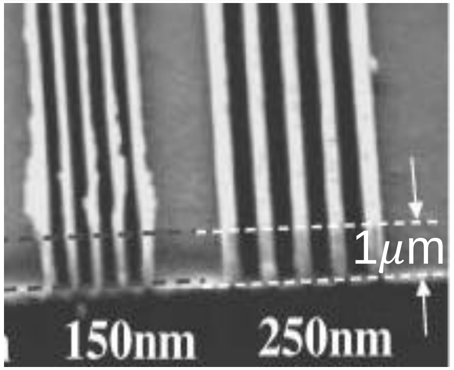
F-bar aided Edge-based Smoothed Finite Element Method with Tetrahedral Elements for Large Deformation Analysis of Nearly Incompressible Materials

Yuki ONISHI
Tokyo Institute of Technology (Japan)

Motivation & Background

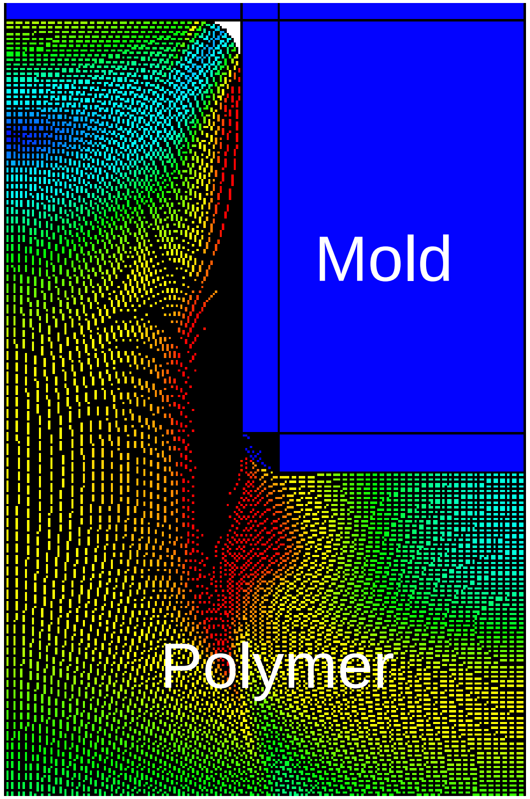
Motivation

We want to analyze **severe large deformation** of nearly incompressible solids ***accurately and stably***!
(Target: automobile tire, thermal nanoimprint, etc.)



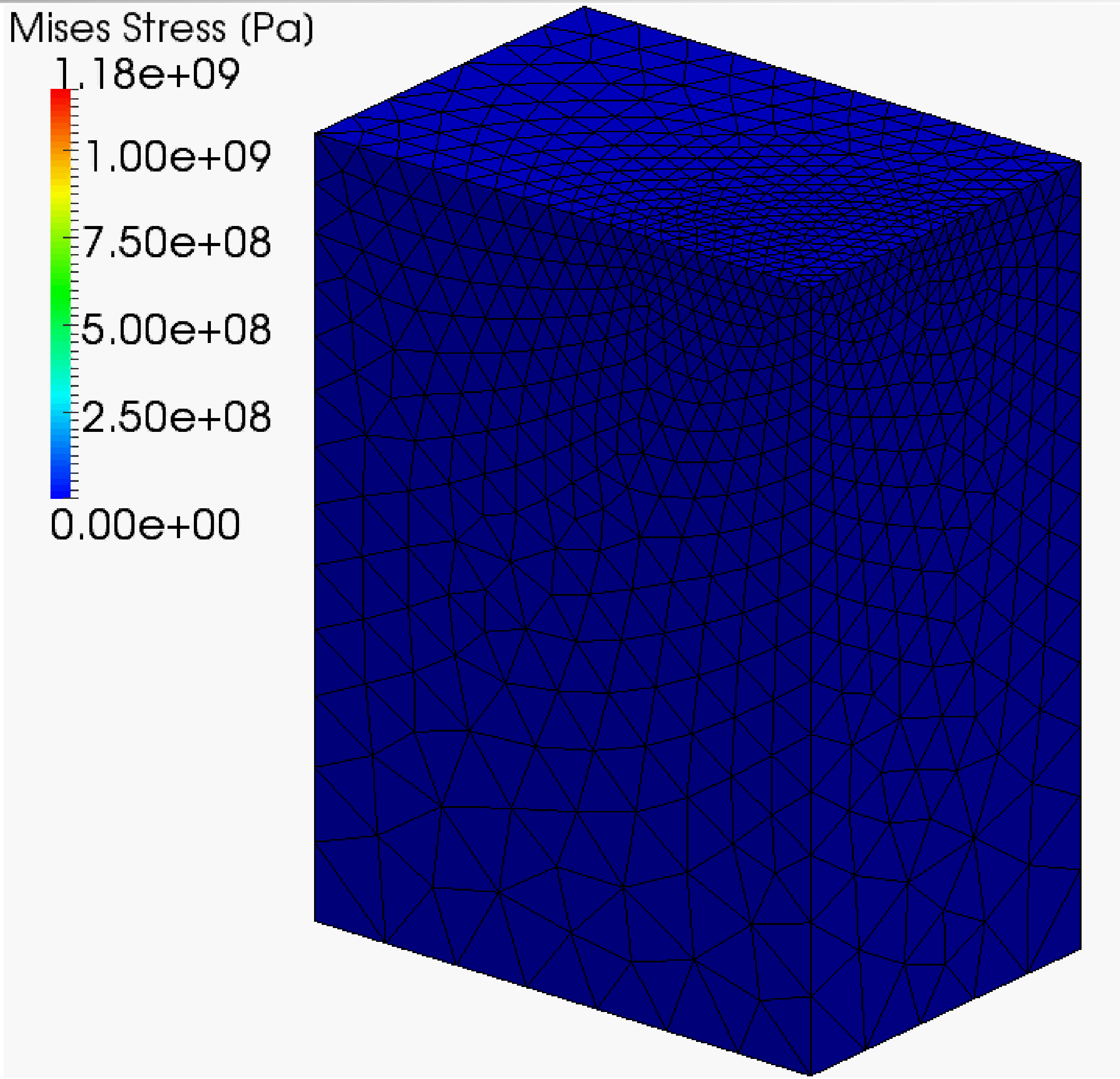
Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.



Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

Our First Result in Advance



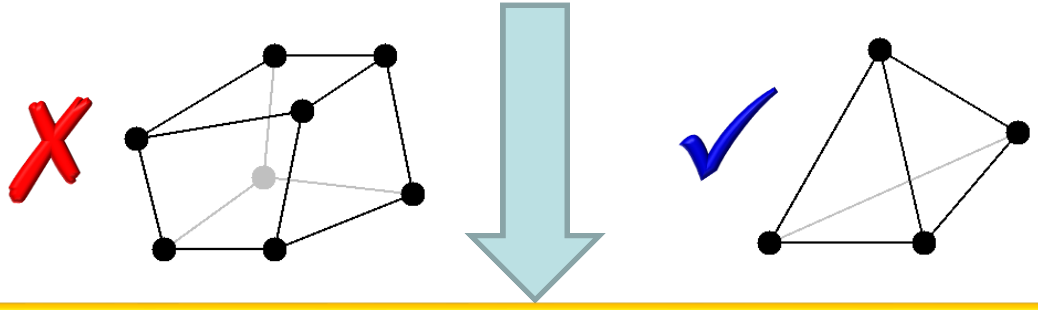
What we want to do:

- Static
- Implicit
- Large deformation
- Mesh rezoning

Issues

The biggest issue
in large deformation mesh rezoning

It is impossible to remesh arbitrary deformed 3D shapes with hexahedral (H8) elements.



We have to use tetrahedral (T4) elements...

However, the *standard* (constant strain) T4 elements easily induce shear and volumetric locking, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Unstable in contact analysis; No good in large deformation due to intermediate nodes.
- EAS method:
  - ✗ Unstable due to spurious zero-energy modes.
- B-bar, F-bar and selective integration method:
  - ✗ Not applicable to T4 mesh directly.
- F-bar patch method:
  - ✗ Difficult to construct good patches. Not shear locking free.
- u/p hybrid (mixed) elements:
  - ✗ No sufficient formulation for T4 mesh so far. (There are almost acceptable hybrid elements such as C3D4H of ABAQUS.)
- Smoothed finite element method (S-FEM):
  - ? Unknown potential (since 2009~). ***It's worth trying!!***

Various Types of S-FEMs

- **Basic type**
  - Node-based S-FEM (NS-FEM)
  - Face-based S-FEM (FS-FEM)
  - Edge-based S-FEM (ES-FEM)

✗ Spurious zero-energy (NS-FEM)

