Accurate viscoelastic large deformation analysis using F-bar aided edge-based smoothed finite element method for 4-node tetrahedral meshes (F-barES-FEM-T4)

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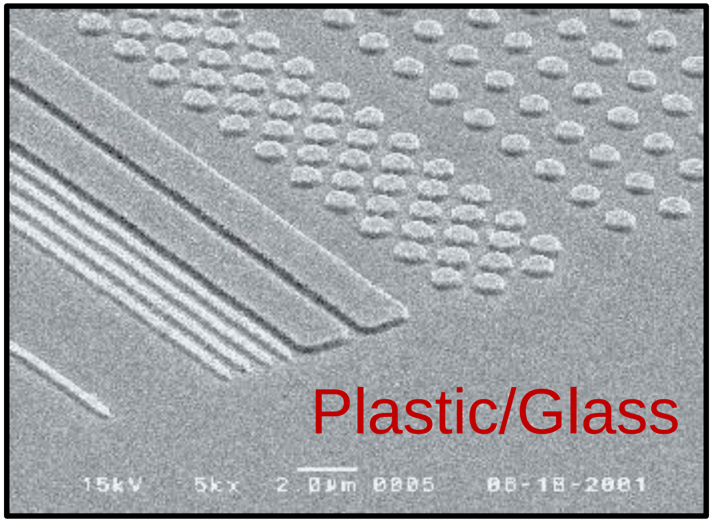
Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



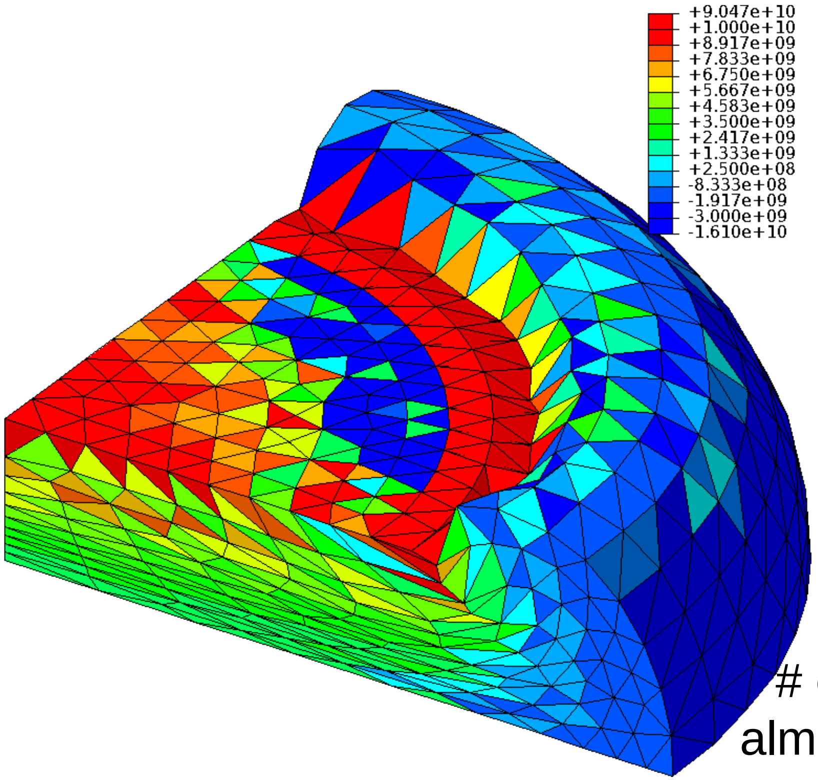
Issues

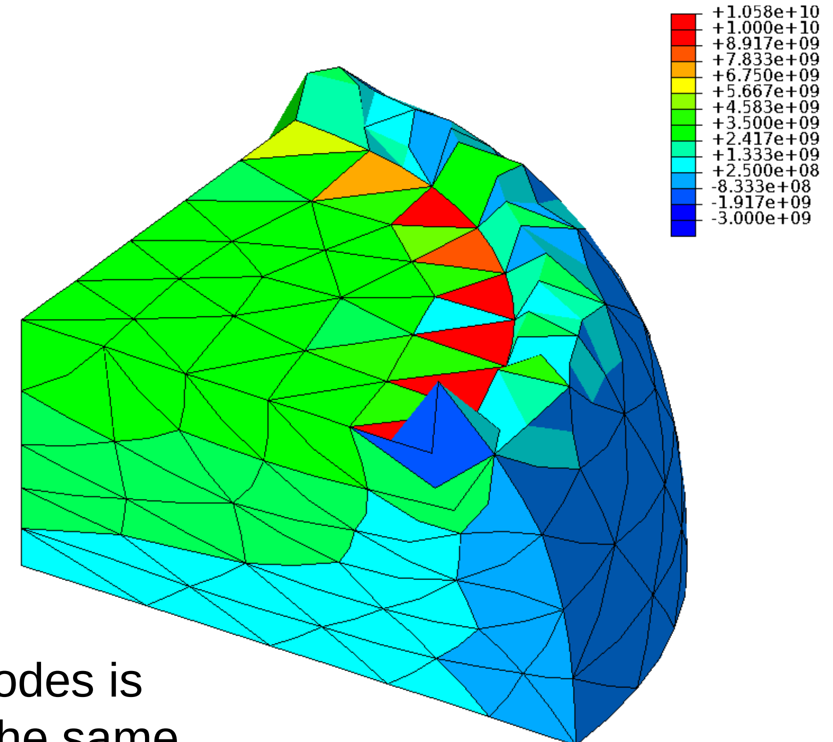
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
    Accuracy loss in large strain due to intermediate nodes.
- Enhanced assumed strain method (EAS):
  - ✗ Spurious low-energy modes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding, Early convergence failure etc..

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

A Recent Solution

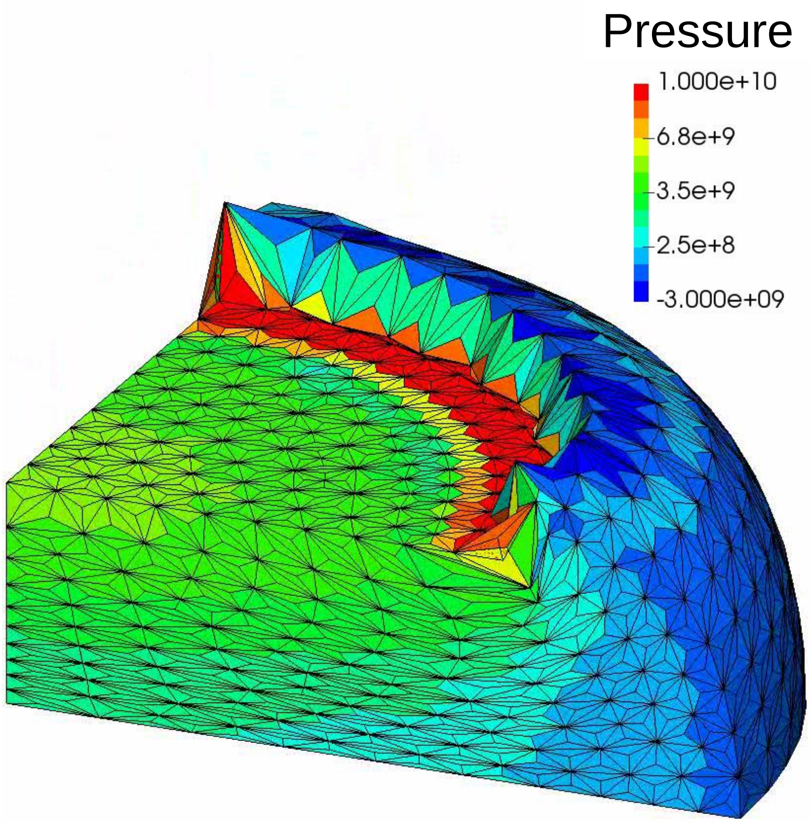
A new idea of FE formulation called "Smoothed Finite Element Method (**S-FEM**)" was recently proposed and is in researching today widely.

