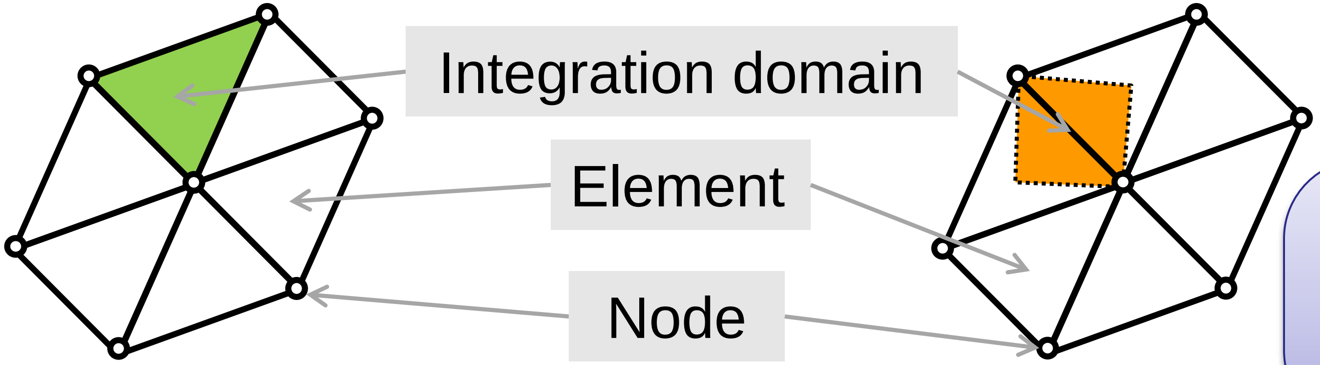
- A kind of strain smoothing method.
- Using element edges as Gauss points.
- Robust against element skew.
- Super-linear mesh convergence rate with T4 mesh.

Standard FEM

assembles each element's value.

ES-FEM

assembles each edge's value.

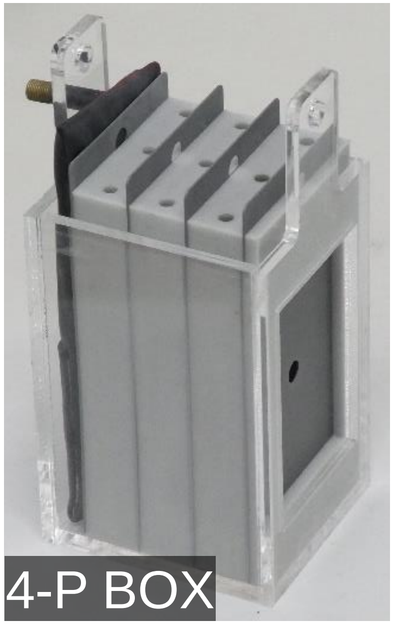


Integration domain is different !!

Analysis Results

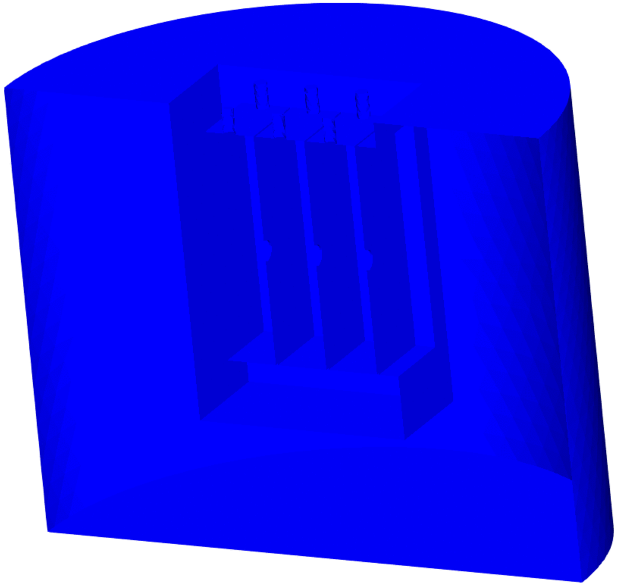
4-Plate BOX Simulation

Outline



(http://plaza.rakuten.co.jp/heroc)

Film Thickness

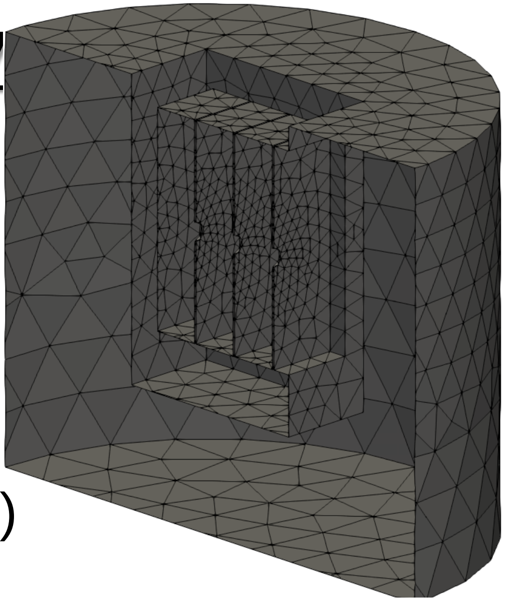


- Imitating a bag-like structure such as side sill in a carbody.
- Accuracy on the innermost surface (leftmost plate surface) is the most important; i.e., “maximize the minimum”.
- Film thickness is calculated with 4 different mesh seed sizes and compared between FEM-T4 and ES-FEM-T4.