✗ Volumetric Locking (FS-FEM, ES-FEM)

- **Selective type**
  - Selective FS/NS-FEM
  - Selective ES/NS-FEM

✗ Limitation of constitutive model, Pressure oscillation, Corner locking

- **Bubble-enhanced or Hat-enhanced type**
  - bFS-FEM, hFS-FEM
  - bES-FEM, hES-FEM

✗ Pressure oscillation, Short-lasting

- **F-bar type**
  - NEW F-barES-FEM

? Unknown potential

Objective

Develop a new S-FEM, F-barES-FEM-T4,
by combining F-bar method and ES-FEM-T4
for large deformation problems
of nearly incompressible solids

Table of Body Contents

- Method: Formulation of F-barES-FEM-T4
- Result: Verification of F-barES-FEM-T4
- Summary

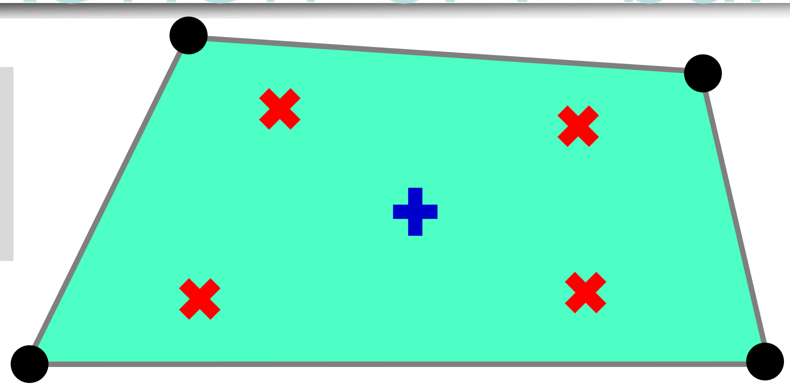
Method

Formulation of F-barES-FEM-T4

(F-barES-FEM-T3 in 2D is explained for simplicity.)

Quick Review of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}$ $(= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\mathrm{iso}}$ $(= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point as

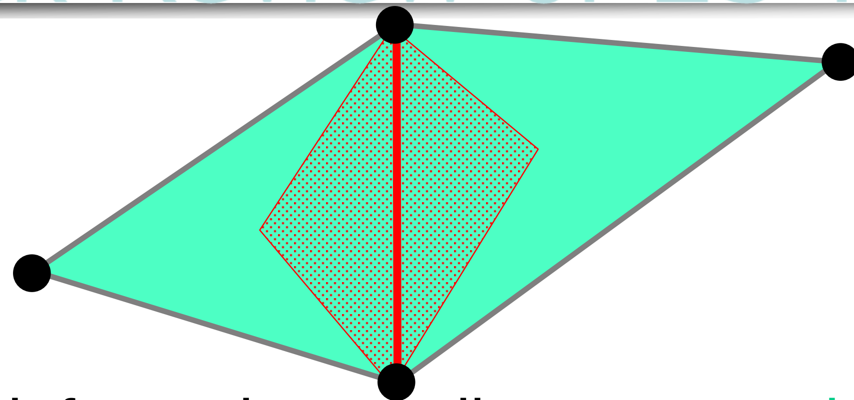
$$\bar{\boldsymbol{F}} = \bar{J}^{\,1/3}\, \boldsymbol{F}^{\mathrm{iso}}.$$

4. Use $\bar{\boldsymbol{F}}$ to calculate the stress, nodal force and so on.

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.

Quick Review of ES-FEM

For triangular (T3) or tetrahedral (T4) elements.