Our group has proposed a latest S-FEM named "**F-barES-FEM-T4**" (detailed later):

- No intermediate node & No additional DOF, (i.e., Purely displacement-based 4-node tetrahedral (T4) element),
- Free from shear, volumetric and corner locking,
- No pressure checkerboarding,
- Long lasting in large deformation.

A Recent Solution (cont.)

e.g.1) Compression of ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$



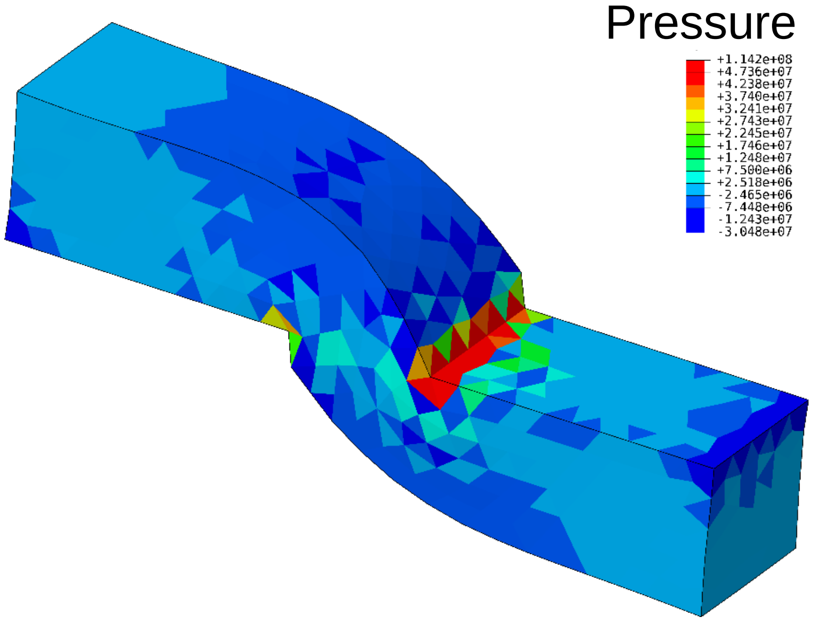
Same mesh as C3D4H case.

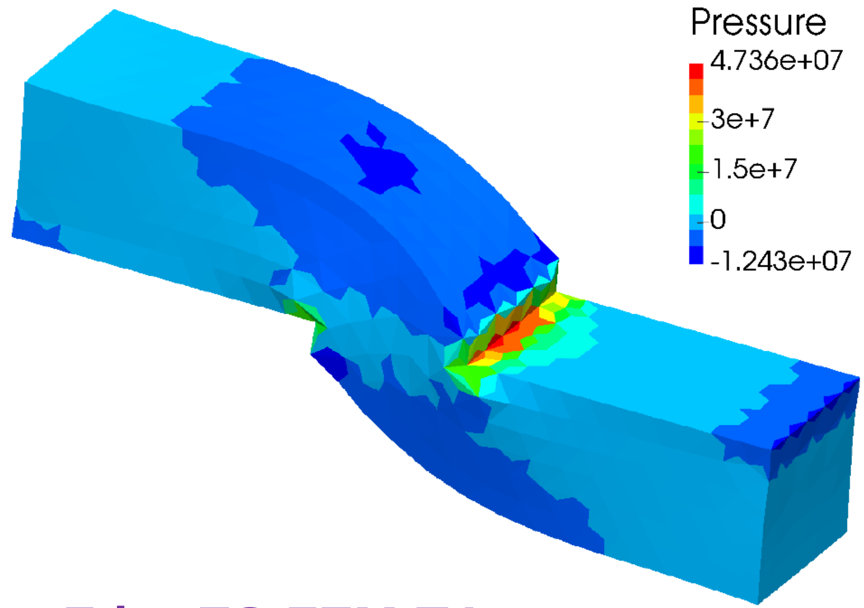
F-barES-FEM-T4 (One of the latest S-FEM)

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding

A Recent Solution (cont.)

e.g.2) Shear of ***elastoplastic*** body with soft hardening coeff.





1st order hybrid T4 (C3D4H)
- ✓ No volumetric locking
- ✗ Shear locking
- ✗ Pressure checkerboarding

F-barES-FEM-T4
- ✓ No volumetric locking
- ✓ No shear locking
- ✓ No pressure checkerboarding

We have evaluated F-barES-FEM-T4 in elastic and elastoplastic cases but NOT in viscoelastic cases yet.

Objective

To apply and demonstrate
the latest S-FEM called **F-barES-FEM-T4**
to viscoelastic large deformation problems.

Table of Body Contents

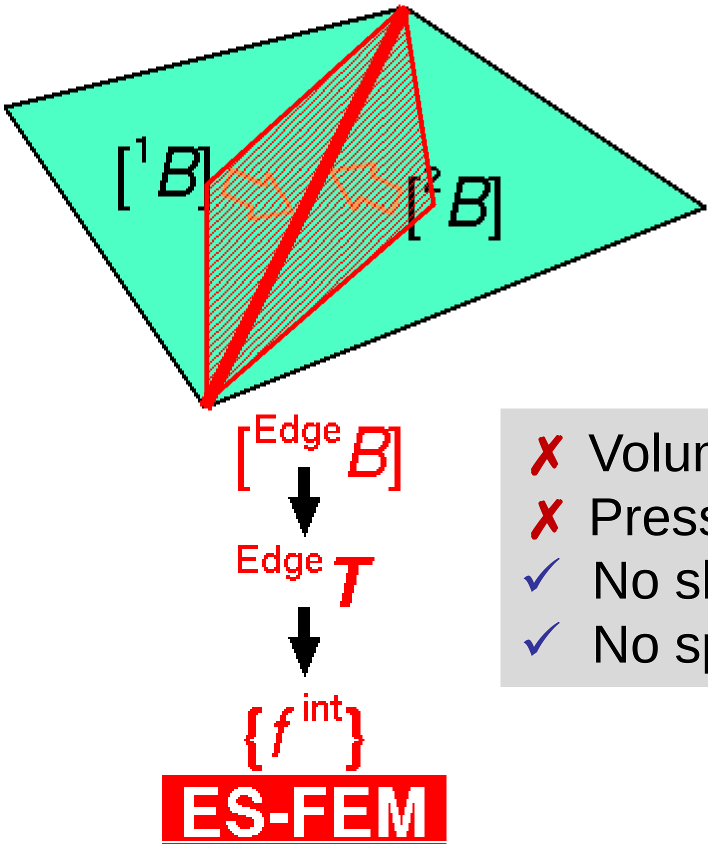
- Introduction of **F-barES-FEM-T4**’s formulation
- Demonstration of **F-barES-FEM-T4**
  in viscoelastic problems
- Summary

