F-bar aided edge-based smoothed finite element methods with 4-node tetrahedral elements for static large deformation hyperelastic and elastoplastic problems

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Motivation

Motivation

We want to accurately and stably analyze **severe large deformation** of solids in **any shape** with finite elements.

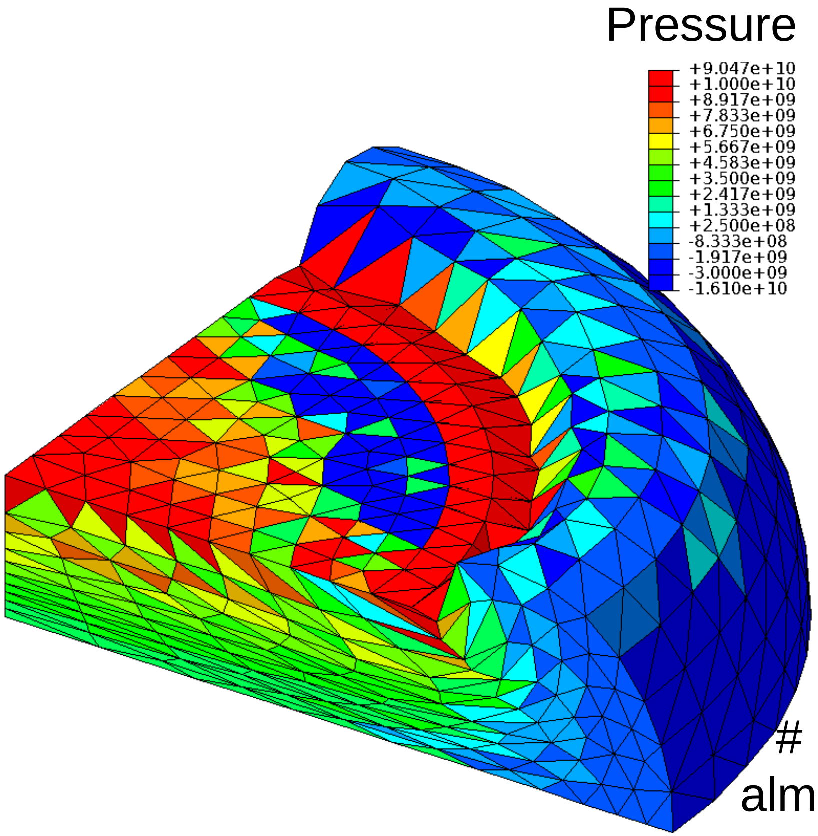
Issues

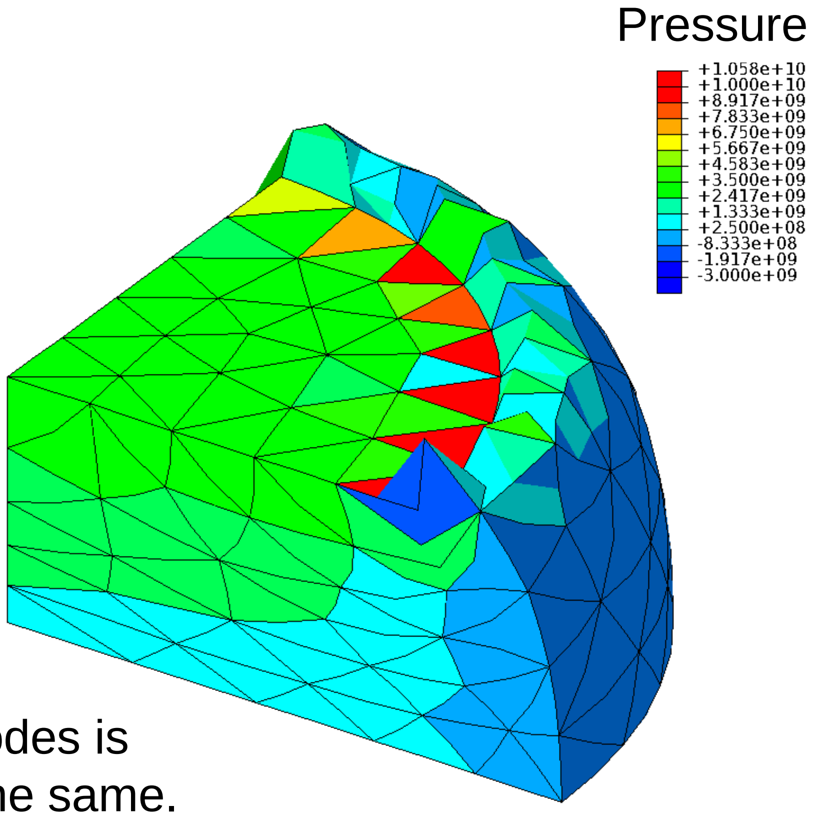
- Only tetra mesh is available for arbitrary body shape.
- The standard 1st / 2nd order tetrahedral element are poor especially when incompressibility is present. Also, all the other u/p hybrid tetrahedral elements (e.g., C3D4H, C3D10MH in ABAQUS) have some issues:
  - pressure oscillation,
  - early convergence failure, etc.

Researches on FE formulations for 1st order tetra (T4) are still active especially for rubber-like or elasto-plastic materials.

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\text{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

✓ No shear/volumetric locking

✗ Pressure oscillation

✗ Corner locking

2nd order modified hybrid T10 (C3D10MH)

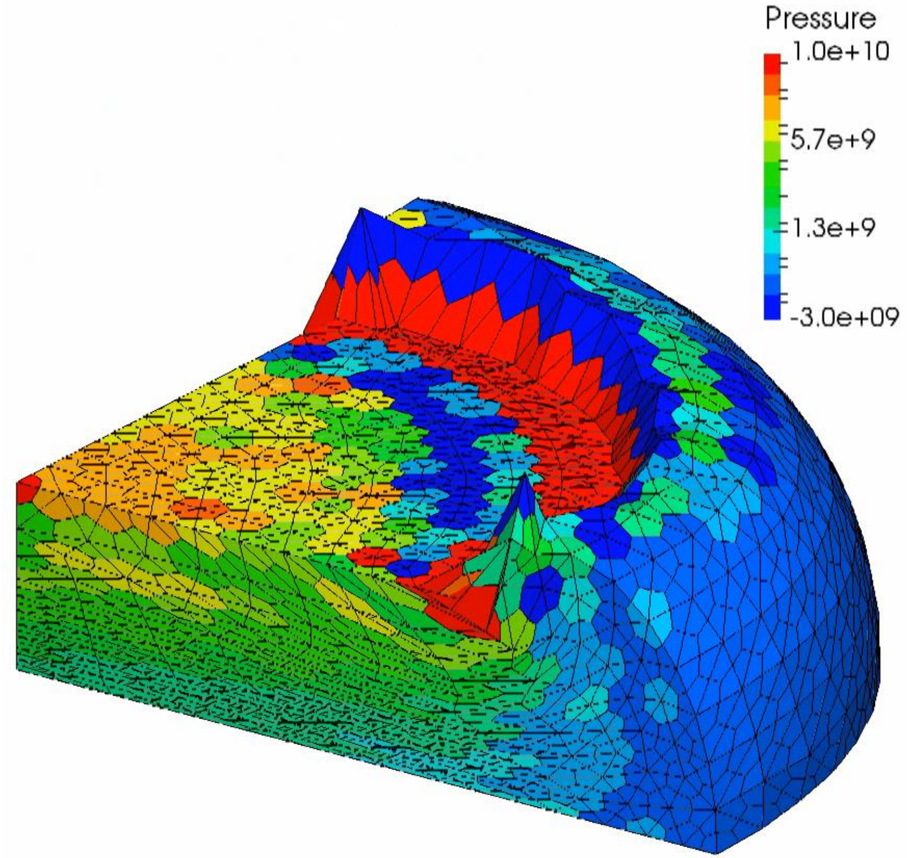
✓ No shear/volumetric locking

✗ Low interpolation accuracy

✗ Early convergence failure

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\mathrm{ini}} = 0.49$



# of Nodes is
exactly the same
as the C3D4H case.

Selective ES/NS-FEM-T4

✓ No shear/volumetric locking
✗ Pressure oscillation
✗ Corner locking

Selective ES/NS-FEM-T4 is not bad
as ABAQUS C3D4H.
Yet, it still has major issues…

Objective

Propose a new type of S-FEM,
F-barES-FEM-T4,
to resolve the pressure oscillation
and the corner locking issues
in hyperelastic and elastoplastic materials.

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- Methods: Quick introduction of F-barES-FEM-T4
- Results: A few example analyses
- Summary

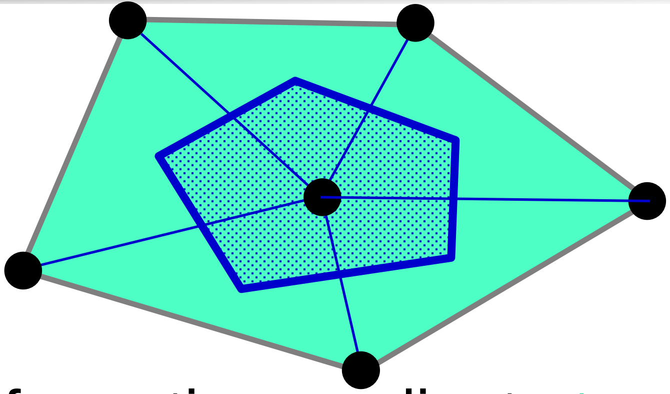
Methods

Quick introduction of F-barES-FEM-T4

(F-barES-FEM-T3 in 2D is explained for simplicity.)

Quick Review of Node-based S-FEM (NS-FEM)

For triangular (T3) or tetrahedral (T4) elements.



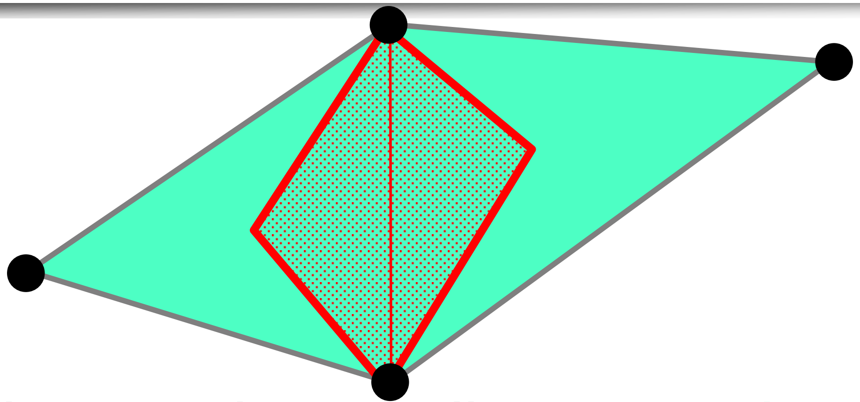
Algorithm:

1. Calculate the deformation gradient at each element, ${}^{\text{Elem}}\boldsymbol{F}$, as usual.
2. Distribute ${}^{\text{Elem}}\boldsymbol{F}$ s to the connecting nodes with area weights to make ${}^{\text{Node}}\tilde{\boldsymbol{F}}$ at each node.
3. Use ${}^{\text{Node}}\tilde{\boldsymbol{F}}$ s to calculate the stress, nodal force and so on.