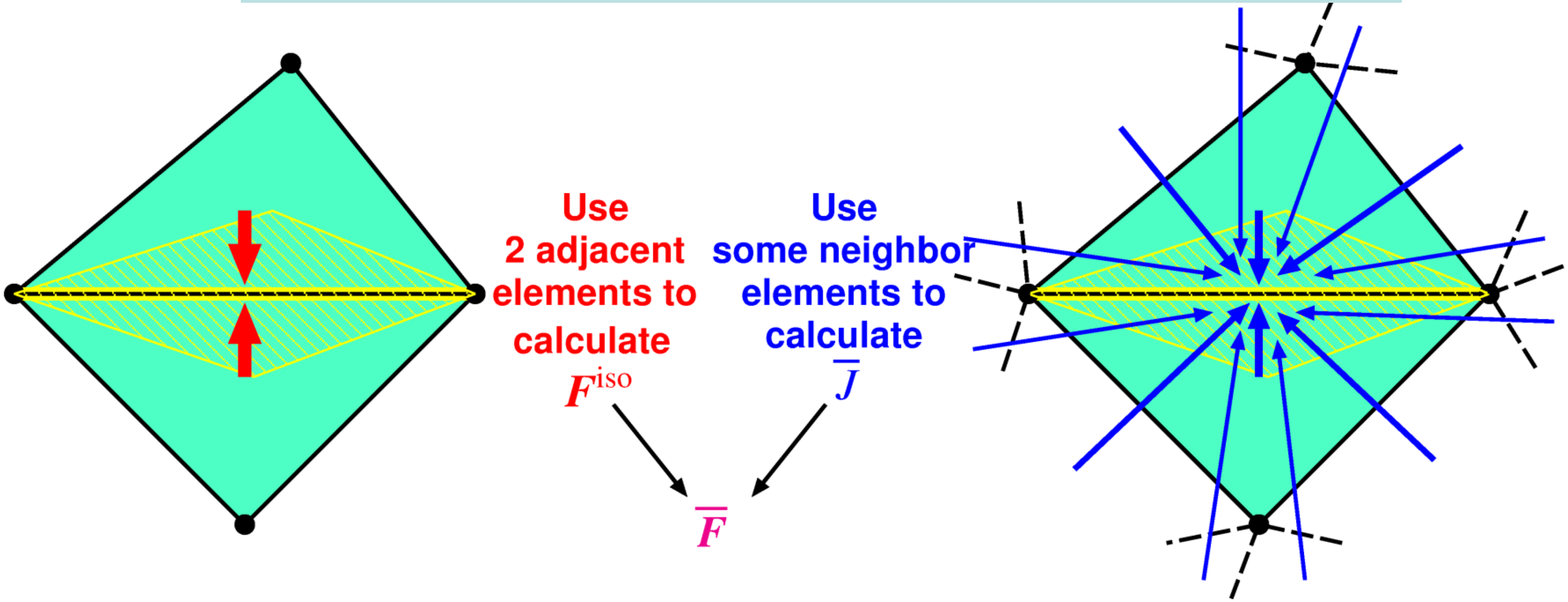
Algorithm:

1. Calculate the deformation gradient $\boldsymbol{F}$ at each element as usual.
2. Distribute the deformation gradient $\boldsymbol{F}$ to the connecting edges with area weights to make ${}^{\mathrm{Edge}}\boldsymbol{F}$ at each edge.
3. Use ${}^{\mathrm{Edge}}\boldsymbol{F}$ to calculate the stress, nodal force and so on.

ES-FEM is used to **avoid shear locking** in T3 or T4 elements. Yet, it **cannot avoid volumetric locking**.

Outline of F-barES-FEM

Concept Combination of F-bar method and ES-FEM



- ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\overline{J}$ is given by Cyclic Smoothing (detailed later).
- ${}^{\mathrm{Edge}}\overline{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

$$ {}^{\mathrm{Edge}}\overline{\boldsymbol{F}} = {}^{\mathrm{Edge}}\overline{J}^{\,1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Outline of F-barES-FEM

Brief Formulation

1. Calculate ${}^{\mathrm{Elem}}J$ as usual.
2. Smooth ${}^{\mathrm{Elem}}J$ at nodes and get ${}^{\mathrm{Node}}\tilde{J}$.
3. Smooth ${}^{\mathrm{Node}}\tilde{J}$ at elements and get ${}^{\mathrm{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

5. Smooth ${}^{\mathrm{Elem}}\tilde{\tilde{J}}$ (c layers of ~) at edges to make ${}^{\mathrm{Edge}}\bar{J}$.
6. Combine ${}^{\mathrm{Edge}}\bar{J}$ and ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ of ES-FEM as

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{\,1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Hereafter, F-barES-FEM-T4 with c-time cyclic smoothing is called "F-barES-FEM-T4(c)".