Introduction of F-barES-FEM-T4's formulation

1. Brief of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each edge smoothing domain.

As if putting an integration point on each edge center

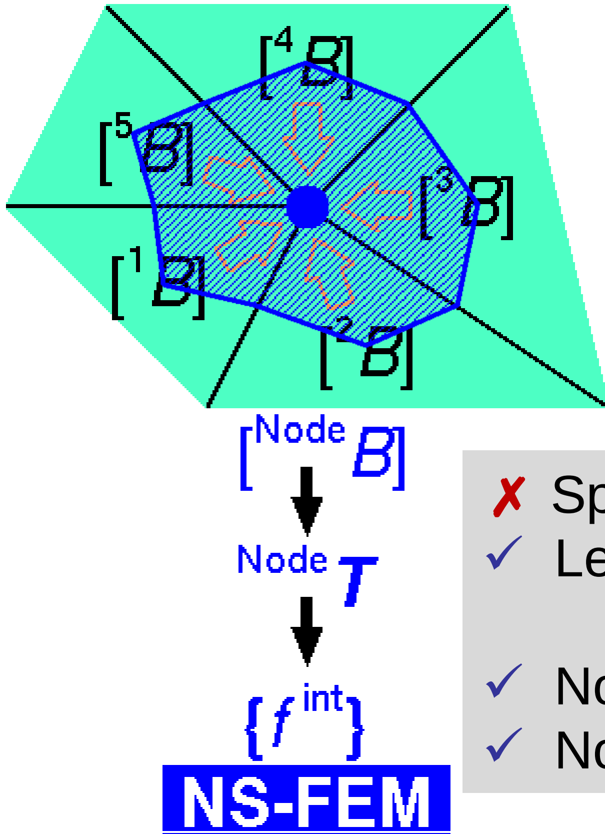


- ✗ Volumetric locking
- ✗ Pressure checkerboarding
- ✓ No shear locking
- ✓ No spurious modes

2. Brief of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[{}^{\mathrm{Node}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each node smoothing domain.

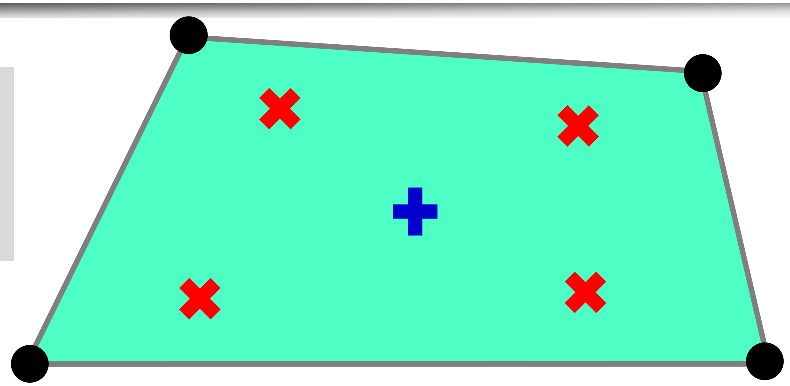
As if putting an integration point on each node



- ✗ Spurious low-energy mode
- ✓ Less pressure checkerboarding
- ✓ No shear locking
- ✓ No volumetric locking

3. Brief of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}$ $(= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\text{iso}}$ $(= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point to obtain $\bar{\boldsymbol{F}}$ as

$$\bar{\boldsymbol{F}} = \bar{J}^{1/3}\, \boldsymbol{F}^{\text{iso}}.$$

A kind of low-pass filter for J

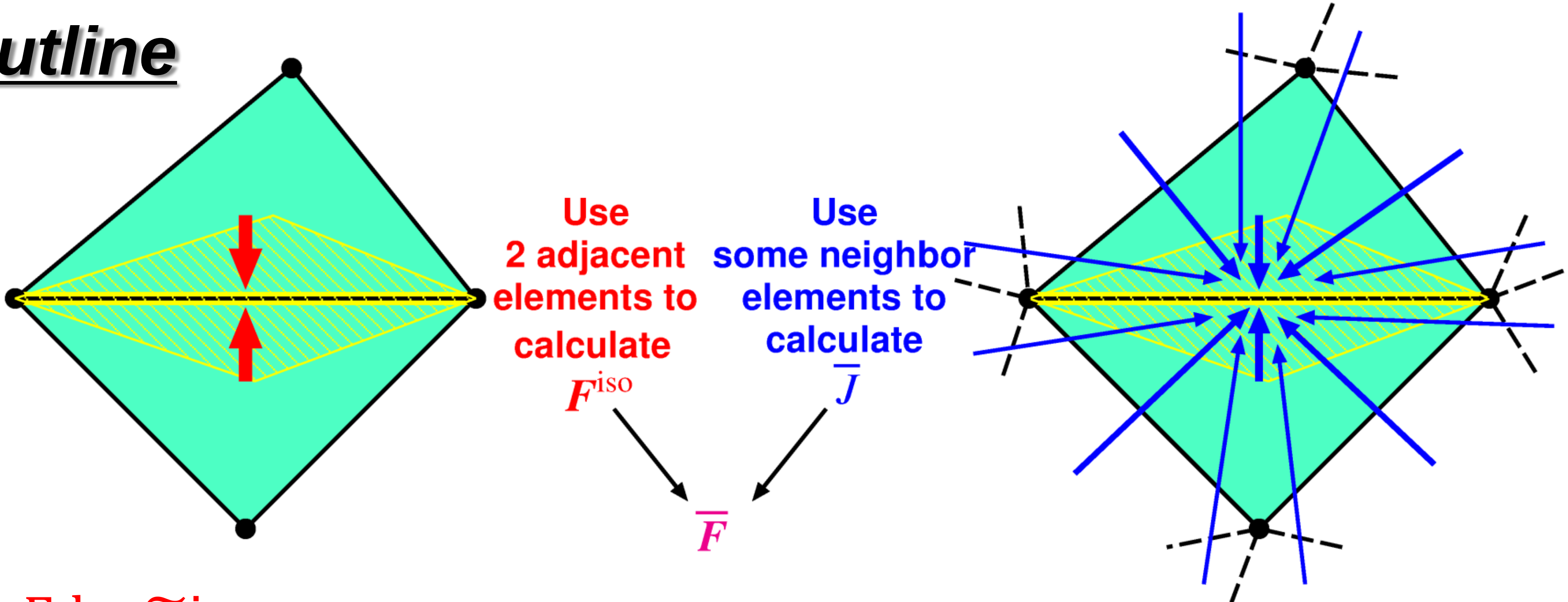
4. Use $\bar{\boldsymbol{F}}$ to calculate the stress $\boldsymbol{T}$, nodal force $\{f^{\text{int}}\}$ etc..

- ✗ Shear locking
- ✓ Less pressure checkerboarding
- ✓ No volumetric locking
- ✓ No spurious modes

Outline of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

Outline



- ${}^{\mathrm{Edge}}\tilde{F}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\bar{J}$ is given by cyclically (*c* times) applied NS-FEM.
- ${}^{\mathrm{Edge}}\bar{F}$ is calculated in the manner of F-bar method:

$$ {}^{\mathrm{Edge}}\bar{F} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\tilde{F}^{\mathrm{iso}}. $$

How to Treat Viscoelastic Model

The target constitutive model to treat is the **Hencky's viscoelastic model** based on the **generalized Maxwell model**.

The most standard one.

- Stress

Bulk modulus — Hencky (Logarithmic) strain

$$\begin{cases} \boldsymbol{T}^{\mathrm{hyd}} = K\,\mathrm{tr}(\boldsymbol{H})\,\boldsymbol{I}, \\ \boldsymbol{T}^{\mathrm{dev}} = 2G_0\left(\boldsymbol{H}^{\mathrm{dev}} - \sum g_i \boldsymbol{H}_i^{\mathrm{v}}\right). \end{cases}$$

Viscosity only in deviatoric stress.