NS-FEM avoids shear & volumetric locking in T3/T4 elements and also **alleviates pressure oscillation**.
Yet, it suffers from spurious low-energy modes, corner locking and **minor pressure oscillation….**

Quick Review of Edge-based S-FEM (ES-FEM)

For triangular (T3) or tetrahedral (T4) elements.



Algorithm:

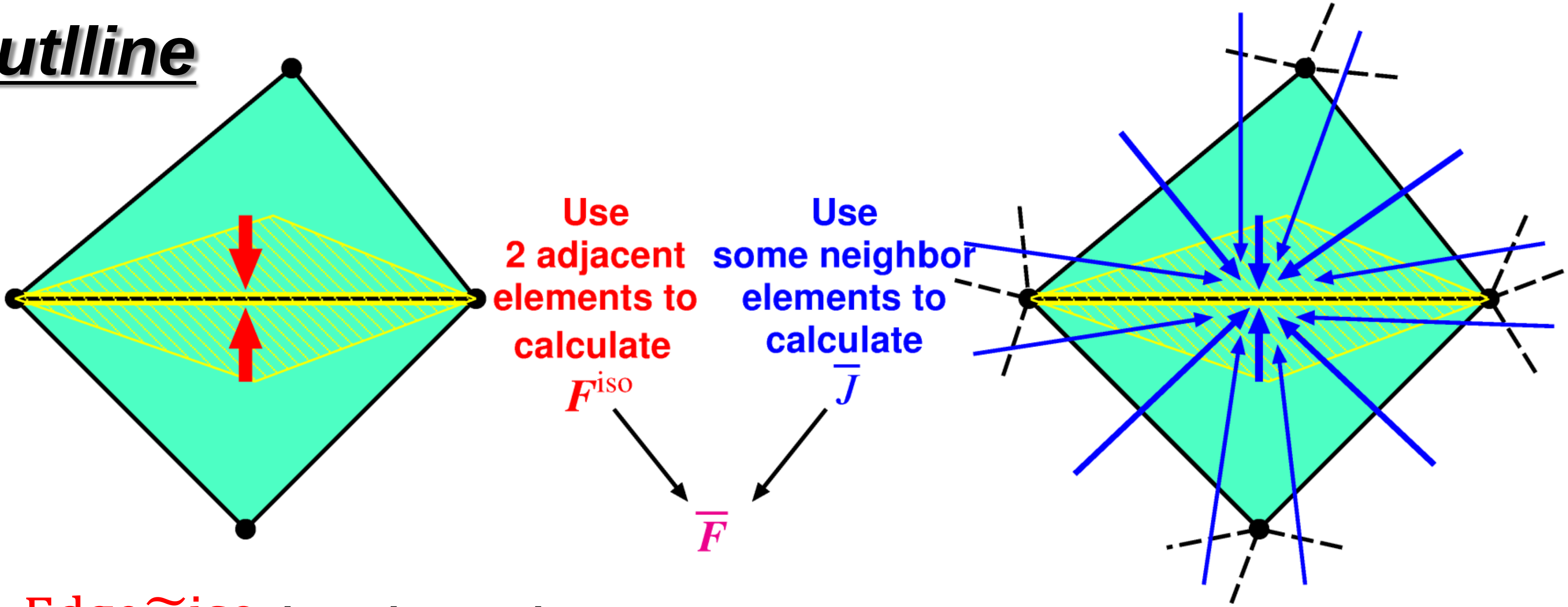
1. Calculate the deformation gradient at each element, ${}^{\mathrm{Elem}}\boldsymbol{F}$, as usual.
2. Distribute ${}^{\mathrm{Elem}}\boldsymbol{F}$ s to the connecting edges with area weights to make ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}$ at each edge.
3. Use ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}$ s to calculate the stress, nodal force and so on.

ES-FEM avoids shear locking in T3/T4 elements.
Yet, it suffers from volumetric locking, corner locking,
and **major pressure oscillation…**

Quick Introduction of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

<u>Outlline</u>



Use
2 adjacent
elements to
calculate
F^{iso}

Use
some neighbor
elements to
calculate
$\bar{J}$

$\bar{F}$

- ${}^{\text{Edge}}\tilde{F}^{\text{iso}}$ is given by ES-FEM.
- ${}^{\text{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- ${}^{\text{Edge}}\bar{F}$ is calculated in the manner of F-bar method:

$$ {}^{\text{Edge}}\bar{F} = {}^{\text{Edge}}\bar{J}^{\,1/3}\;{}^{\text{Edge}}\tilde{F}^{\text{iso}}. $$

Outline of F-barES-FEM

Brief Formulation

A kind of
low-pass filter
for J

1. Calculate $^{\text{Elem}}J$ as usual.
2. Smooth $^{\text{Elem}}J$ at nodes and get $^{\text{Node}}\tilde{J}$.
3. Smooth $^{\text{Node}}\tilde{J}$ at elements and get $^{\text{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic
Smoothing
of J

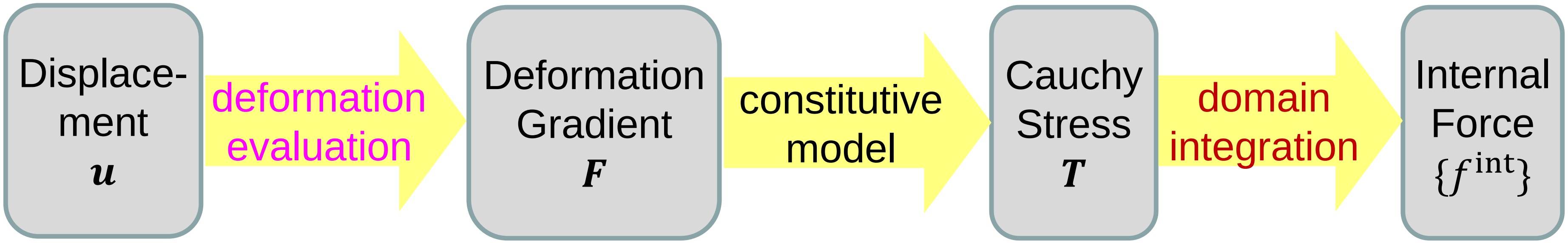
5. Smooth $^{\text{Elem}}\overset{\vdots}{\tilde{\tilde{J}}}$ (c layers of ~) at edges to make $^{\text{Edge}}\bar{J}$.
6. Combine $^{\text{Edge}}\bar{J}$ and $^{\text{Edge}}\boldsymbol{F}^{\text{iso}}$ of ES-FEM as

$$^{\text{Edge}}\bar{\boldsymbol{F}} = {}^{\text{Edge}}\bar{J}^{1/3}\ {}^{\text{Edge}}\boldsymbol{F}^{\text{iso}}.$$

Hereafter, F-barES-FEM-T4 with $\boldsymbol{c}$ cycles of smoothing is called “F-barES-FEM-T4($\boldsymbol{c}$)”.

Additional Point of F-barES-FEM

Typical Flow of FE Solver



Displace-ment $\boldsymbol{u}$

deformation evaluation

Deformation Gradient $\boldsymbol{F}$

constitutive model

Cauchy Stress $\boldsymbol{T}$

domain integration

Internal Force $\{f^{\mathrm{int}}\}$

- Selective ES/NS-FEM
  splits $\boldsymbol{T}$ into $\boldsymbol{T}^{\mathrm{hyd}}$ and $\boldsymbol{T}^{\mathrm{dev}}$
  and merges $\{f^{\mathrm{int}}_{\mathrm{hyd}}\}$ and $\{f^{\mathrm{int}}_{\mathrm{dev}}\}$ into $\{f^{\mathrm{int}}\}$.
- F-barES-FEM
  builds $\boldsymbol{F}^{\mathrm{vol}}$ and $\boldsymbol{F}^{\mathrm{iso}}$ separately
  and combines $\boldsymbol{F}^{\mathrm{vol}}$ and $\boldsymbol{F}^{\mathrm{iso}}$ into $\boldsymbol{F}$.

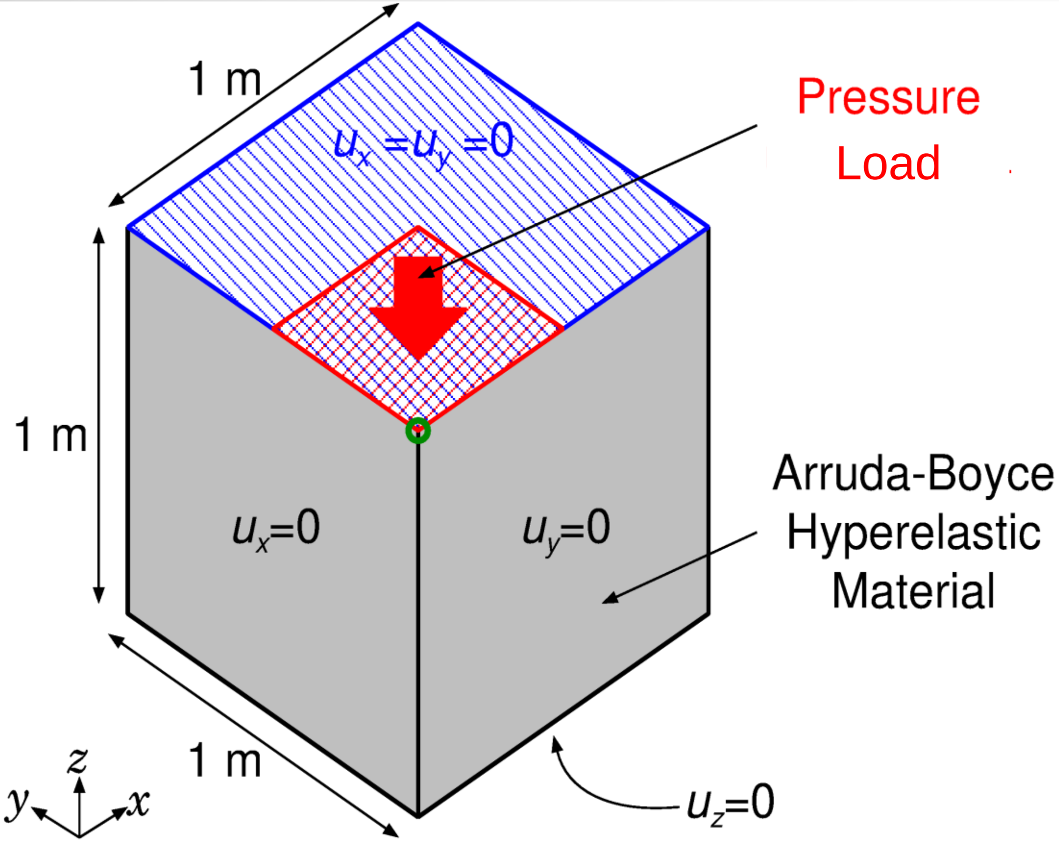
F-barES-FEM can handle
any kind of material constitutive model.