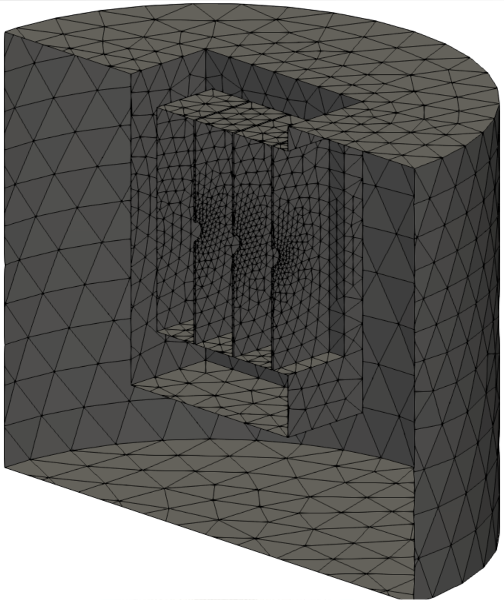
4-Plate BOX Simulation

Overview of Meshes

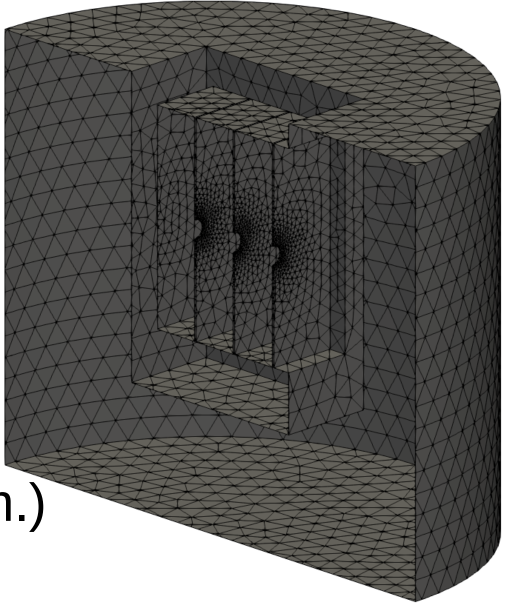
Only the surface meshes are shown.



3.2 mm Mesh
Seed Size
(31k T4 elem.)



1.6 mm Mesh
Seed Size
(65k T4 elem.)