Instantaneous shear modulus — Prony coeff — Viscous strain

- Time advance of viscous strain

$$\boldsymbol{H}_i^{\mathrm{v}+} = \boldsymbol{R} \cdot \boldsymbol{H}_i^{\mathrm{v}} \cdot \boldsymbol{R}^{\mathrm{T}} + \Delta \boldsymbol{H}_i^{\mathrm{v}}.$$

Rigid rotation in an increment — Viscous strain increment

- Equation to solve

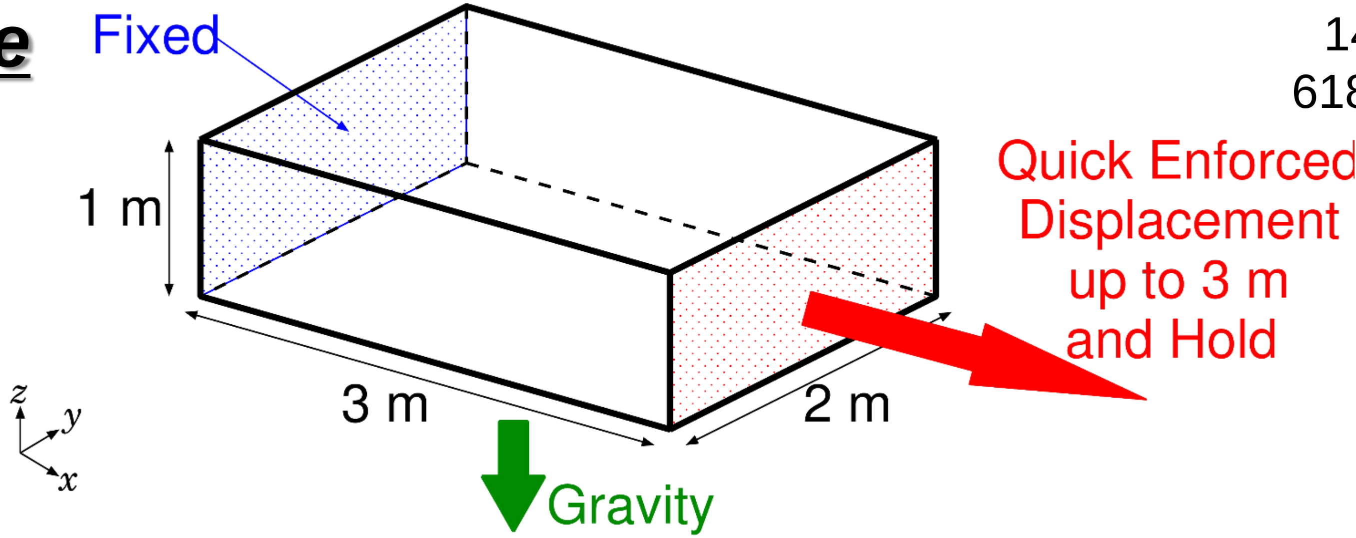
$$[\boldsymbol{K}]\{\boldsymbol{u}\} = \{\boldsymbol{f}\}$$

Same as static problems due to the absence of inertia.

Demonstration of F-barES-FEM-T4 in viscoelastic problems

Tensile Suspension of Viscoelastic Block

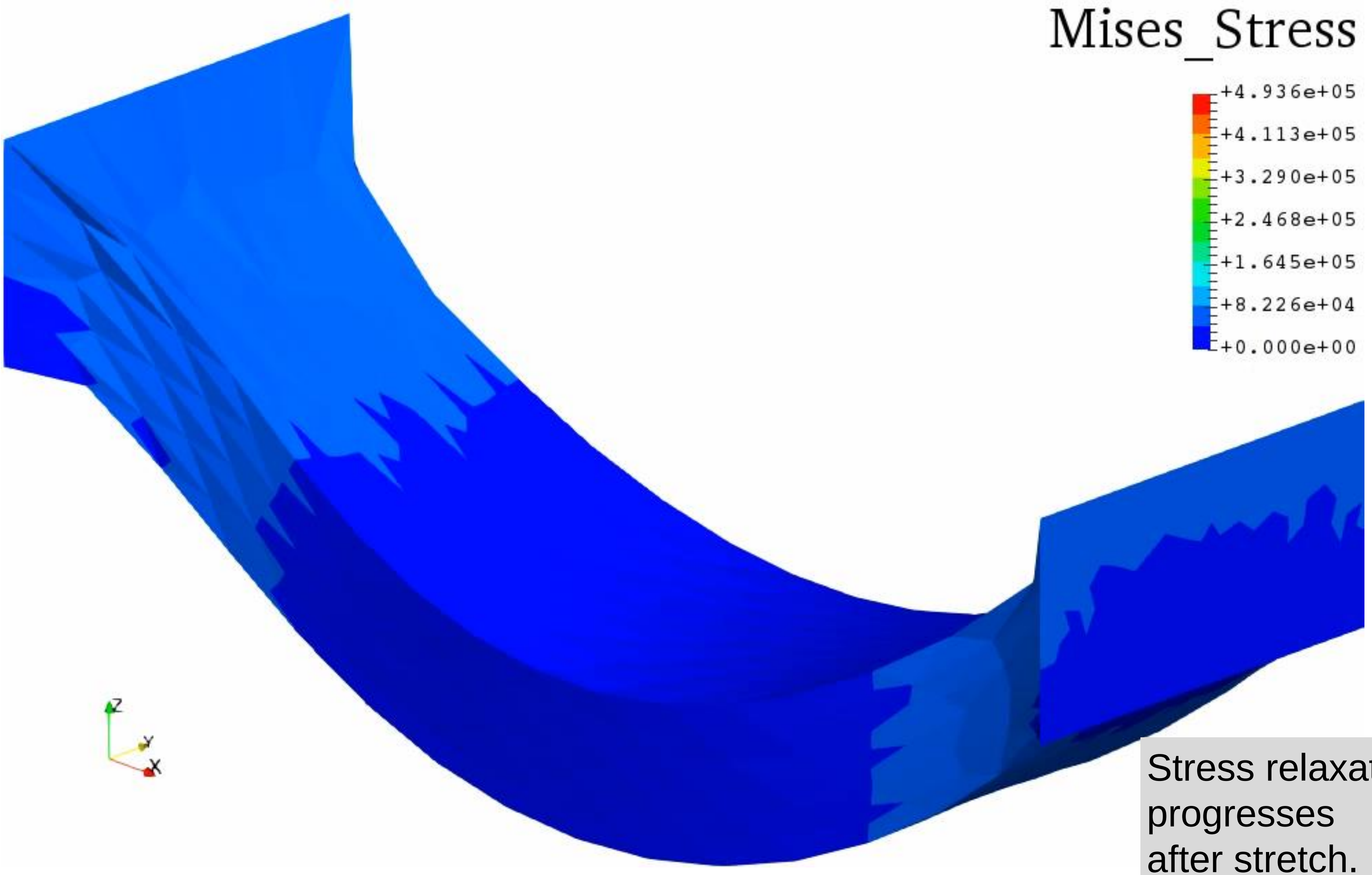
Outline



- $1\ \mathrm{m} \times 2\ \mathrm{m} \times 3\ \mathrm{m}$ block subjected to 100% stretch in 10 s, and hold the enforced displacement for 1000 s under the gravity.
- Hencky's viscoelastic body based on the generalized Maxwell model with 1 maxwell element & 1 long-term spring.
  - Poisson's ratio: $\nu_0 = 0.3$, and $\nu_\infty = 0.49$.
  - Relaxation time: $\tau = 10\ \mathrm{s}$.
- Compare the results of F-barES-FEM-T4(2), ABAQUS C3D4, C3D4H, and C3D8.