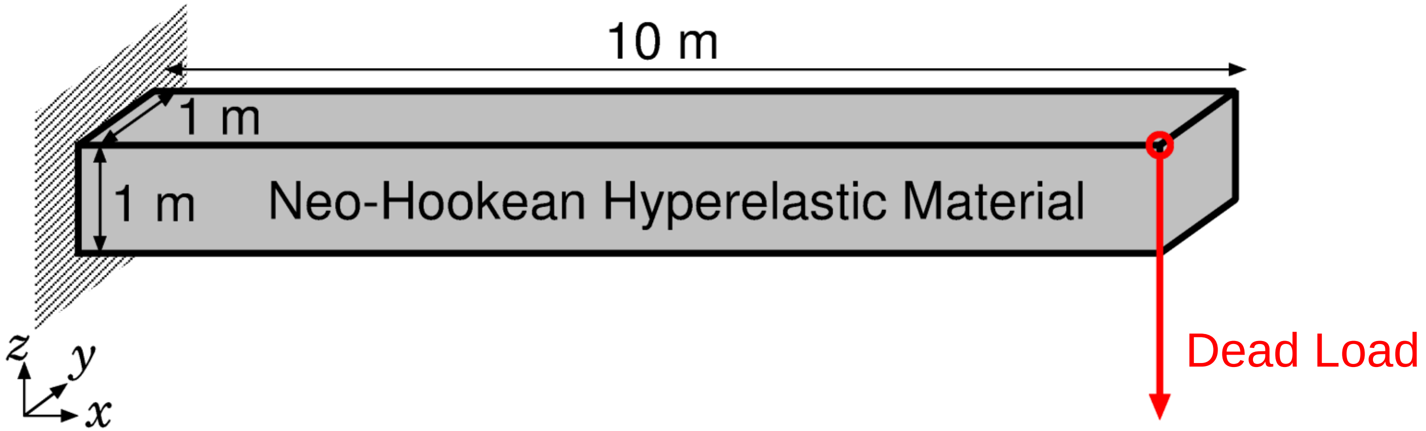
Result

Verification of F-barES-FEM-T4

(Analyses without mesh rezoning are presented for pure verification.)

#1: Bending of a Cantilever

Outline



- Neo-Hookean hyperelastic material

$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s so that the initial Poisson's ratios are 0.49 and 0.499.

- Two types of tetra meshes: structured and unstructured.
- Compared to ABAQUS C3D4H (1st-order hybrid tetrahedral element).