Results

A few example analyses

Compression of Rubber Block

Outline



- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = \mathbf{0.499}$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

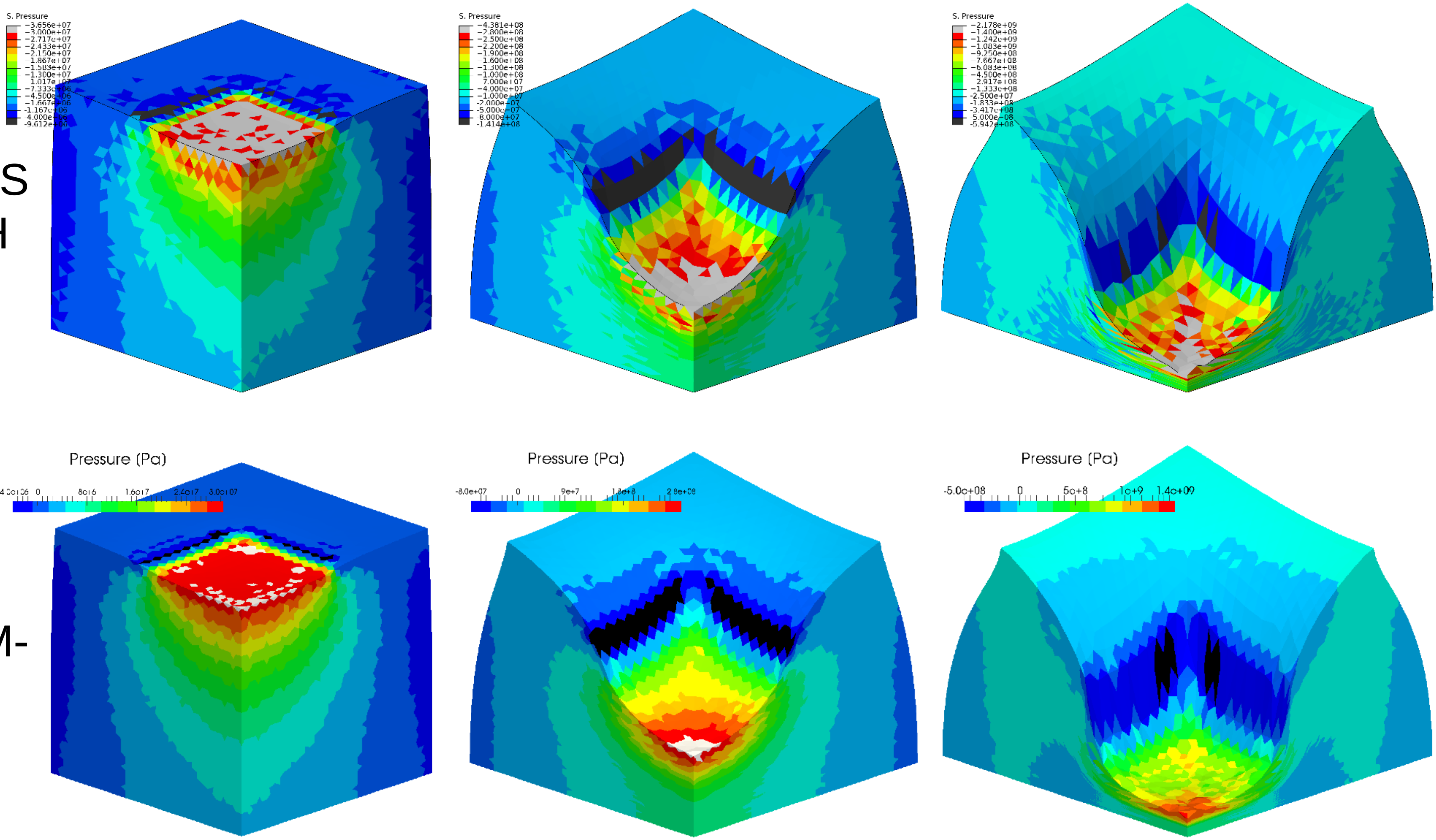
Compression of Rubber Block

Pressure Distribution

Early stage | Middle stage | Later stage

ABAQUS
C3D4H

F-bar
ES-FEM-
T4(2)

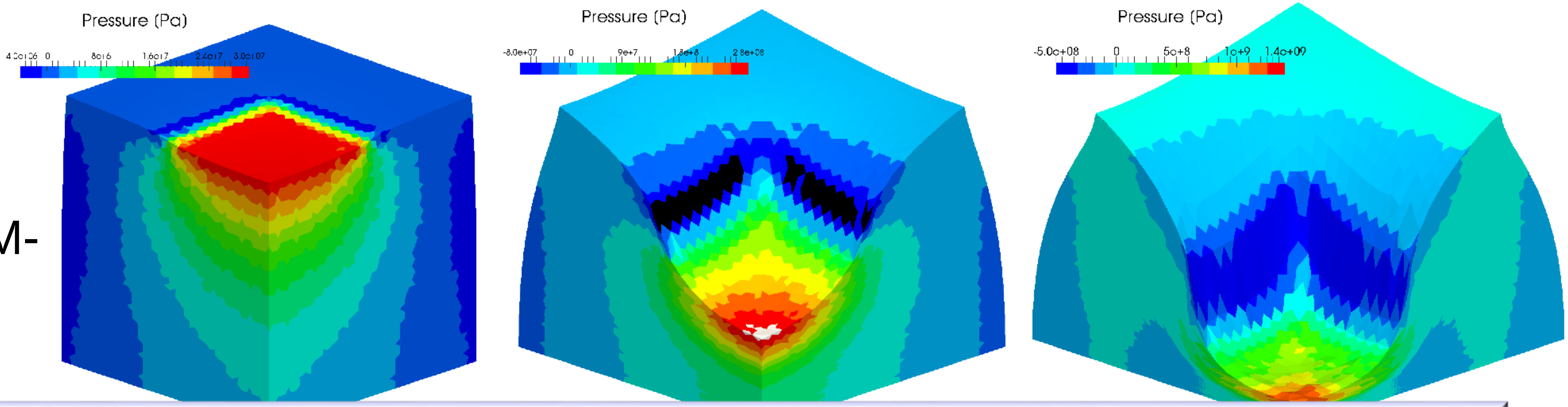


Compression of Rubber Block

Pressure Distribution

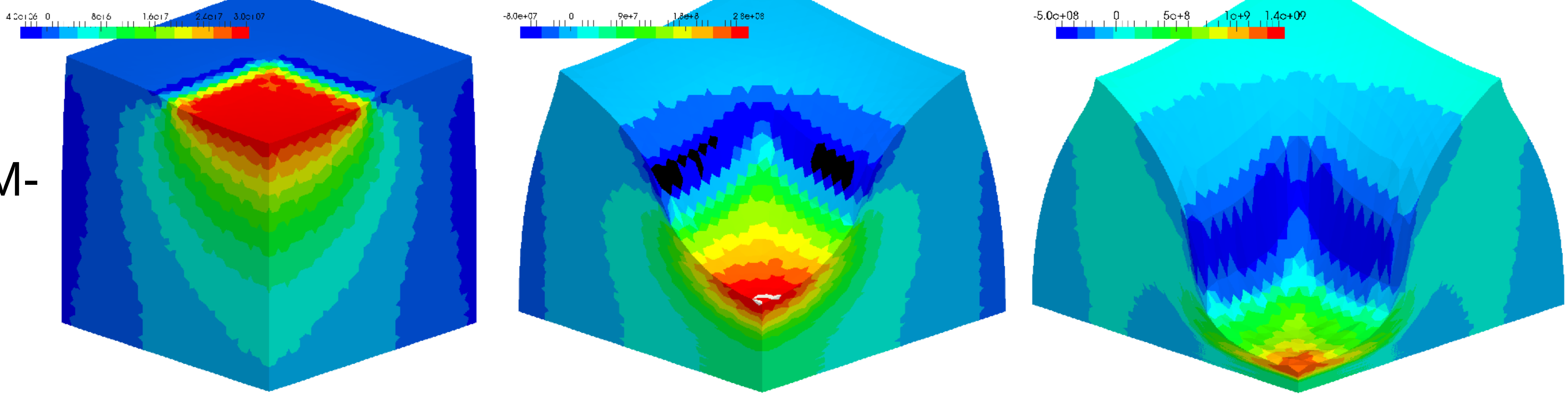
Early stage　Middle stage　Later stage

F-bar ES-FEM-T4(3)



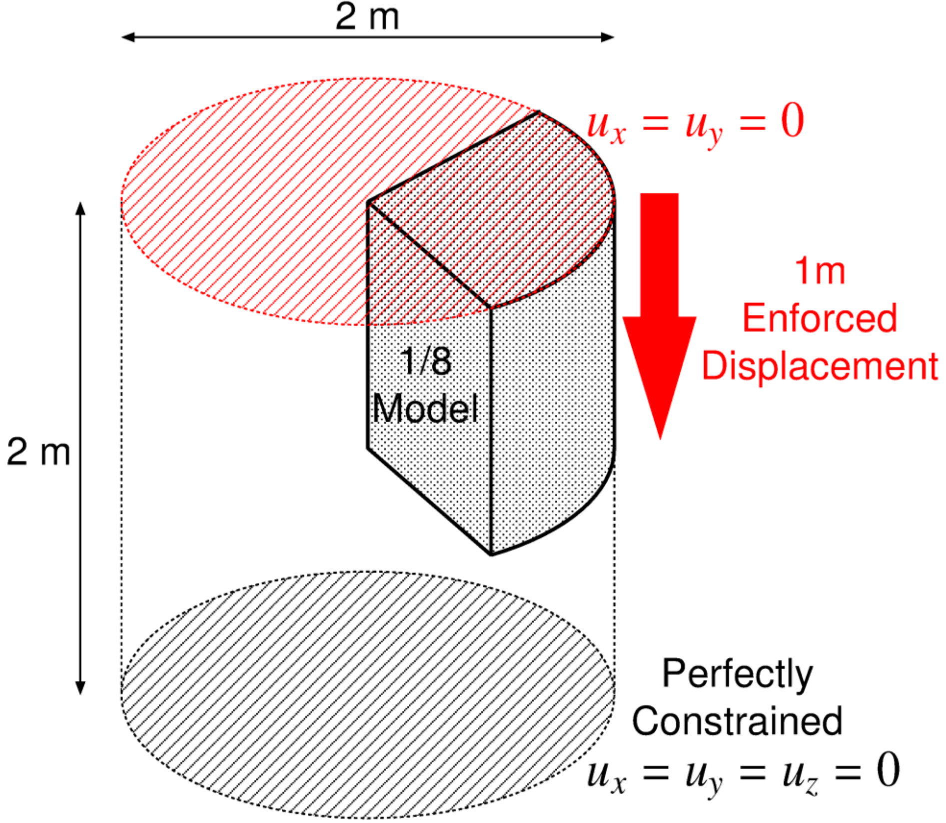
F-barES-FEM-T4 resolves the pressure oscillation issue!