Tensile Suspension of Viscoelastic Block

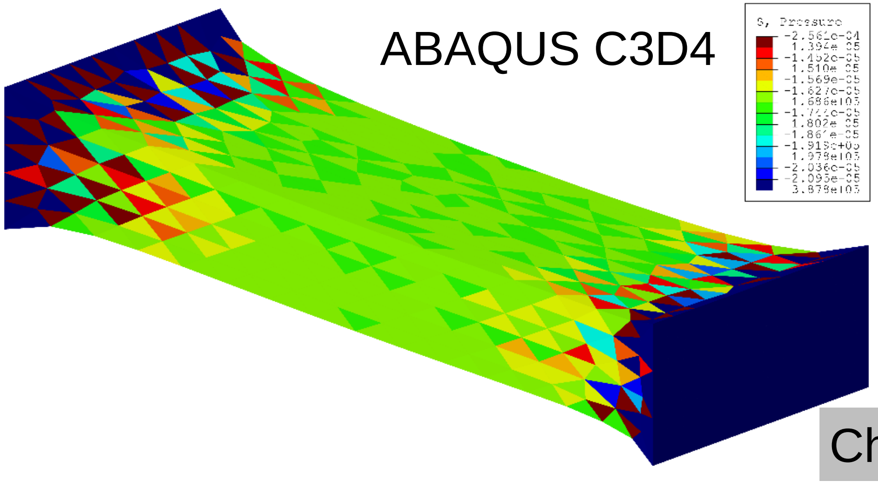
Animation of Mises Stress (F-barES-FEM-T4(2))

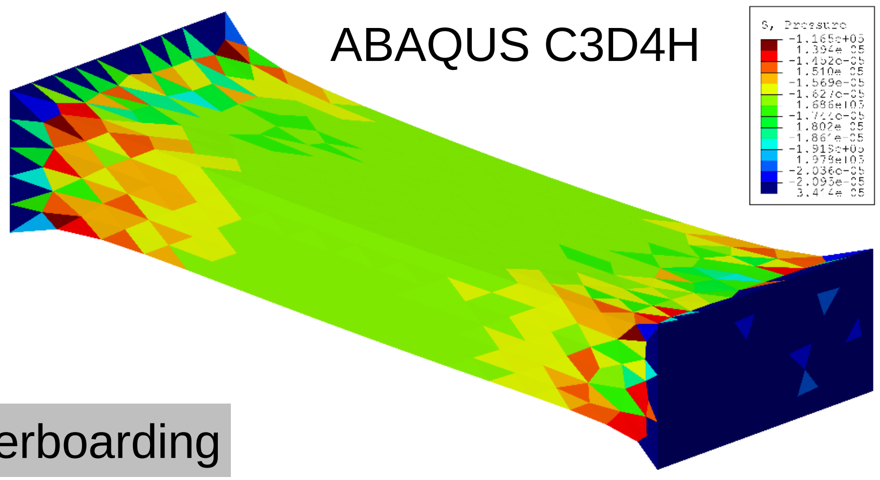


Stress relaxation progresses after stretch.

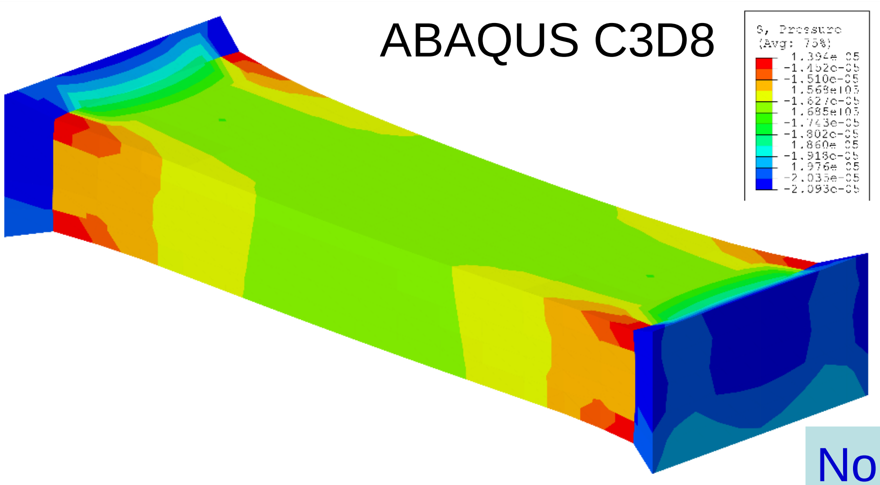
Tensile Suspension of Viscoelastic Block

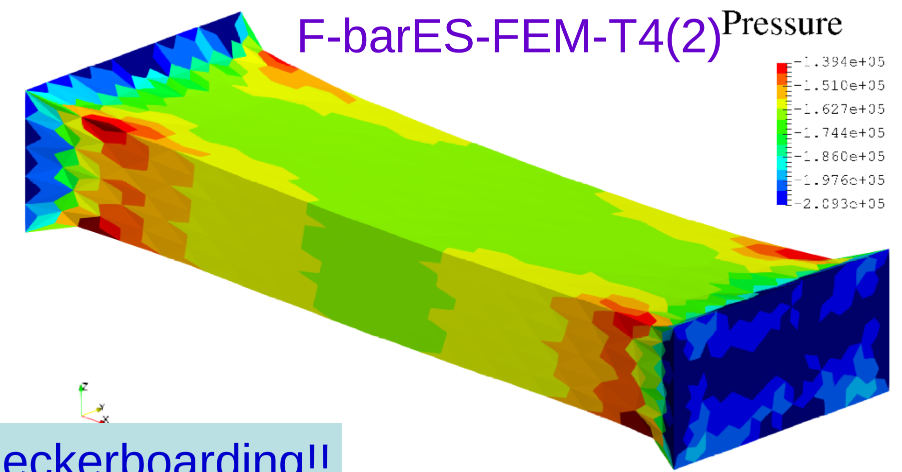
***Pressure* at the end of stretch (common contour range)**