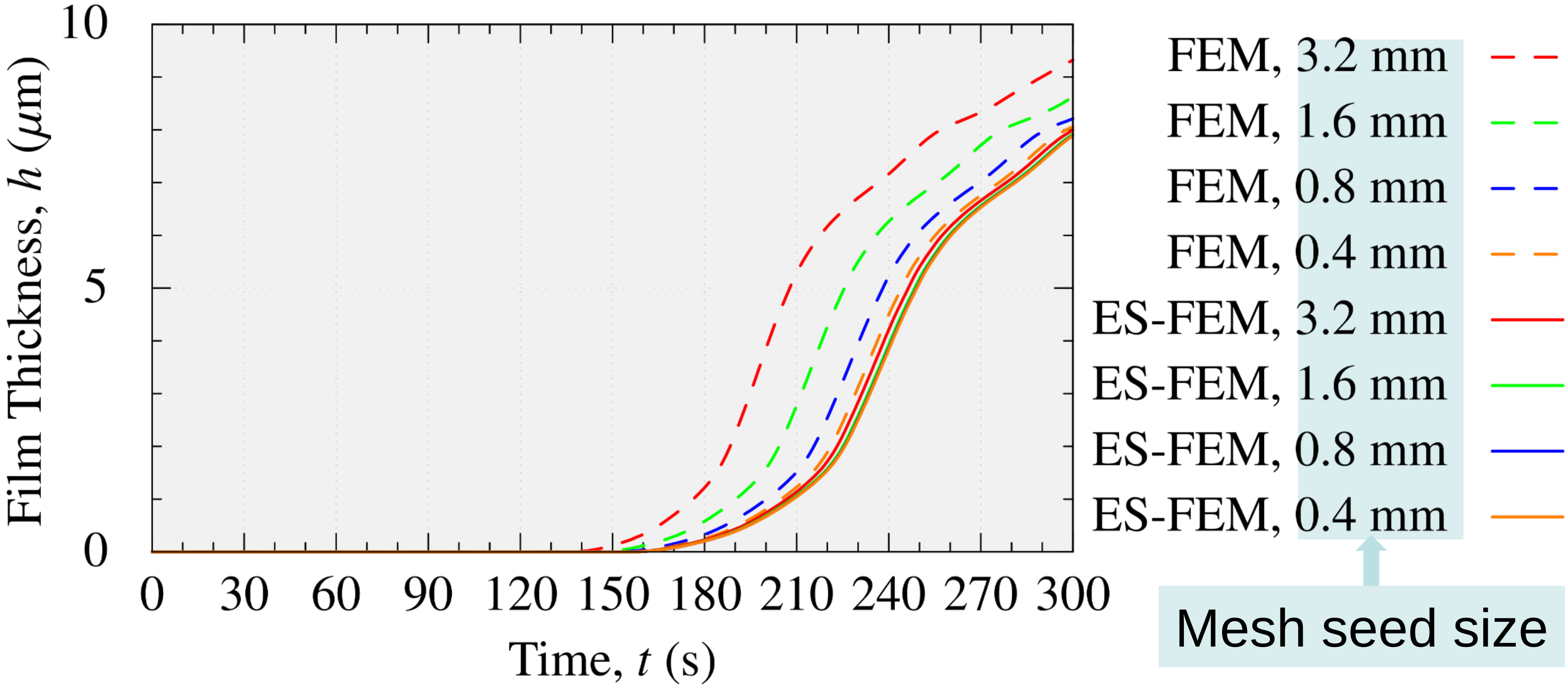


0.8 mm Mesh
Seed Size
(169k T4 elem.)

0.4 mm Mesh
Seed Size
(716k T4 elem.)

4-Plate BOX Simulation

Film Thickness of G-Plate (innermost surface)

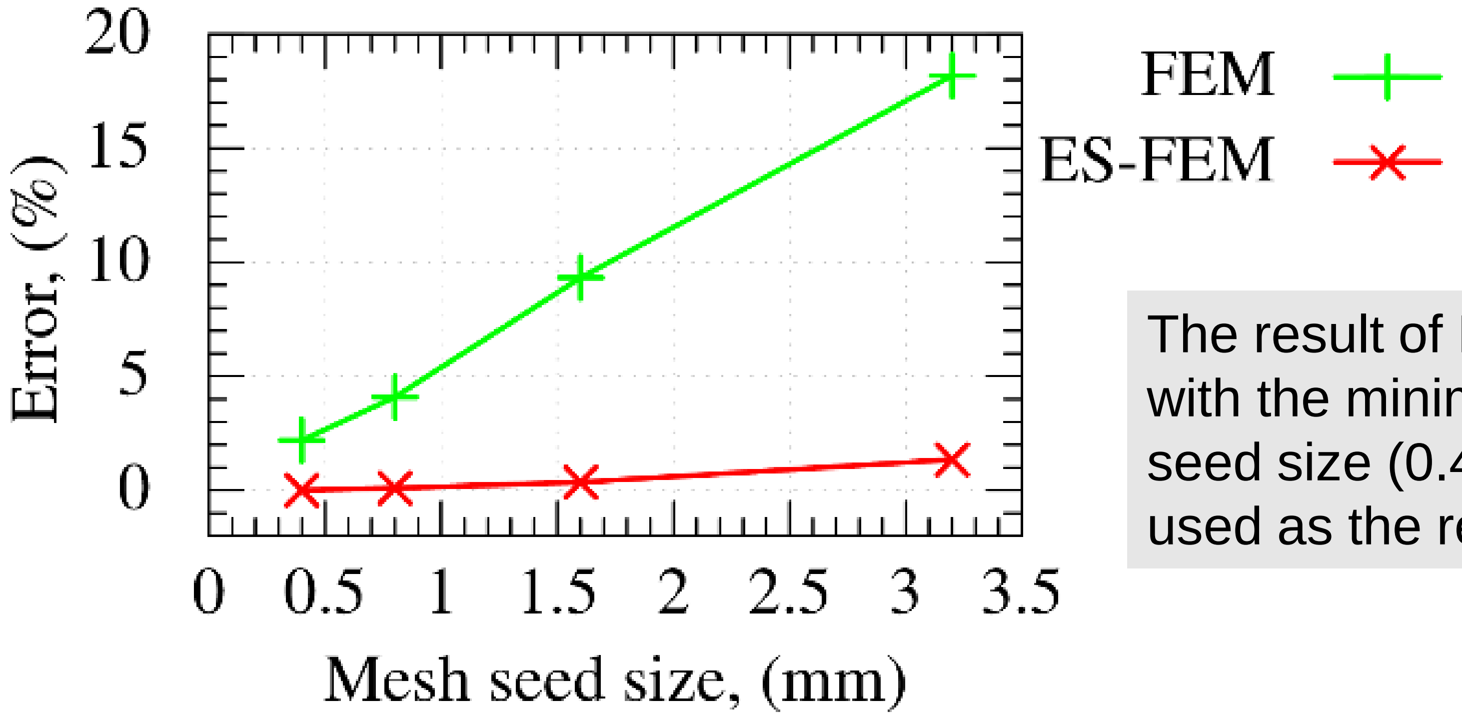


Mesh seed size

FEM results (dashed lines) have ***large*** errors
due to mesh coarseness.
Meanwhile, ES-FEM (solid lines) results have no such errors.