F-bar ES-FEM-T4(4)



Barreling of 1/8 Rubber Cylinder

Outline



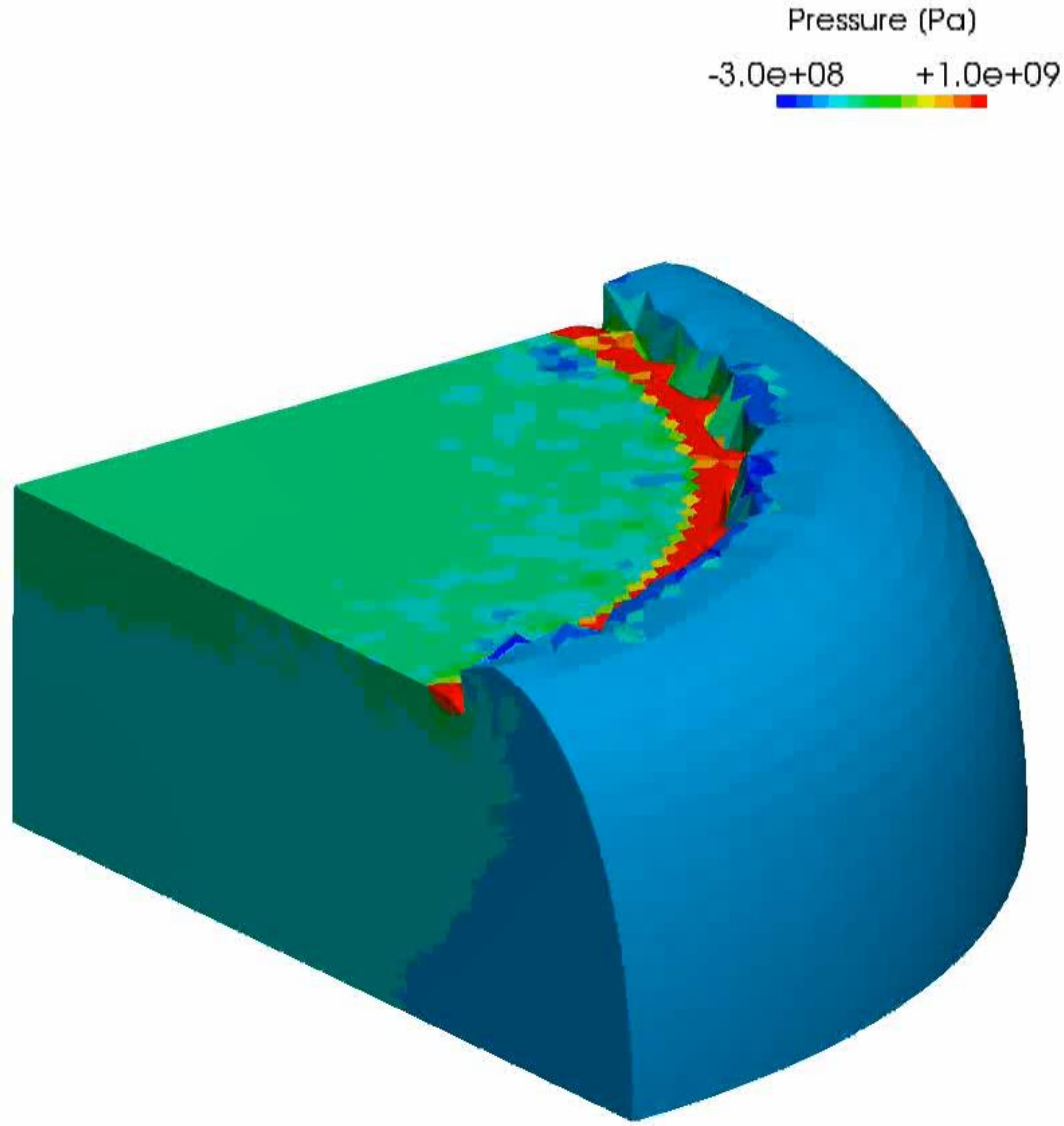
- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = \mathbf{0.499}$).
- Enforced displacement is applied to the top surface.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Barreling of 1/8 Rubber Cylinder

Result of F-bar ES-FEM(2) (Pressure)

50% nominal compression

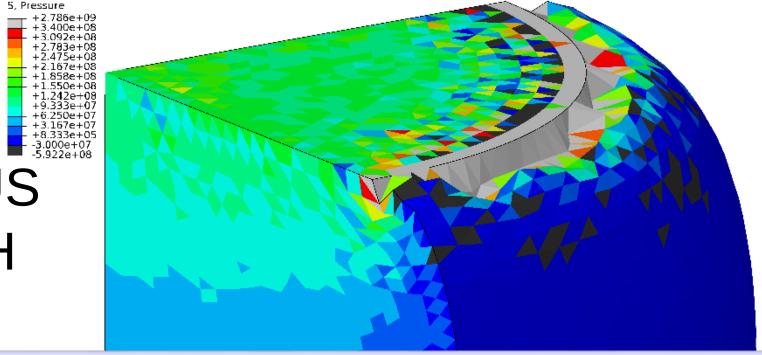
Almost smooth pressure distribution is obtained except just around the rim.



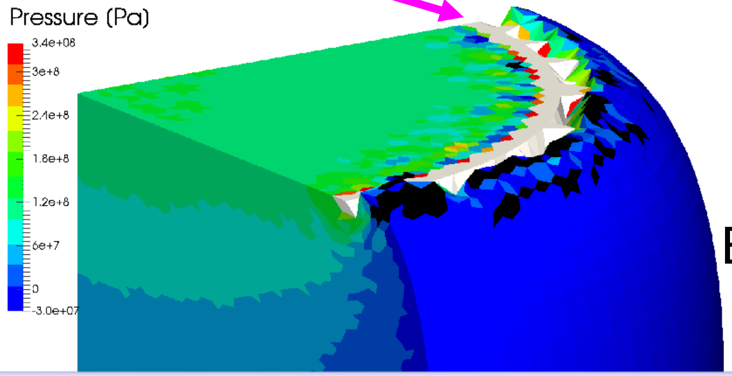
Barreling of 1/8 Rubber Cylinder

Pressure Distribution

Strange deformation (corner locking) around the rim

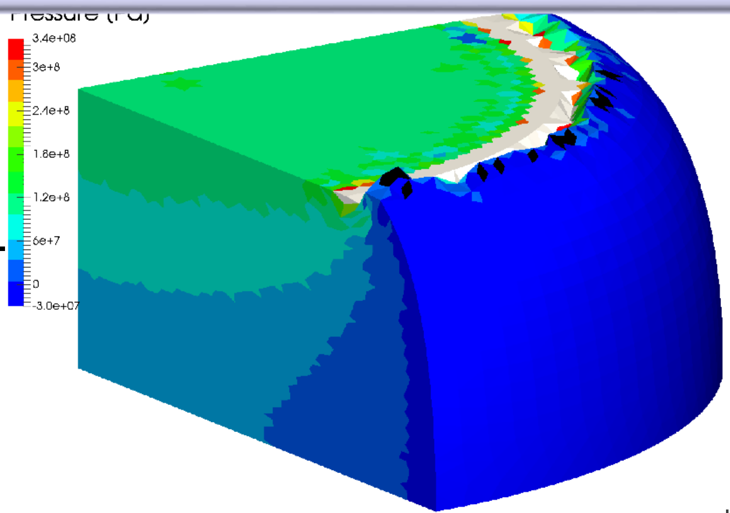


ABAQUS
C3D4H

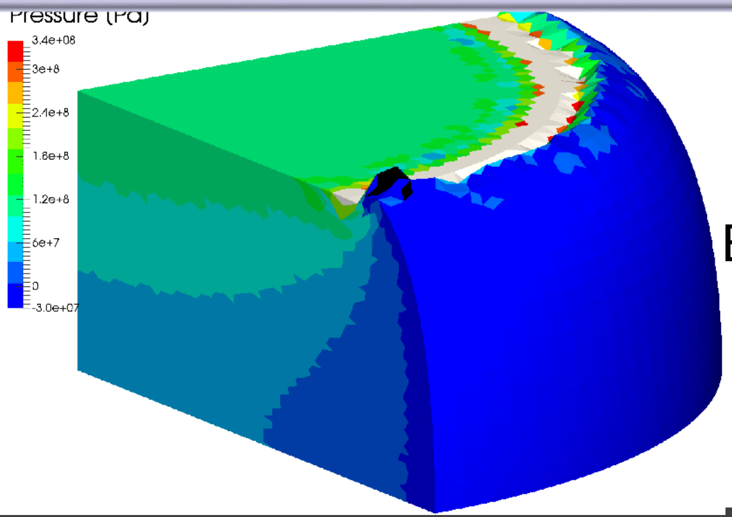


F-bar
ES-FEM-
T4(2)

F-barES-FEM-T4 with a sufficient cyclic smoothing resolves the corner locking issue!