Checkerboarding

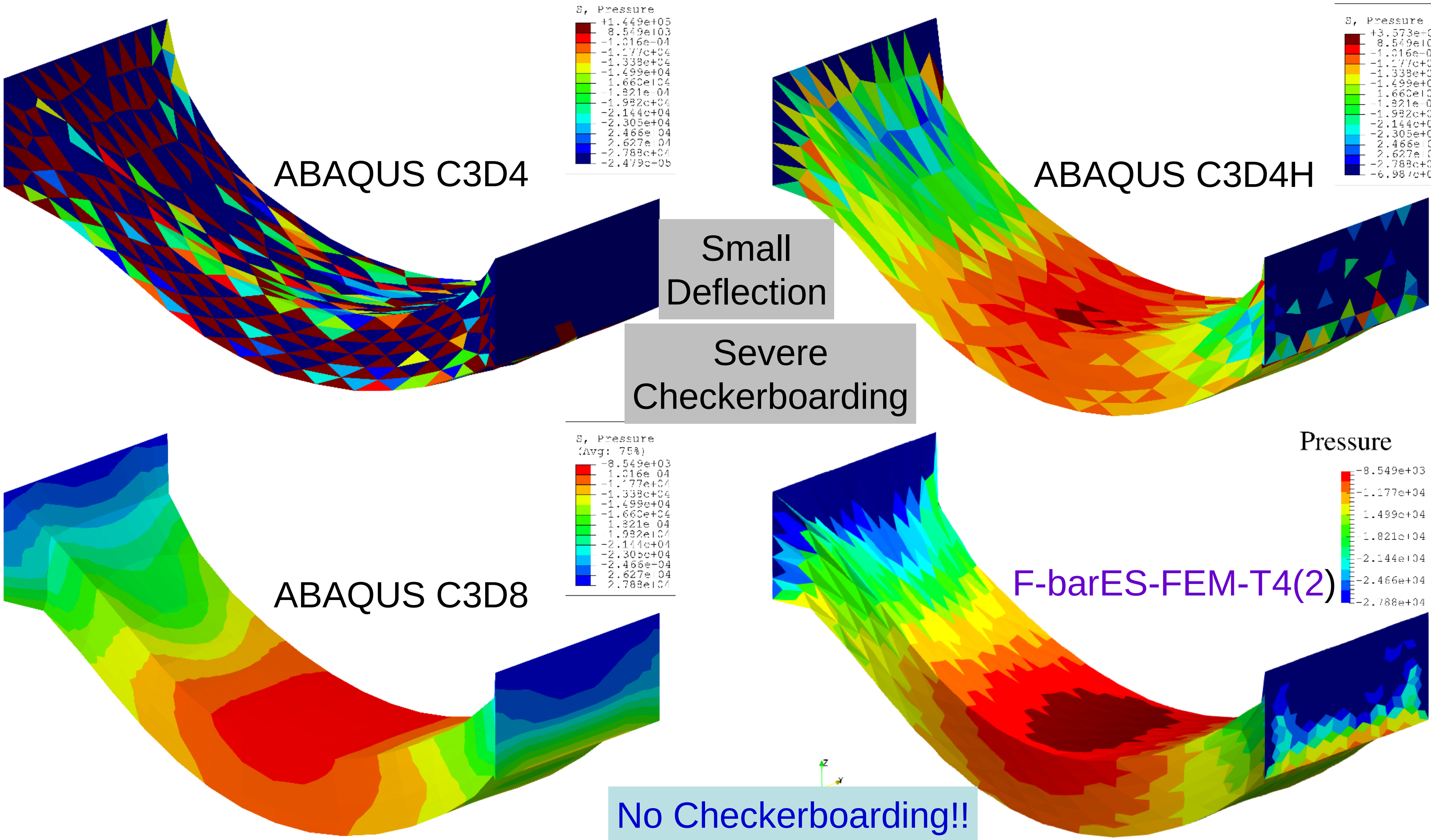




No Checkerboarding!!

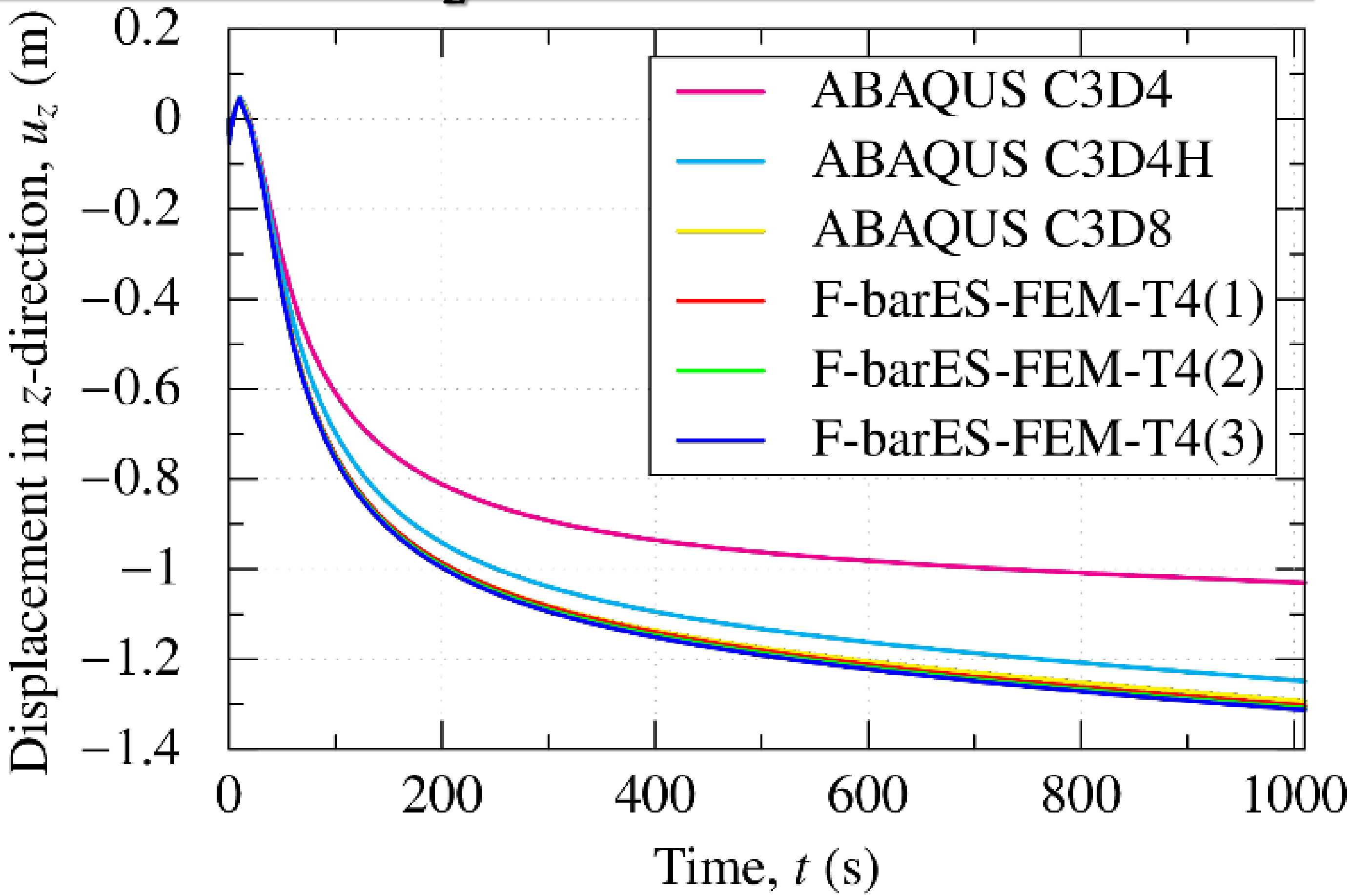
Tensile Suspension of Viscoelastic Block