#1: Bending of a Cantilever

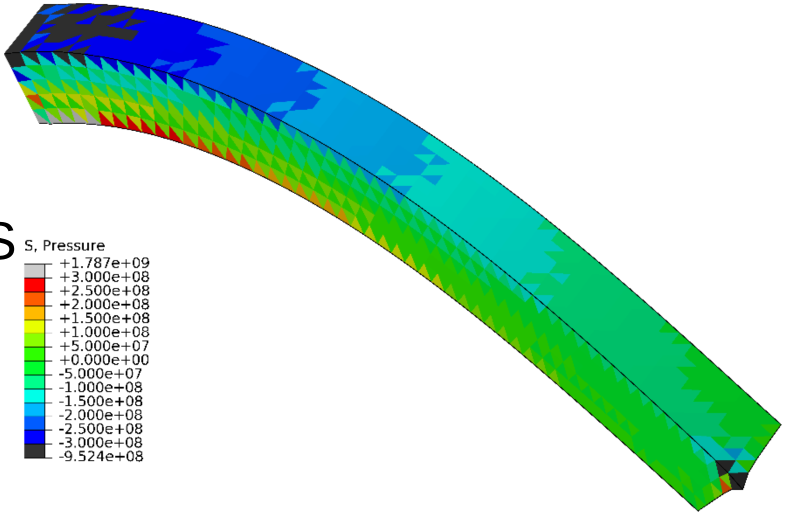
Pressure Distributions

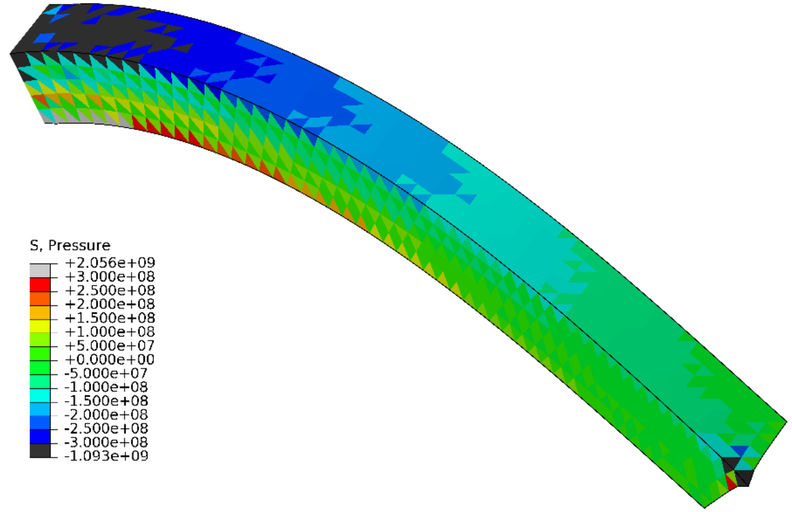
$\nu^{\text{ini}} = 0.49$

Structured Mesh

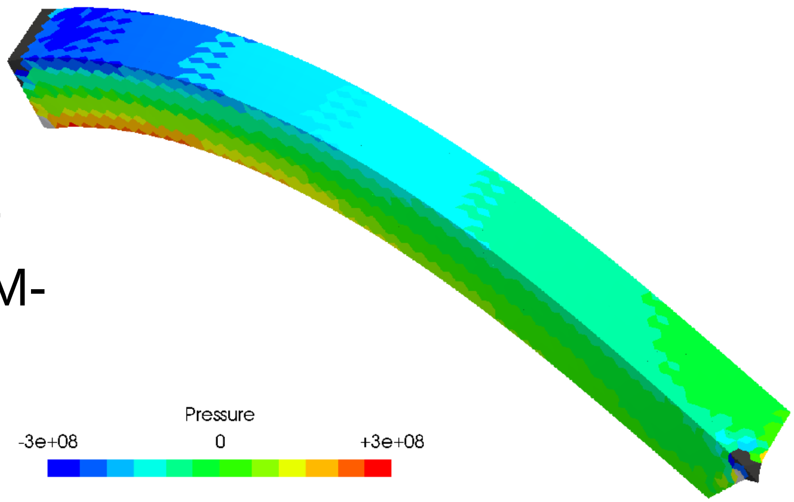
$\nu^{\text{ini}} = 0.499$

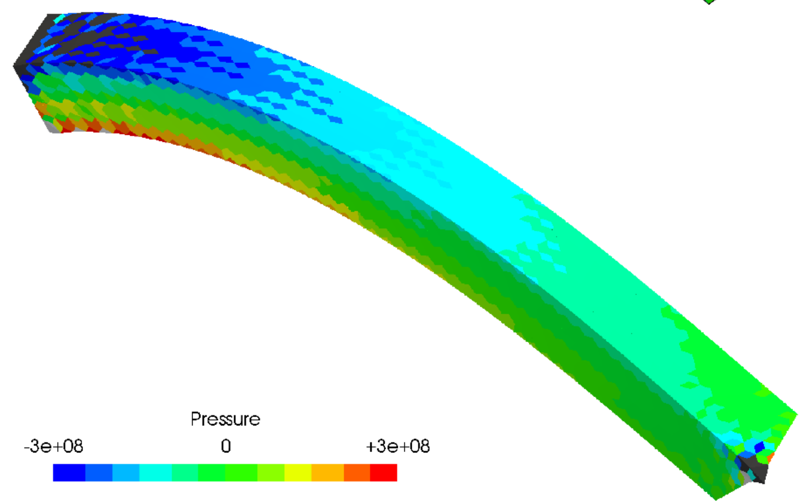
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