4-Plate BOX Simulation

Error of Final Film Thickness on G-Plate

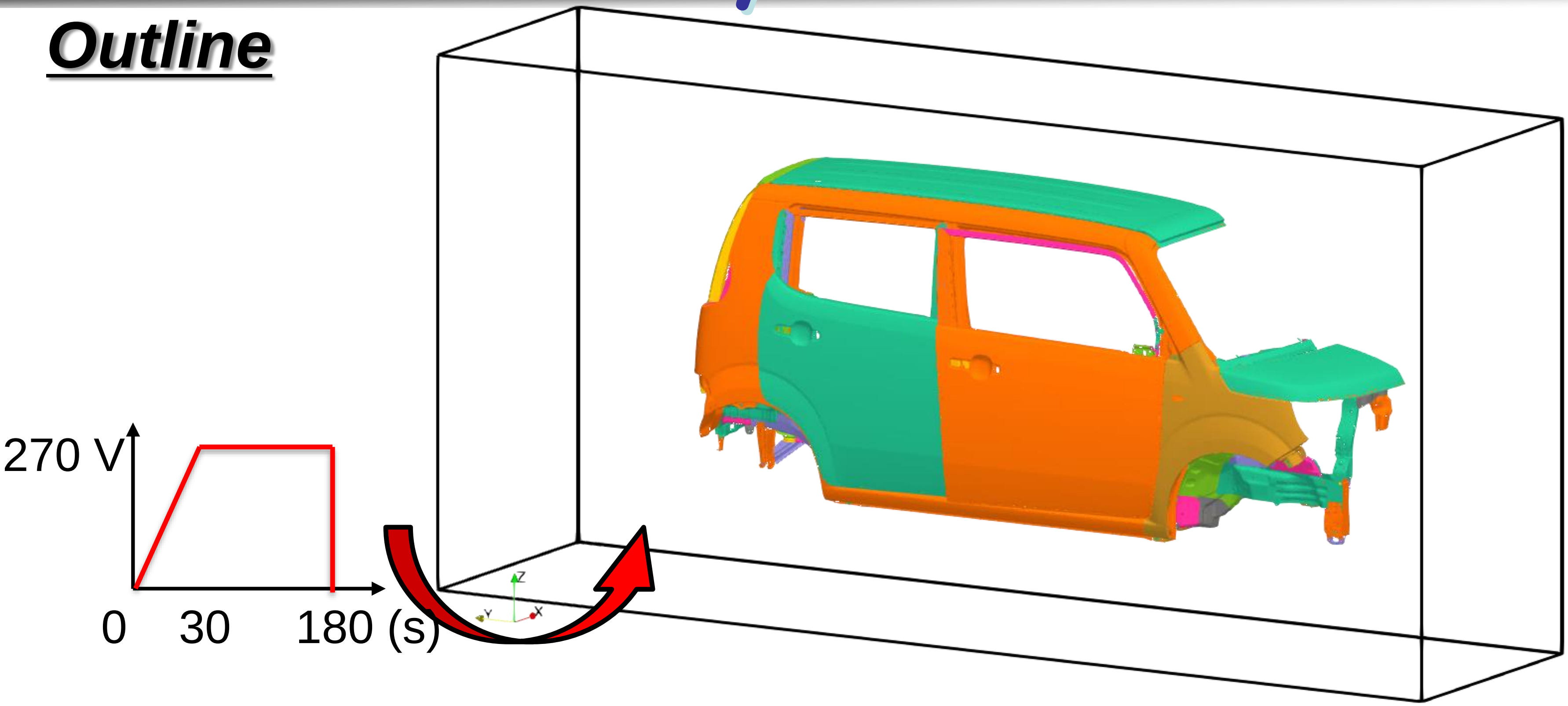


The result of ES-FEM with the minimum mesh seed size (0.4 mm) is used as the reference.

ES-FEM-T4 has far better mesh convergence rate than FEM-T4 !!