***Pressure** at the final state* (common contour range)



Tensile Suspension of Viscoelastic Block

Time histories of u_z at the center of bottom face



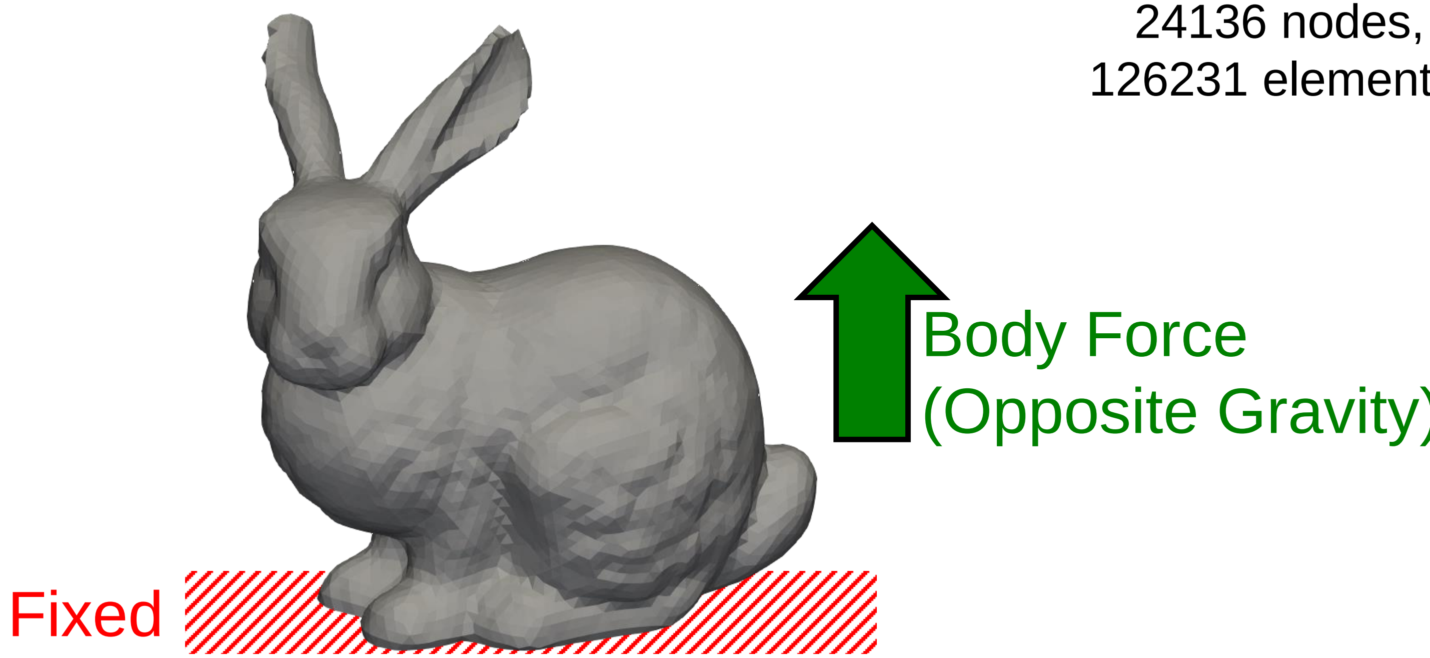
ABAQUS C3D4 and C3D4H have shear locking.

ABAQUS's T4 elements cannot avoid shear locking, whereas F-barES-FEM-T4 has good accuracy as H8-SRI element.

Body Forced Stretch of Viscoelastic Bunny

Outline

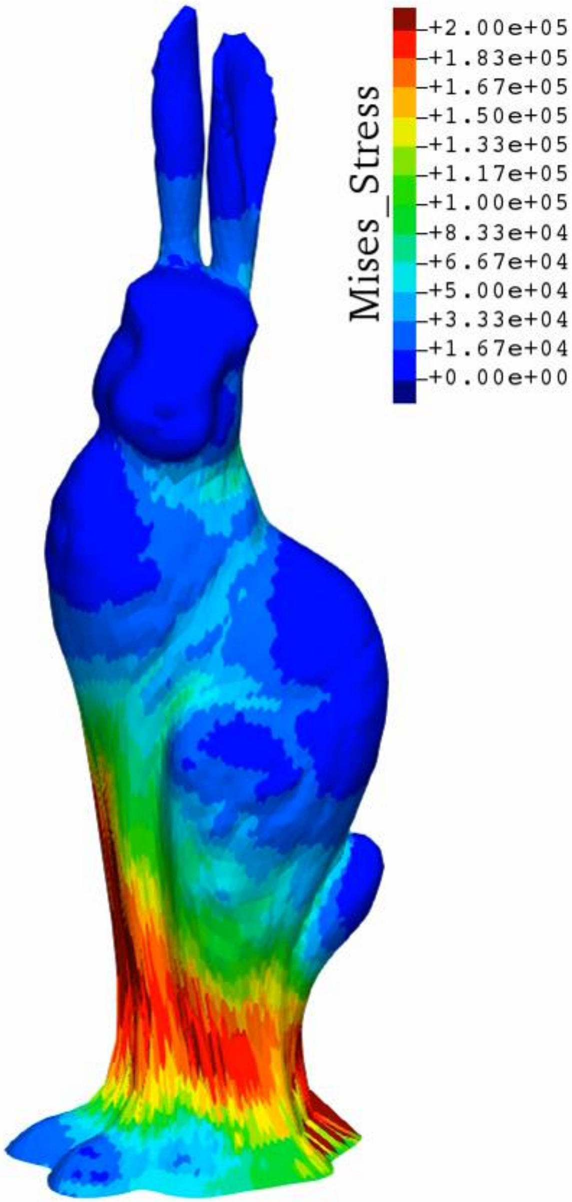
24136 nodes,
126231 elements.



- A Stanford bunny subjected to stretch under the body force.
- Same viscoelastic properties: $\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10\ s$.
- Hexahedral mesh is difficult to build.
- Compare the results of F-barES-FEM-T4(2) and ABAQUS C3D4H.

Body Forced Stretch of Viscoelastic Bunny

Animation of Mises stress (F-bar ES-FEM-T4(2))



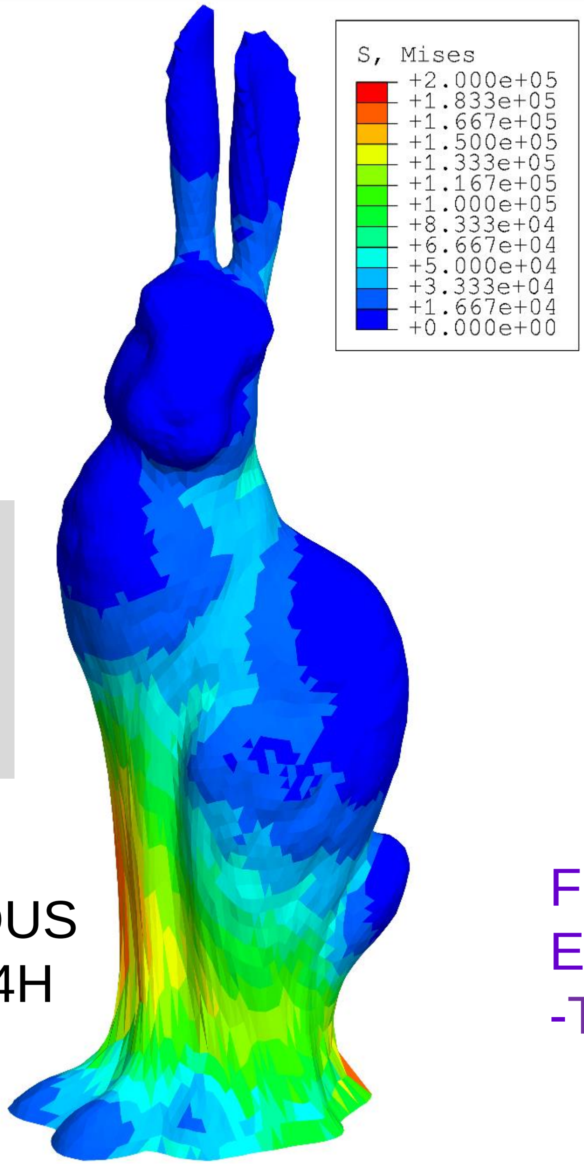
Smooth Mises stress distribution is obtained.

Necking occurred.