#1: Bending of a Cantilever

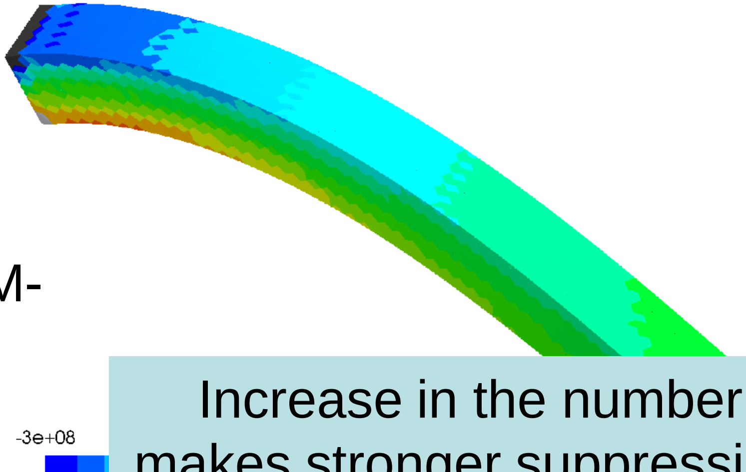
Pressure Distributions

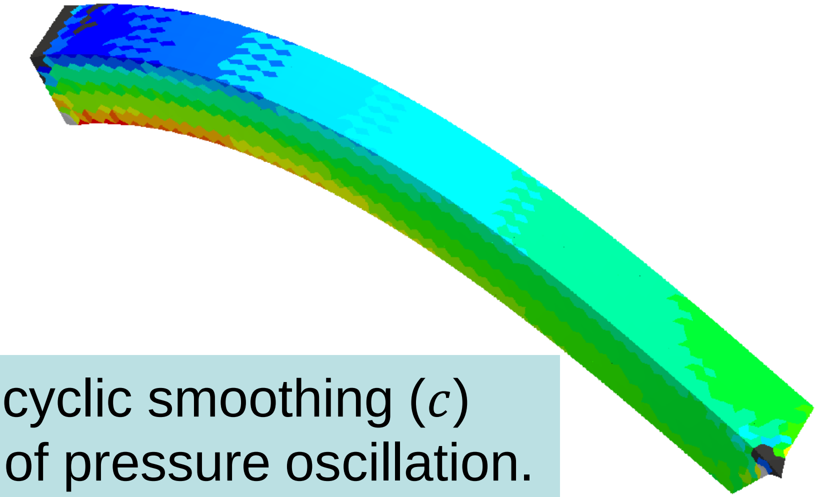
$\nu^{\text{ini}} = 0.49$

Structured Mesh

$\nu^{\text{ini}} = 0.499$

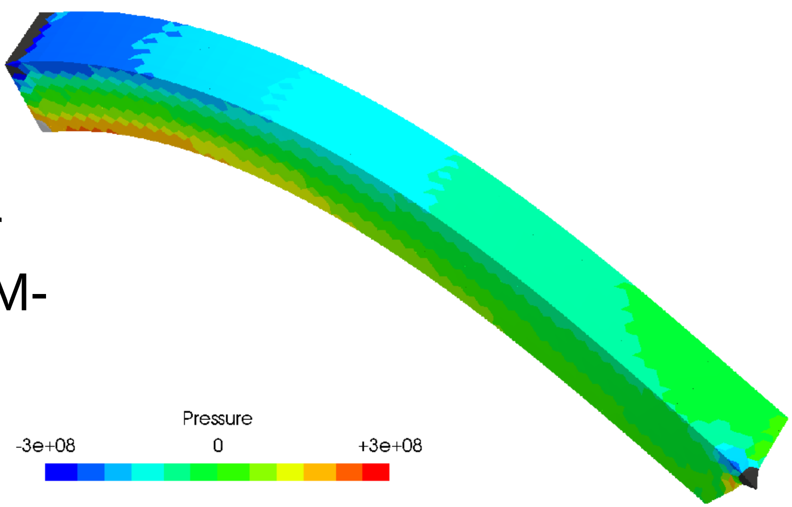
F-bar ES-FEM-T4(2)

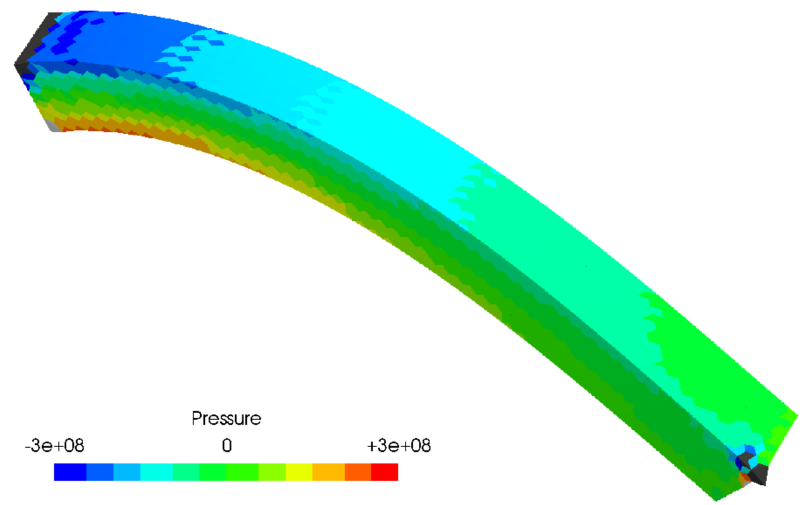




Increase in the number of cyclic smoothing (c) makes stronger suppression of pressure oscillation.