Carbody Simulation

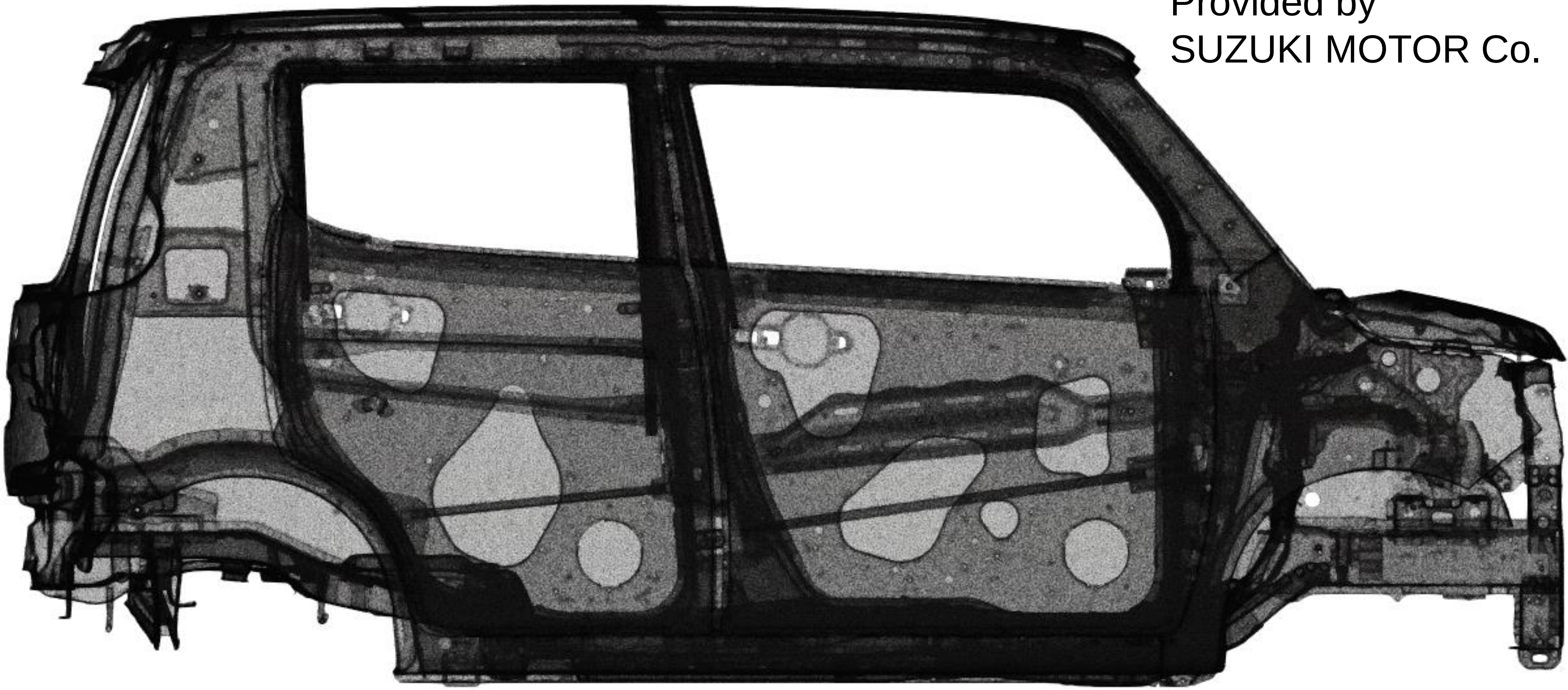
Outline



- A half carbody fixed in a box pool.
- The side wall is treated as an anode surface.
- Compare the time-developed film thickness between FEM-T4 and ES-FEM-T4 with a same mesh.