Body Forced Stretch of Viscoelastic Bunny

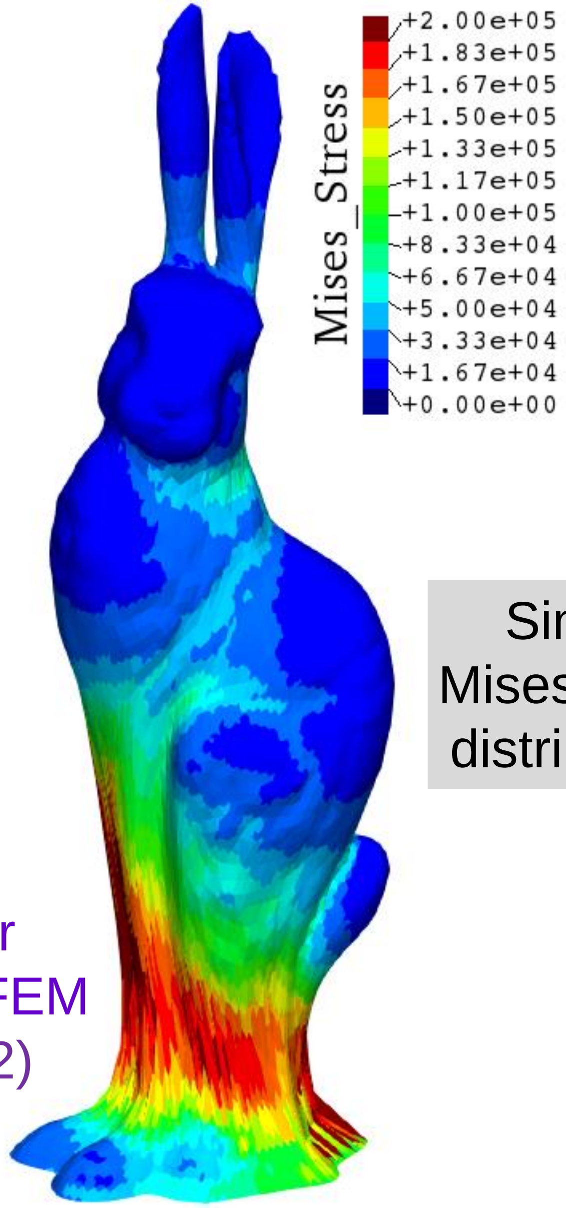
Mises stress at the final state

Slightly small deformation due to shear locking.



ABAQUS
C3D4H

F-bar
ES-FEM
-T4(2)

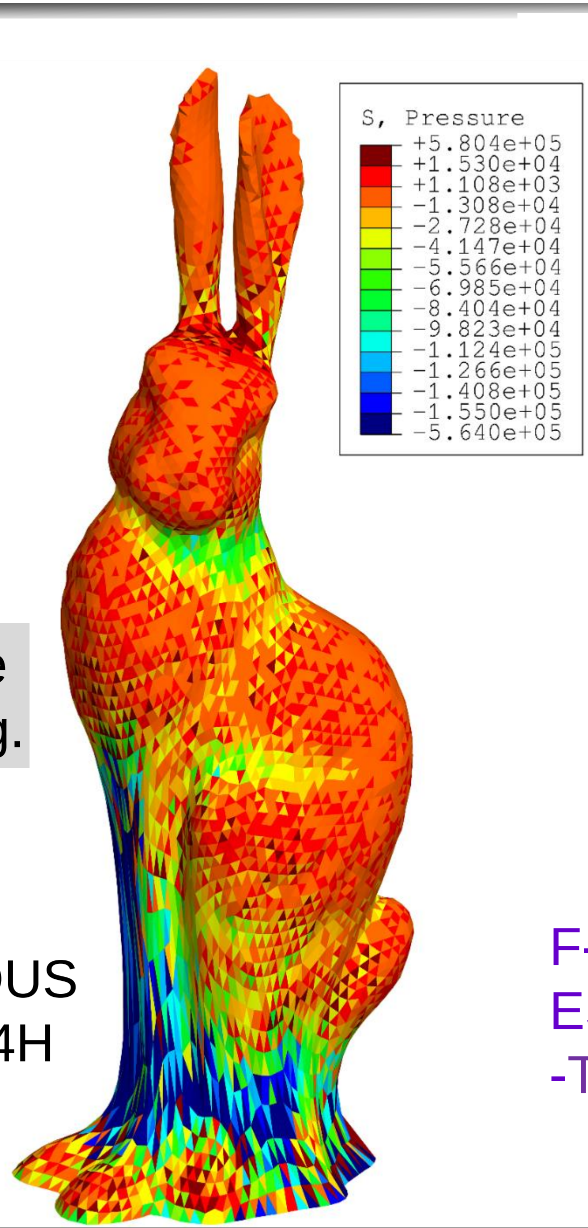


Similar Mises stress distribution.

Body Forced Stretch of Viscoelastic Bunny

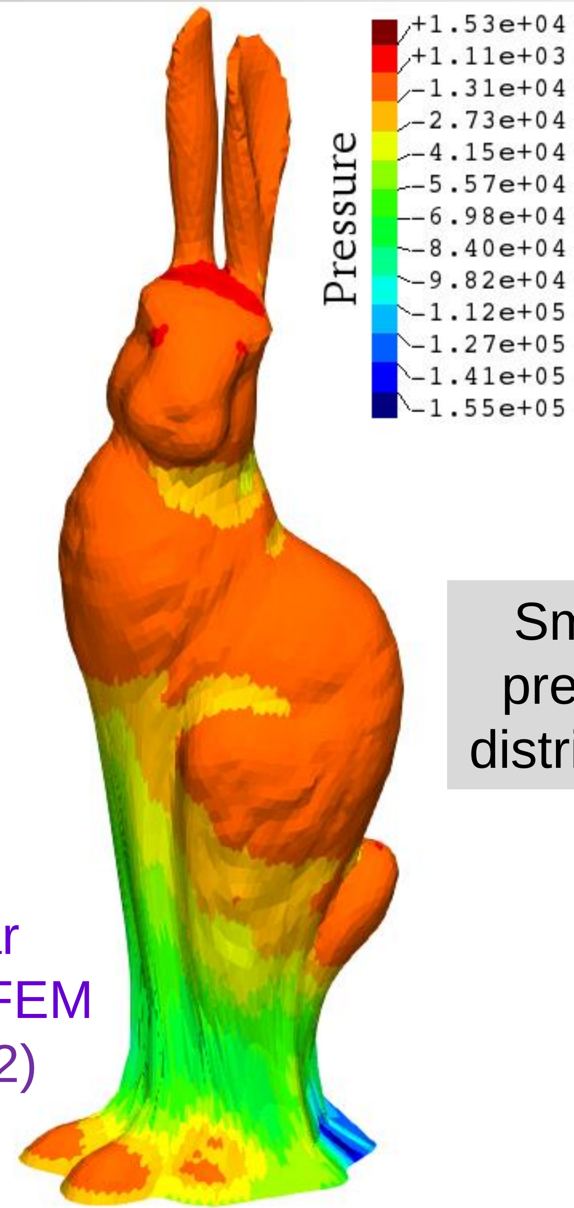
Pressure at the final state

Severe pressure checkerboarding.



ABAQUS
C3D4H

F-bar
ES-FEM
-T4(2)



Smooth pressure distribution.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

- ✓ Locking-free with 1st order tetra meshes.
  No difficulty in severe strain or contact analysis.
- ✓ No increase in DOF.
  Purely displacement-based formulation.
- ✓ Long lasting.
- ✓ Less pressure checkerboarding.

More stable & accurate than other T4 elements!!!

Drawbacks

- ✗ The more cyclic smoothing necessitates the more CPU time due to the wider bandwidth.

Slower than other T4 elements…

Take-Home Messages

F-barES-FEM-T4 is the current best T4 FE formulation for the large deformation with near incompressibility:

- Hyperelastic materials,
- Elastoplastic materials, and
- Viscoelastic materials.

Thank you for your kind attention!