F-bar
ES-FEM-
T4(3)

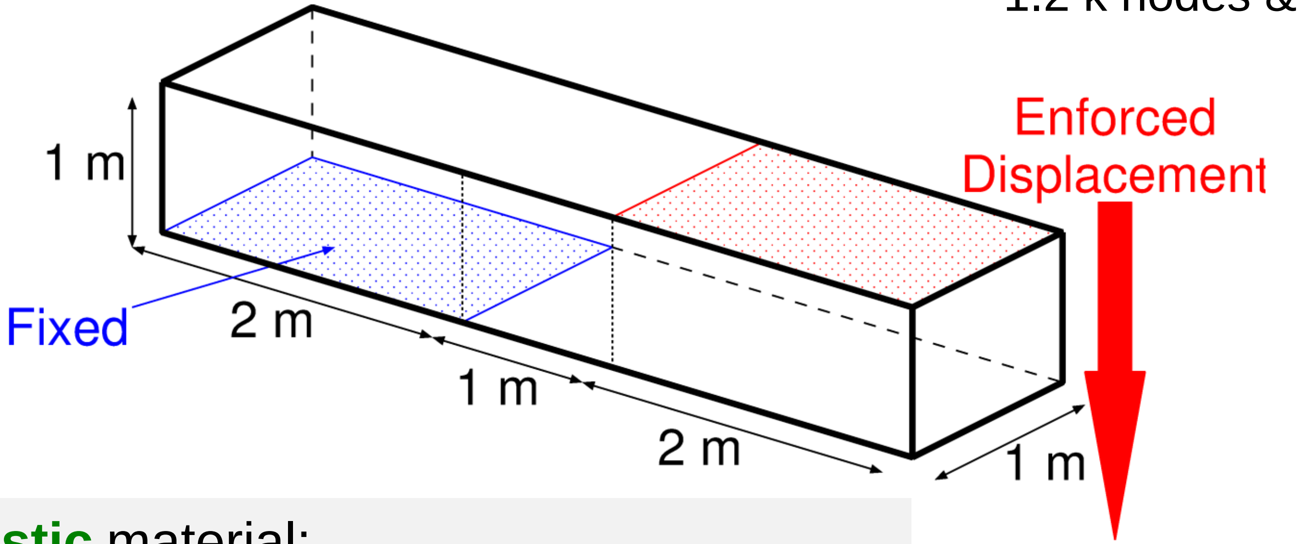


F-bar
ES-FEM-
T4(4)

Shearing & Tensioning of Elasto-Plastic Bar

Outline

1.2 k nodes & 4.8 k elems.

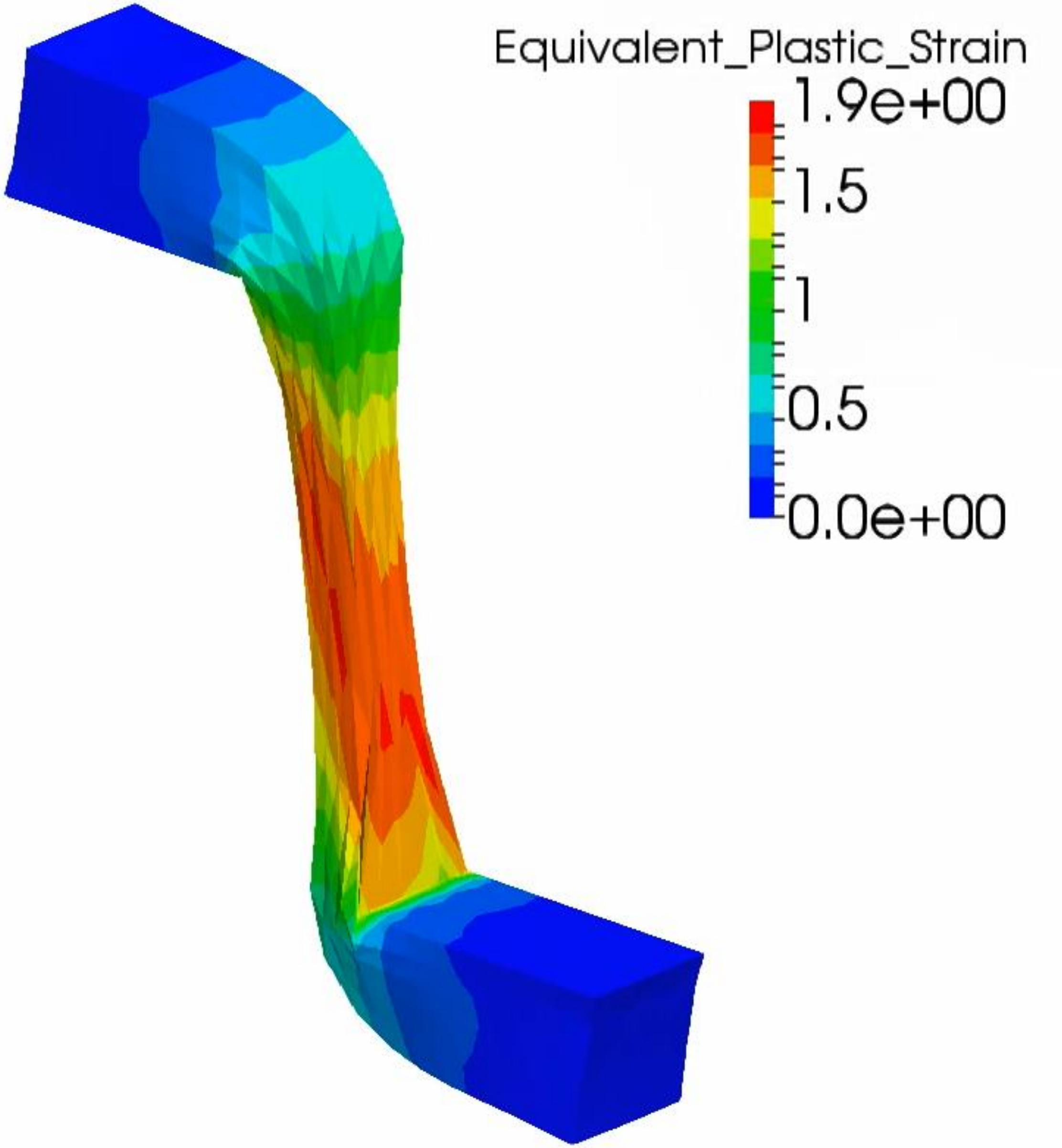


Elasto-plastic material:
- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

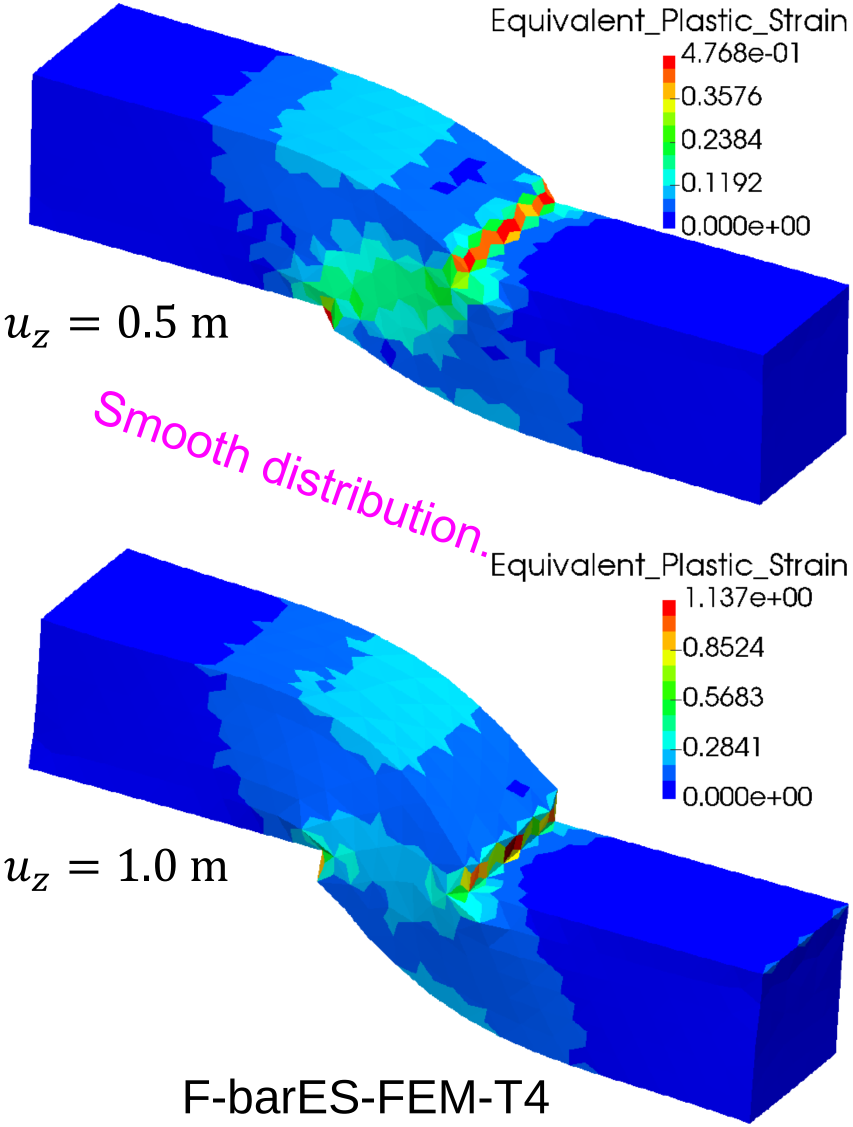
Shearing & Tensioning of Elasto-Plastic Bar

Result of F-bar ES-FEM ***(Equiv. Plastic Strain)***



Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain

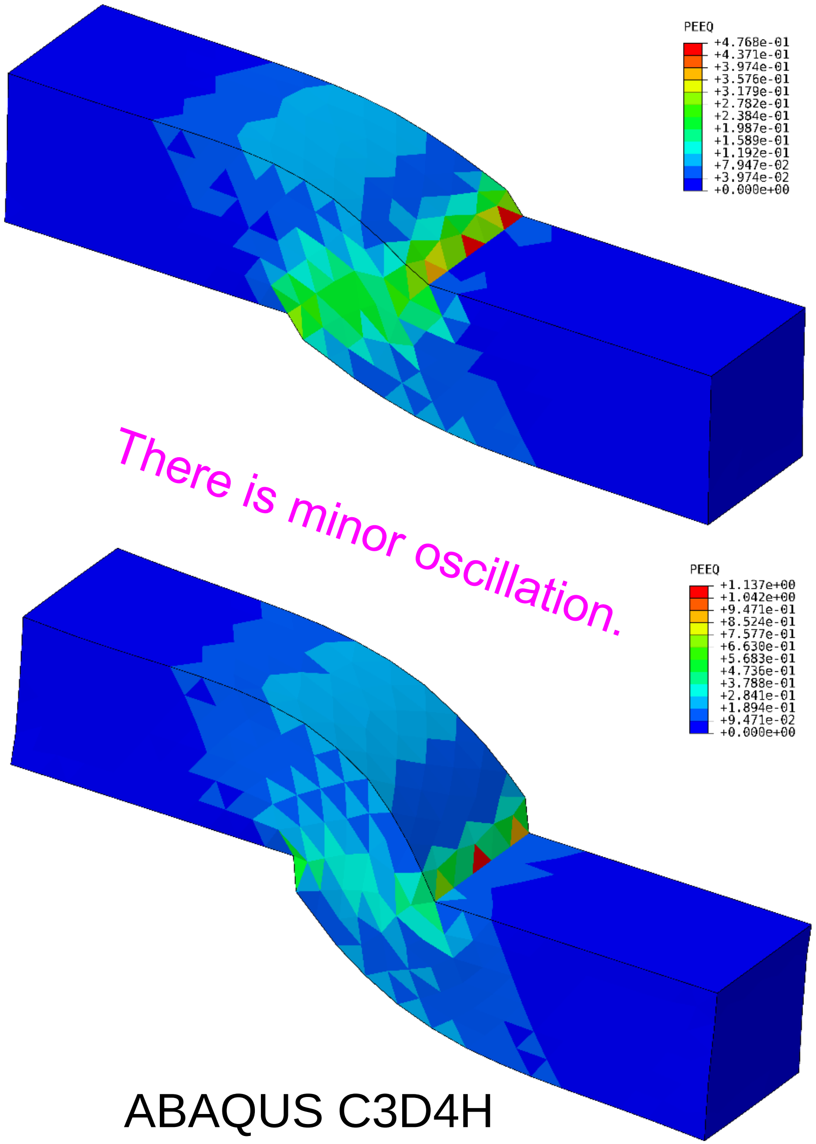


$u_z = 0.5$ m

$u_z = 1.0$ m

Smooth distribution.