Carbody Simulation

Overview of Surface Mesh

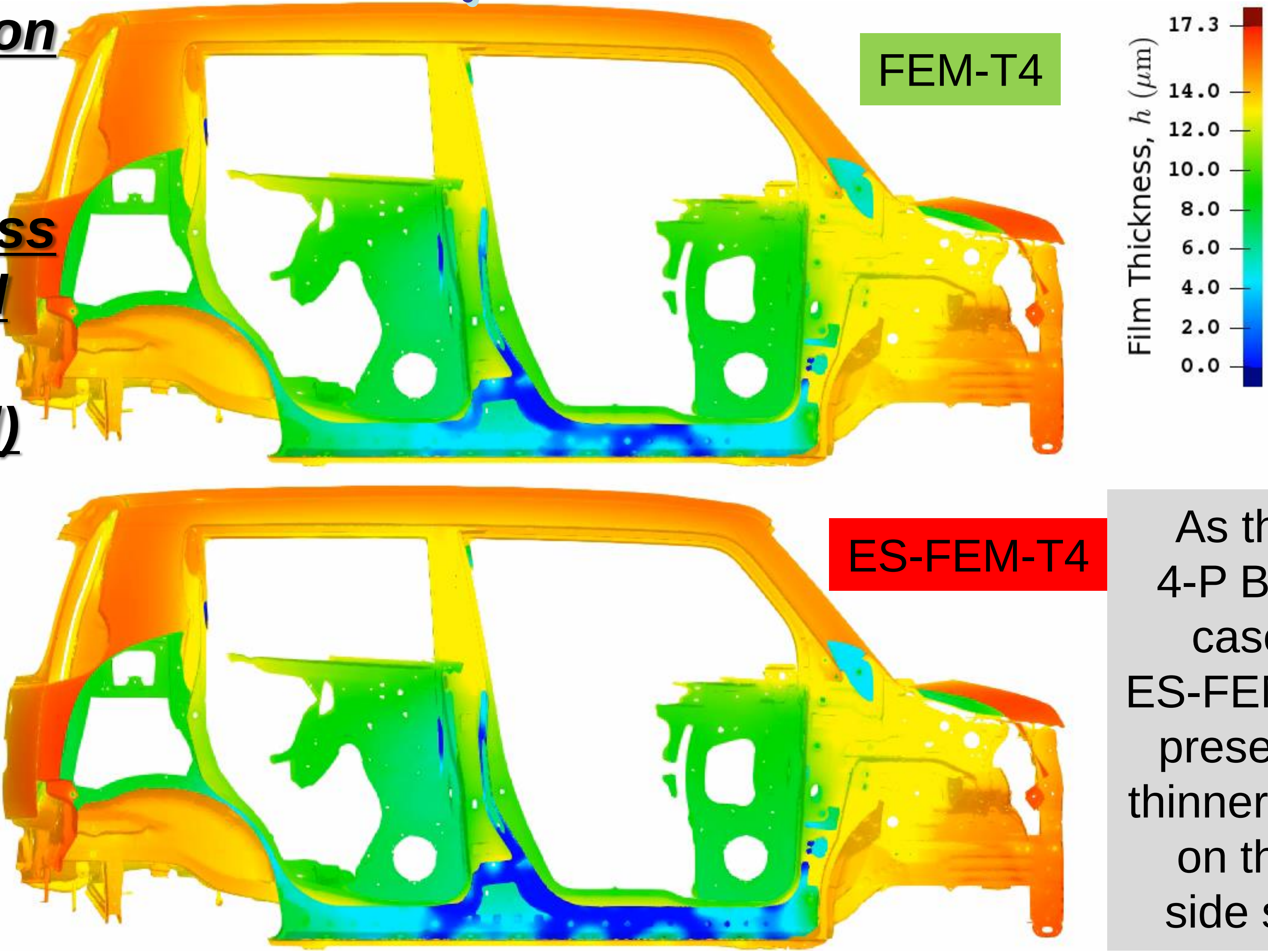


Provided by
SUZUKI MOTOR Co.

- 13M T4 elements (3M nodes & 18M edges) in total in the pool.

Carbody Simulation

Animation of Film Thickness (Clipped View on Side Sill)



As the 4-P BOX case, ES-FEM-T4 presents thinner dist. on the side sill.

Big difference appears on the inner surfaces.

Comparison of Computational Costs

Calculation Time

on a PC with Intel i9-9960X using 10 cores

	FEM-T4	ES-FEM-T4
4-P BOX with 3.2 mm mesh	0.02 h	0.02 h
4-P BOX with 1.6 mm mesh	0.04 h	0.05 h
4-P BOX with 0.8 mm mesh	0.45 h	0.45 h
4-P BOX with 0.4 mm mesh	9.5 h	9.0 h
Carbody	67 h	125 h

Same Accuracy

In case iterative solvers can be used,
there is no big difference in calculation time
although the accuracy of ES-FEM-T4 is much better.

Summary

Benefits and Drawbacks of S-FEMs

Benefits

- ✓ No increase in DOF.

  Purely displacement-based formulation.

- ✓ Locking- & checkerboarding-free with T4 mesh.

  No difficulty in severe strain or contact analysis.

- ✓ Super-linear mesh convergence rate.

  Suitable to for industrial problems with complex shape.

Drawbacks

- ✗ Larger memory consumption.

  Wider matrix bandwidth as T10 element with T4 mesh.

Take-Home Messages

- ES-FEM-T4 is already in practice as an accurate solver using T4 meshes to overcome the slow mesh convergence rate of the standard FEM-T4.
- F-barES-FEM-T4 is the current best T4 FE formulation especially for the large deformation of rubber-like materials, viscoelastic materials, and elastoplastic materials.

Therefore, its practical use will be start shortly.

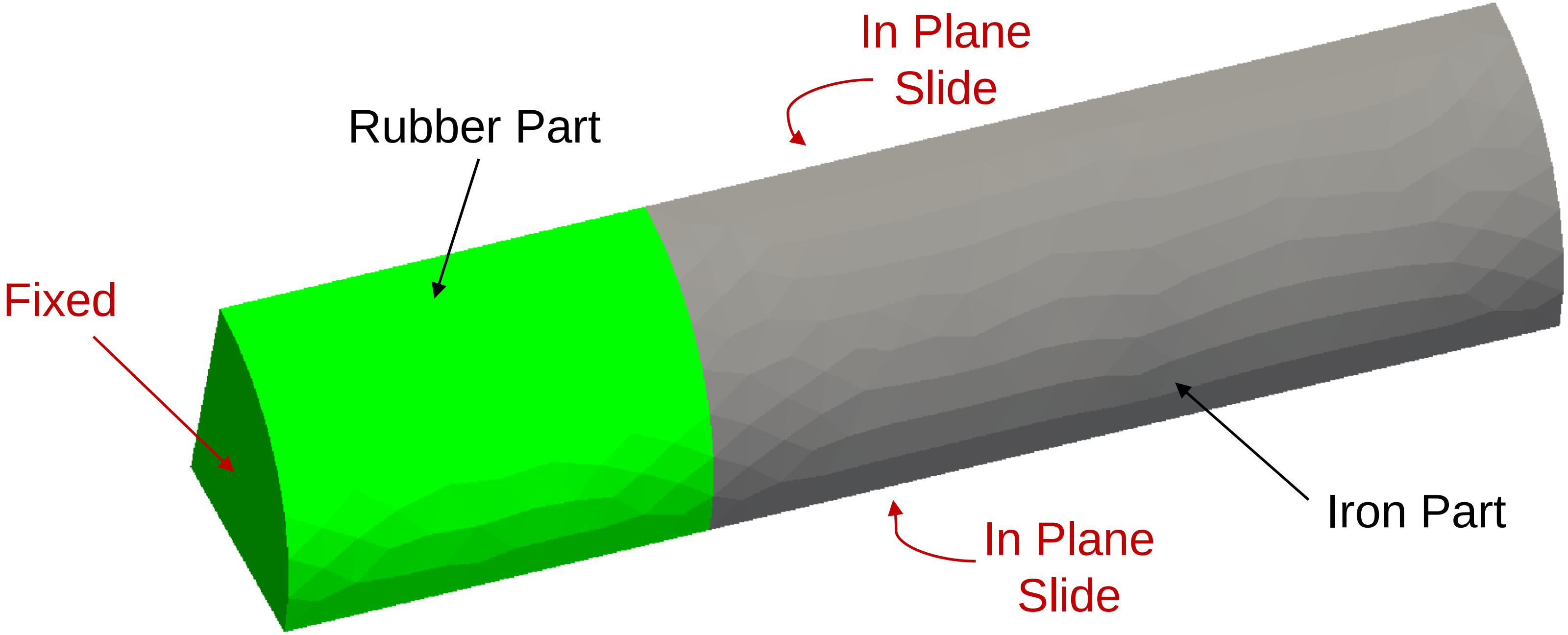
Details of SelectiveCS-FEM-T10 will be presented soon (10:40~) in the S-FEM MS at Room D today!!

Thank you for your kind attention!

Appendix

Natural Modes of ¼ Cylinder

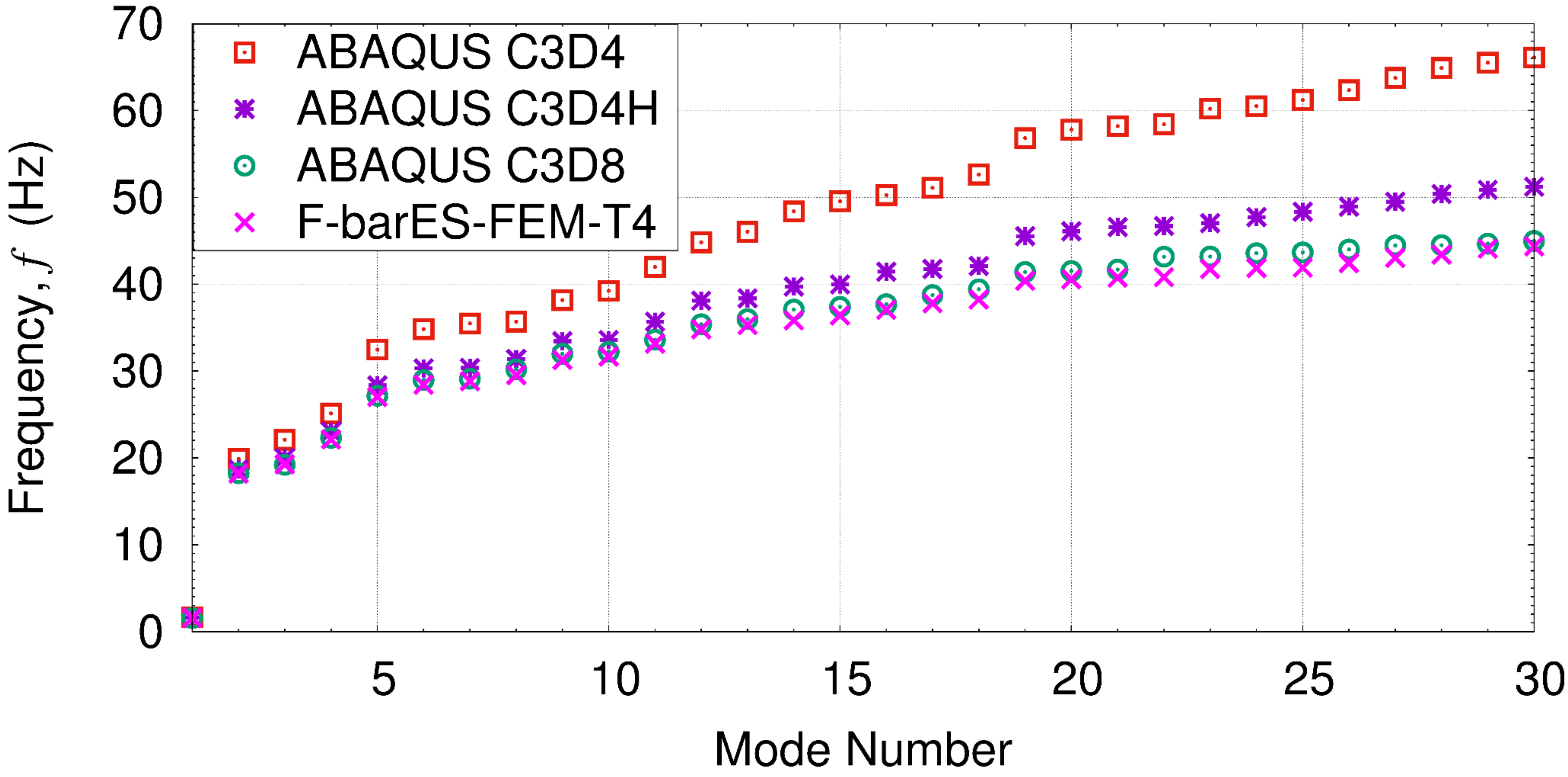
Outline



- Iron part: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{ini} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Elastic, **No cyclic smoothing.**
- Rubber part: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{ini} = \mathbf{0.499},\ \rho = 920\ \mathrm{kg/m^3}$, Elastic, **2 cycles of smoothing.**
- Compared to ABAQUS C3D4, C3D4H, and C3D8.