F-barES-FEM-T4



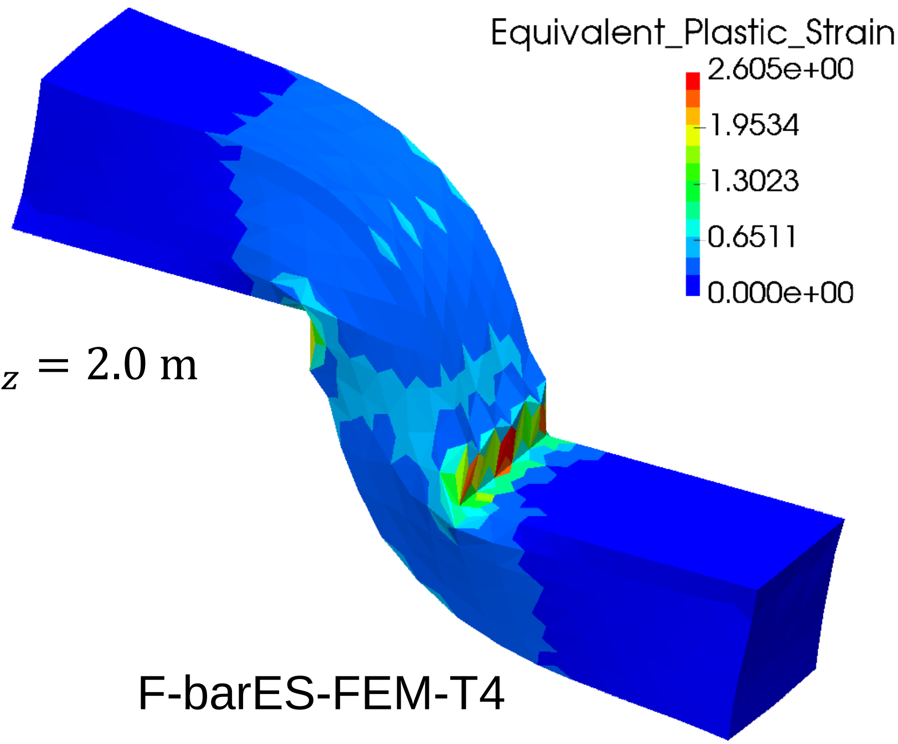
There is minor oscillation.

ABAQUS C3D4H

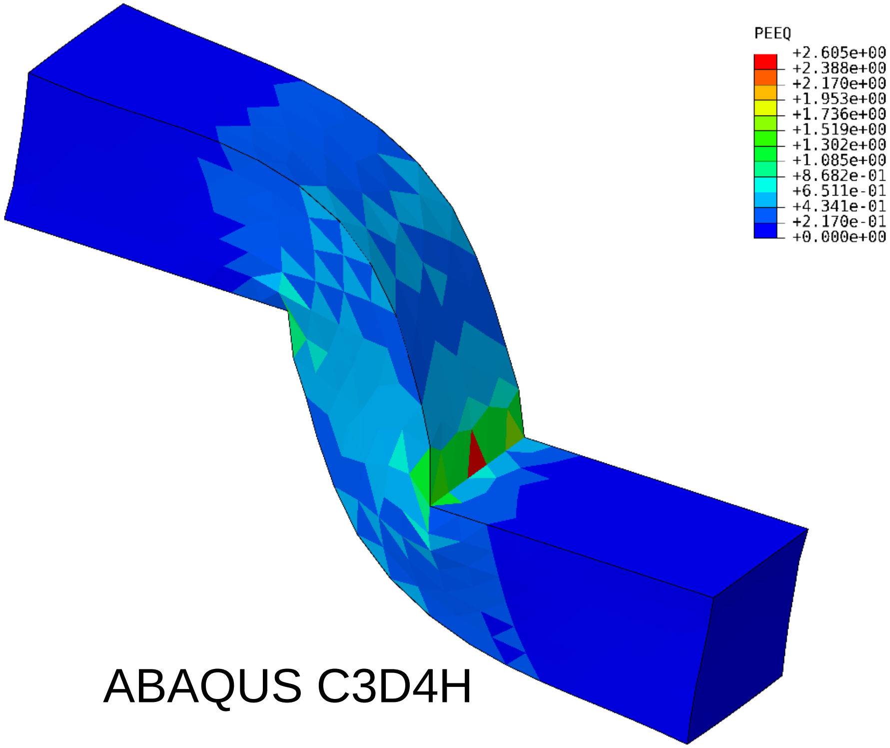
Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain

$u_z = 2.0$ m



F-barES-FEM-T4

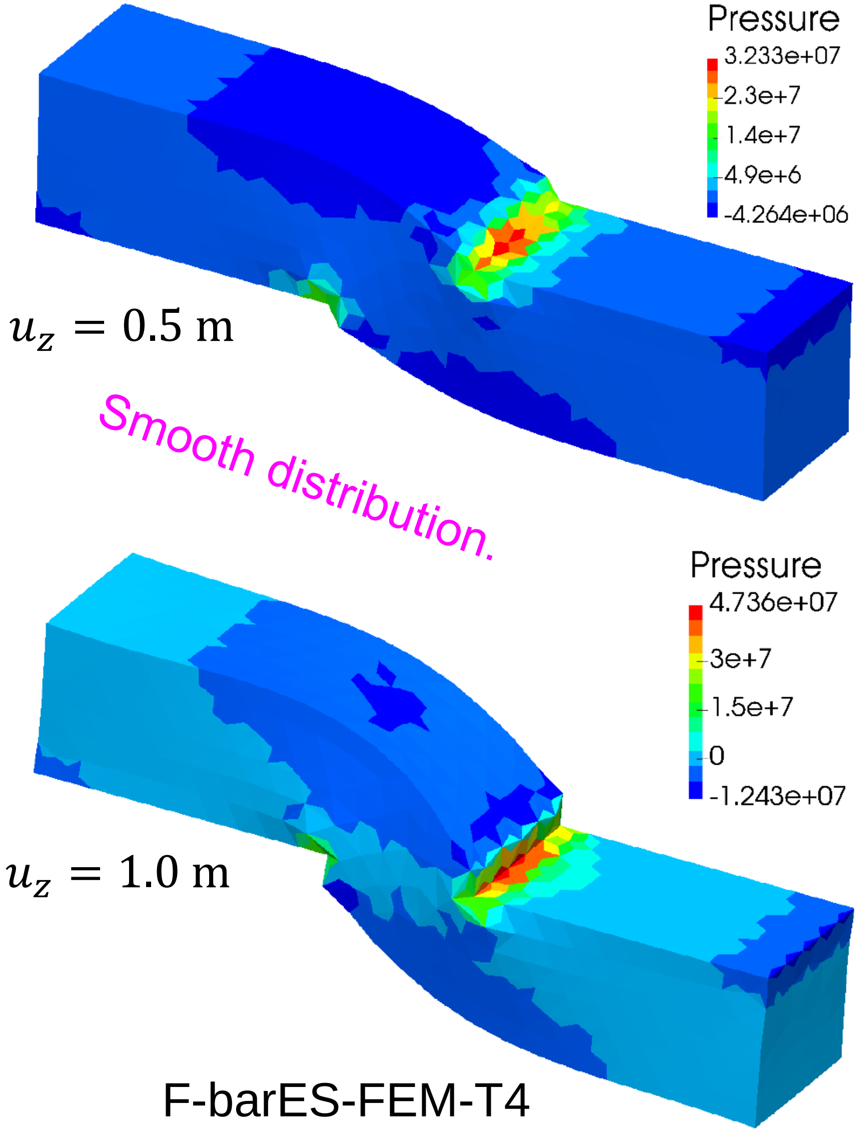


ABAQUS C3D4H

Accuracy of equivalent plastic strain seems no much different.

Shearing & Tensioning of Elasto-Plastic Bar

Pressure

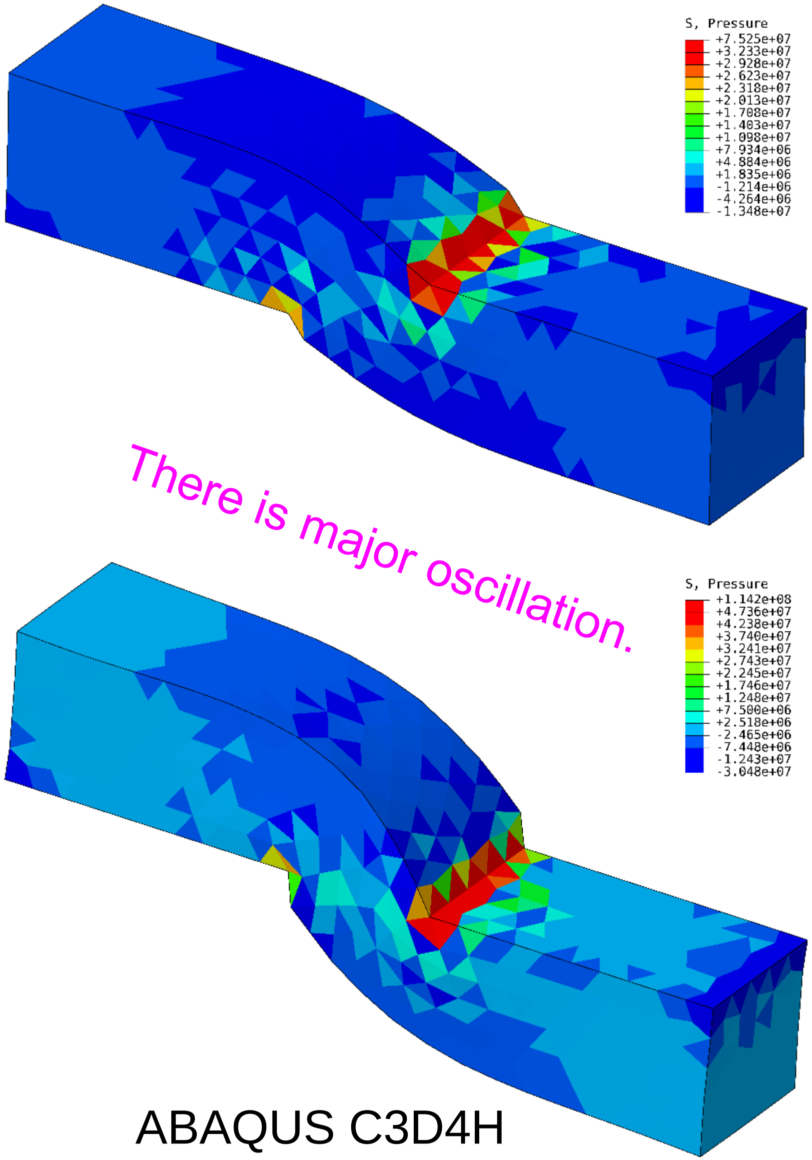


$u_z = 0.5 \text{ m}$

Smooth distribution.