Natural Modes of ¼ Cylinder

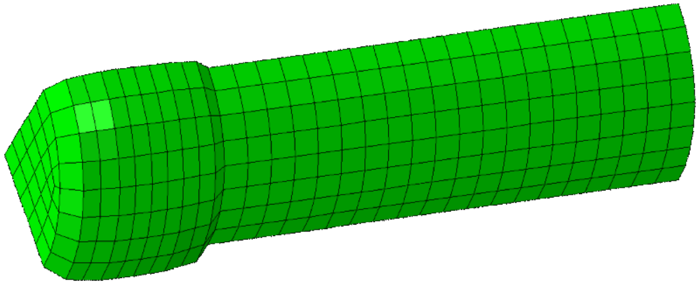
Eigen frequencies



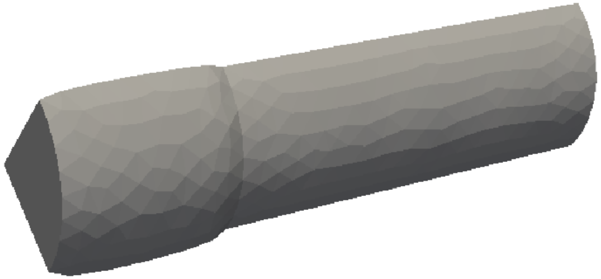
- C3D4 and C3D4H show higher frequencies (stiffer results).
- F-barES-FEM-T4 and C3D8 are in good agreement.

Natural Modes of $\frac{1}{4}$ Cylinder

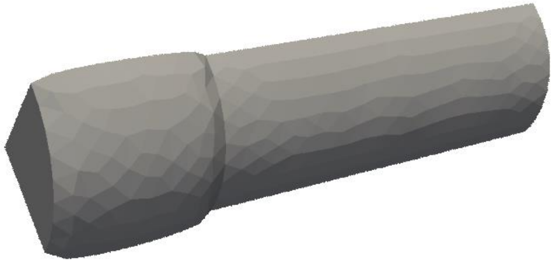
1st eigen mode



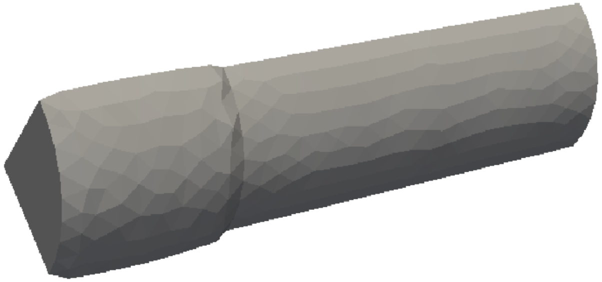
ABAQUS C3D8
(reference)



Selective ES/NS-FEM-T4



F-barES-FEM-T4

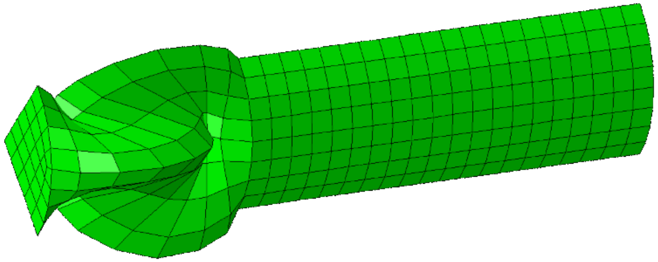


NS-FEM-T4

The 1st modes are all the same as the reference solution.

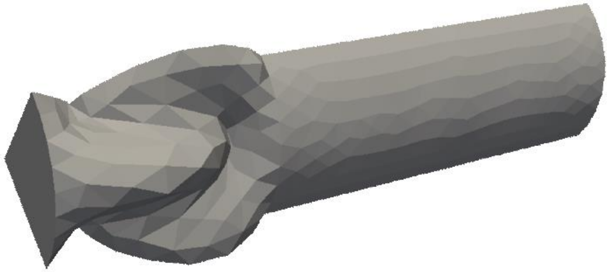
Natural Modes of ¼ Cylinder

11th eigen mode



ABAQUS C3D8
(reference)

Selective ES/NS-FEM-T4



F-barES-FEM-T4

NS-FEM-T4

- NS-FEM-T4 shows strange results due to low-energy mode.
- Selective ES/NS-FEM-T4 & F-barES-FEM-T4 are valid.