$u_z = 1.0 \text{ m}$

F-barES-FEM-T4



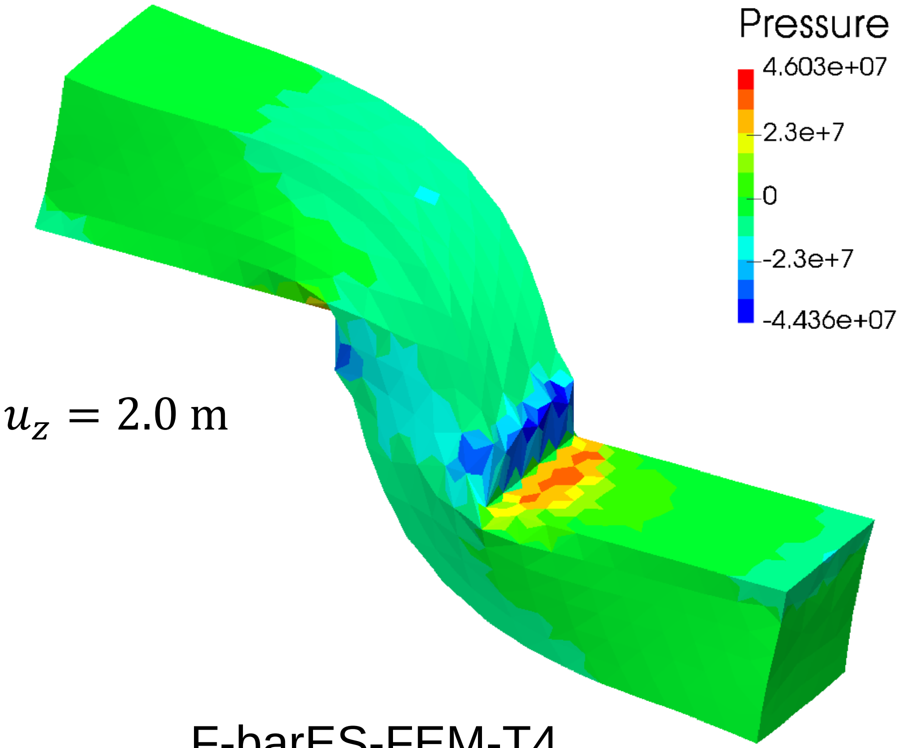
There is major oscillation.

ABAQUS C3D4H

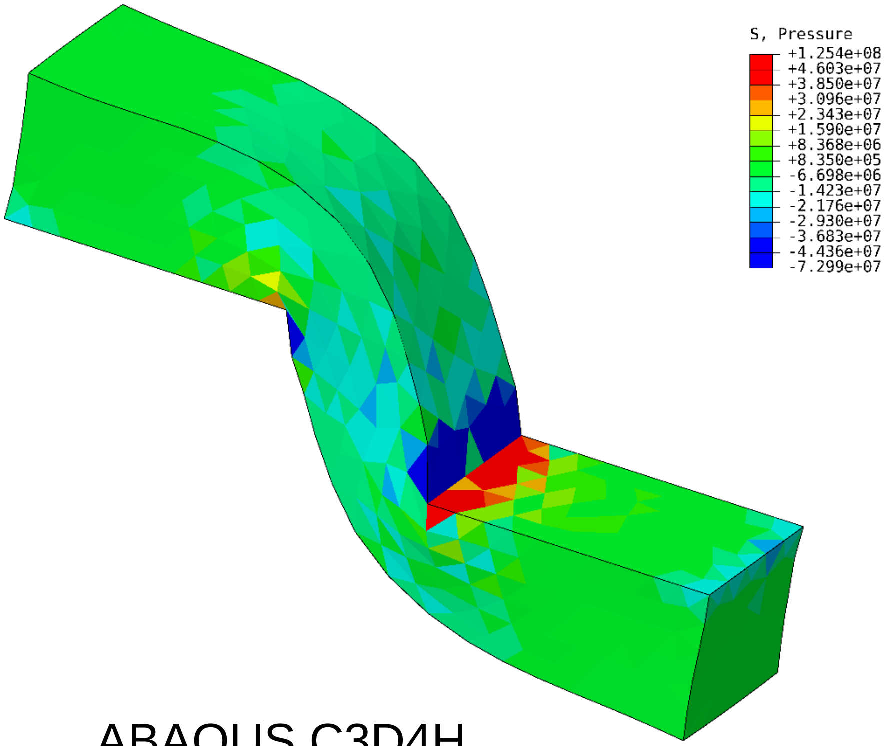
Shearing & Tensioning of Elasto-Plastic Bar

Pressure

$u_z = 2.0$ m



F-barES-FEM-T4

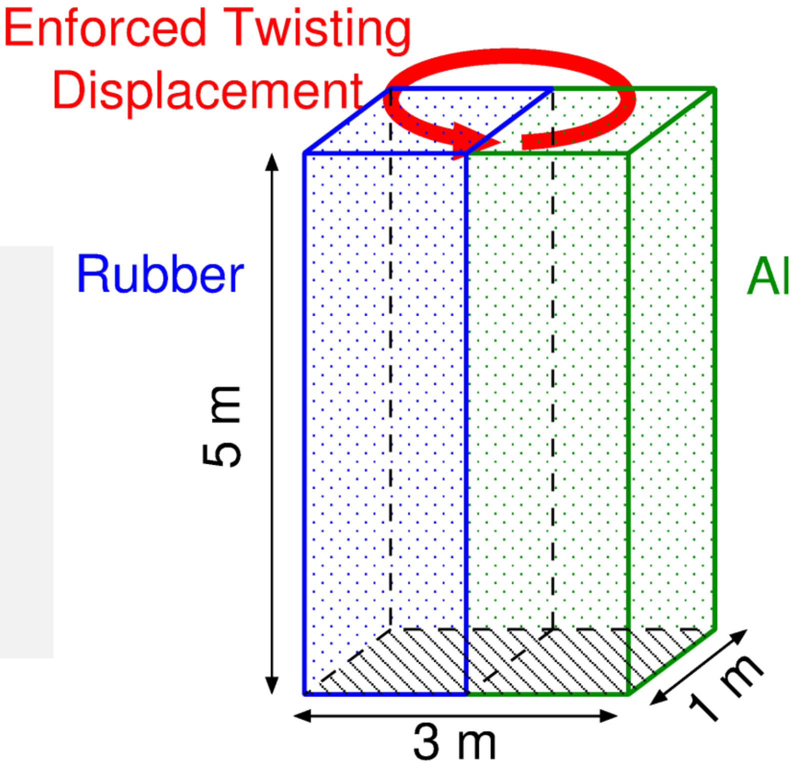


ABAQUS C3D4H

F-barES-FEM-T4 is pressure oscillation free in elastoplastic analysis.

Twist of Rubber/Aluminium Composite Plate

Outline



3 k nodes & 14 k elems.

[Rubber]
Neo-Hook
Hyperelasticity:
$E^{\mathrm{ini}} = 5\ \mathrm{MPa}$,
$\nu^{\mathrm{ini}} = 0.49$,
$(c = 1)$

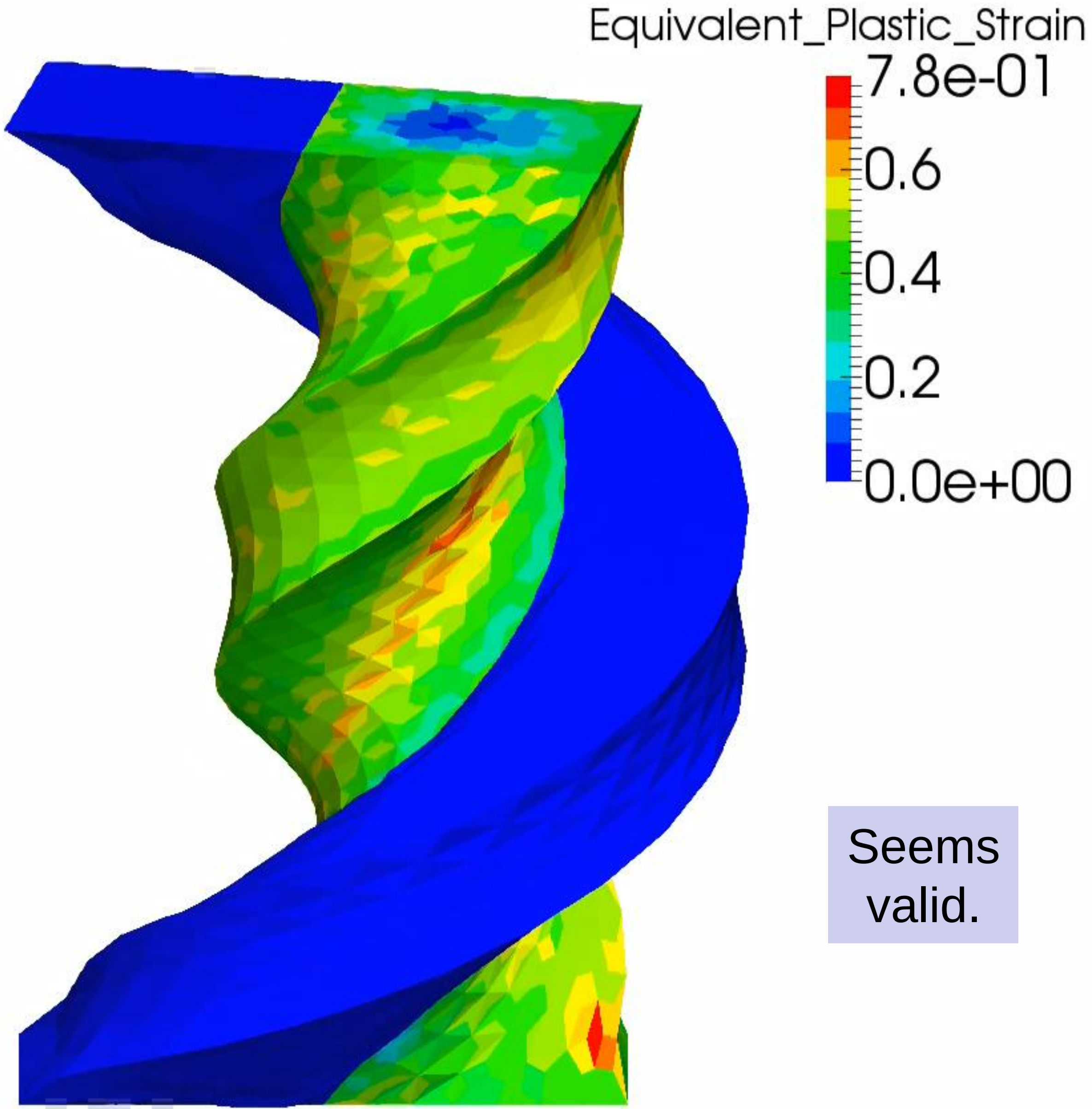
[Aluminium]
Hencky elasticity:
$E = 70\ \mathrm{GPa}$,
$\nu = 0.3$.
Isotropic von Mises
plasticity:
$\sigma_{\mathrm{Y}} = 100\ \mathrm{MPa}$,
$H = 0.7\ \mathrm{GPa}$ (const.),
$(c = 2)$

- Bottom face is perfectly constrained.
- Top face is constrained in the plane
  and twisted 360 deg. around the vertical axis.
- Calculated by F-barES-FEM-T4 only. (Just a demo.)
- Multiple Fs at edges on the material interface.

Twist of Rubber/Aluminium Composite Plate

Result of
F-bar
ES-FEM-T4

Equivalent Plastic Strain

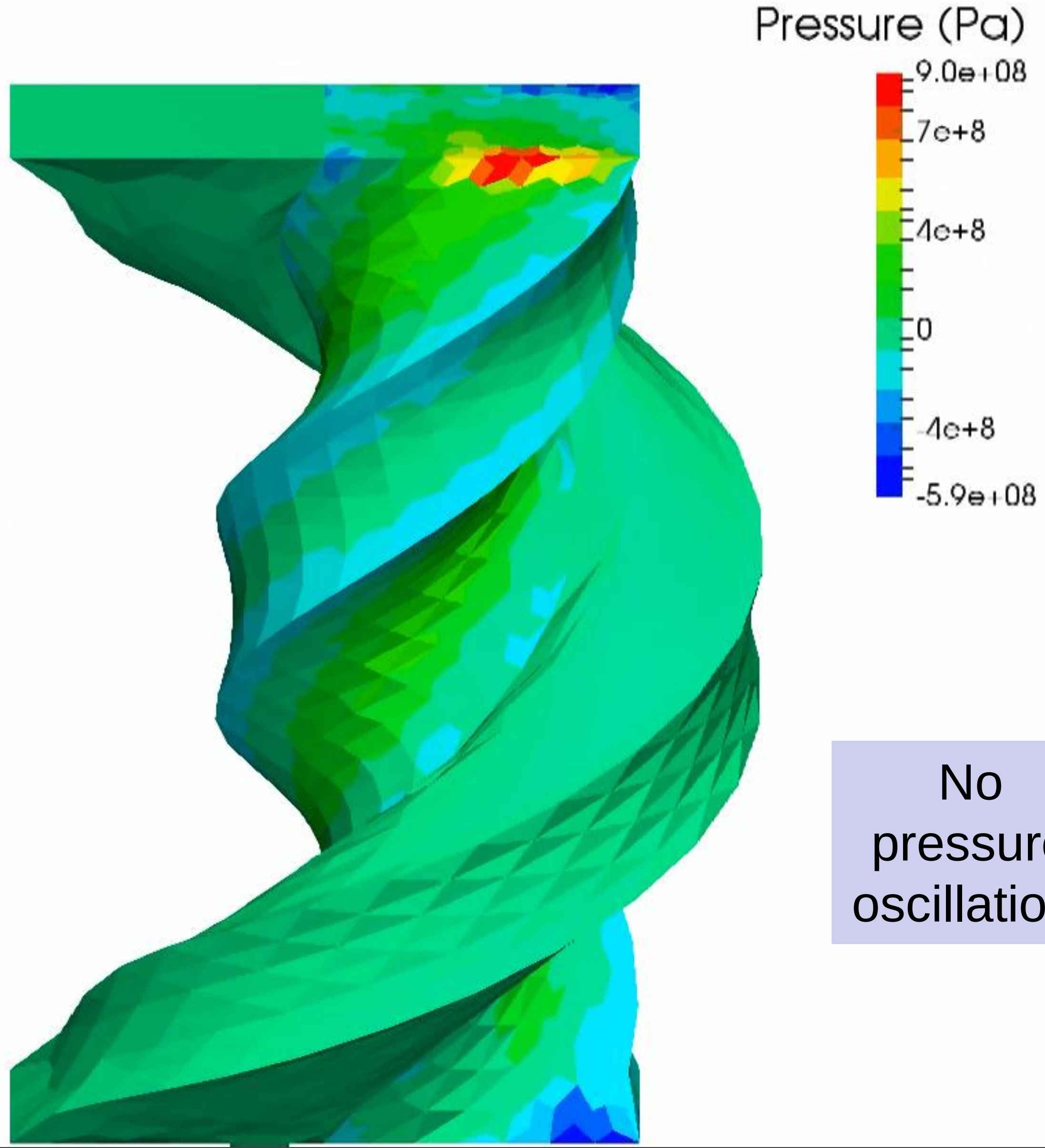


Seems valid.

Twist of Rubber/Aluminium Composite Plate

Result of
F-bar
ES-FEM-T4

Pressure

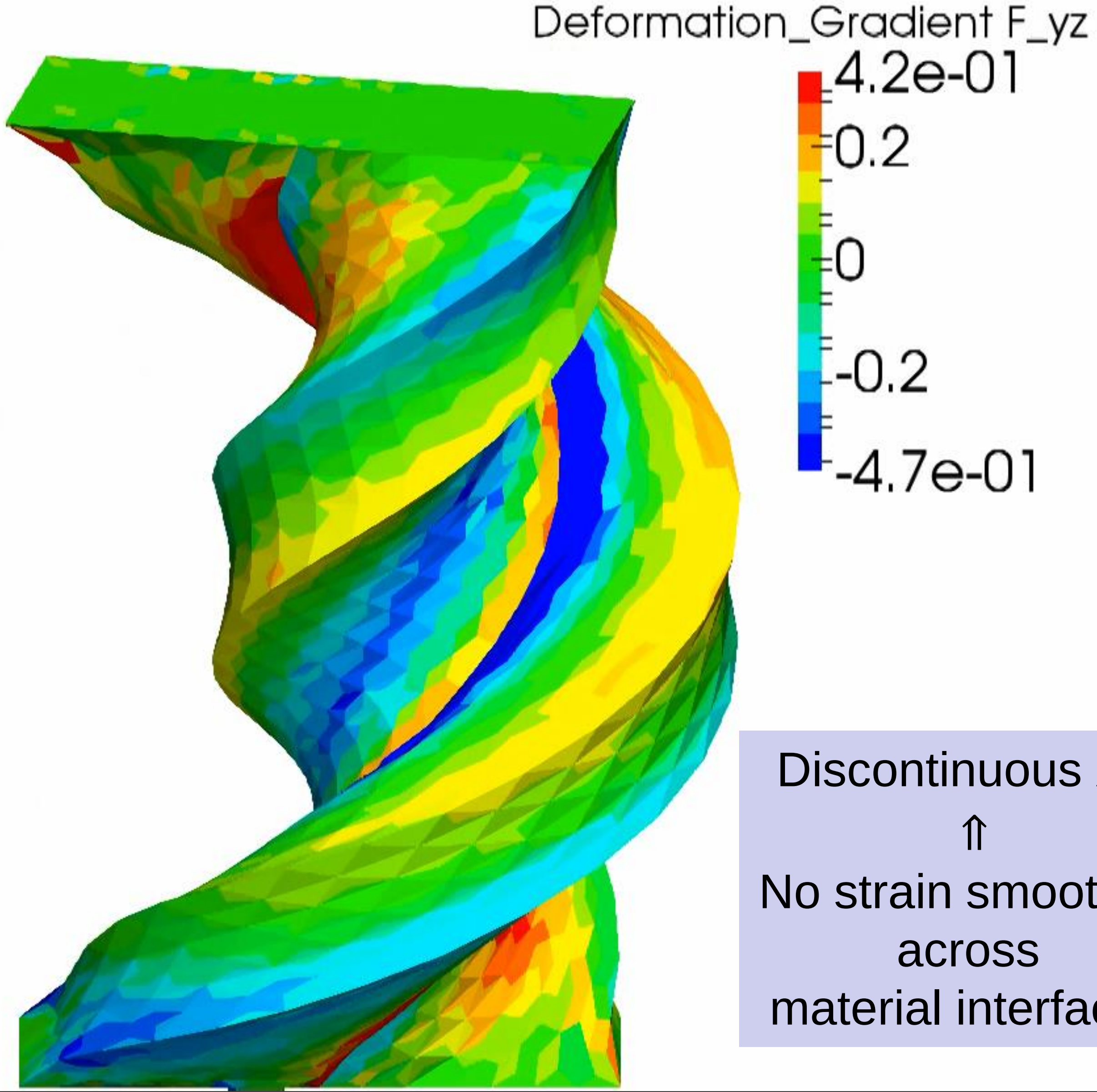


No pressure oscillation.

Twist of Rubber/Aluminium Composite Plate

Result of
F-bar
ES-FEM-T4

Deformation Gradient F_{yz}



Discontinuous F_{yz}.

⇑

No strain smoothing across material interfaces.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

✓ Locking-free with 1st order tetra meshes.

No difficulty in severe strain or contact analysis.

✓ No increase in DOF.

Purely displacement-based formulation.

✓ No restriction of material constitutive model.

Pressure dependent models are acceptable.

✓ Less corner locking and pressure oscillation.

More accurate than Selective ES/NS-FEM!

Drawbacks

✗ The more cyclic smoothing necessitates the more CPU time due to the wider bandwidth.

Slower than Selective ES/NS-FEM…

FYI

If you are interested in F-barES-FEM-T4,
please refer to the following paper:

“F-bar aided edge-based smoothed finite element method using tetrahedral elements for finite deformation analysis of nearly incompressible solids, *International Journal for Numerical Methods in Engineering* (***IJNME***), **Jul. 2016**.

Thank you for your kind attention!