F-bar ES-FEM-T4(3)





#1: Bending of a Cantilever

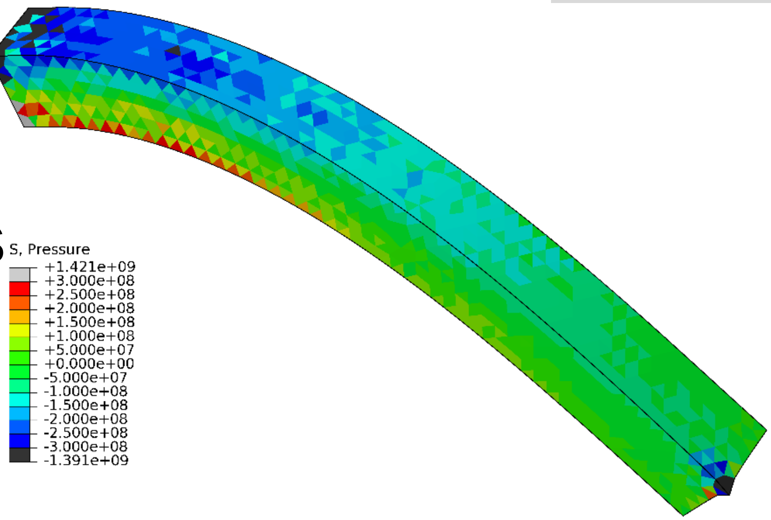
Pressure Distributions

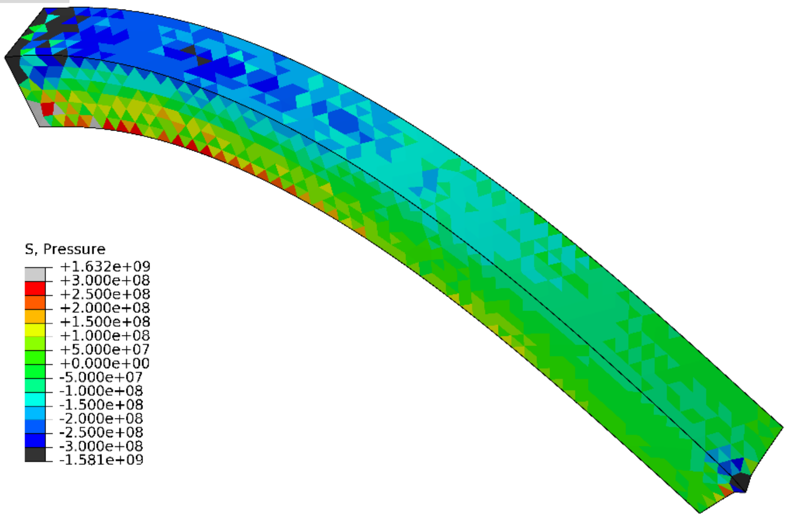
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

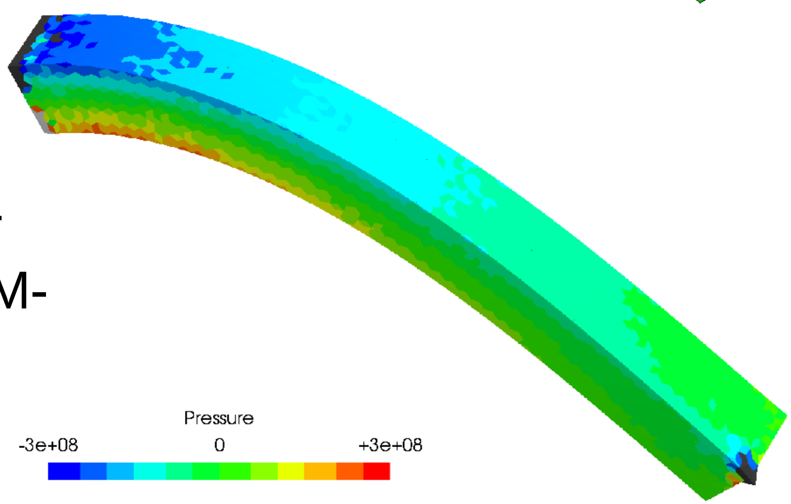
$\nu^{\text{ini}} = 0.499$

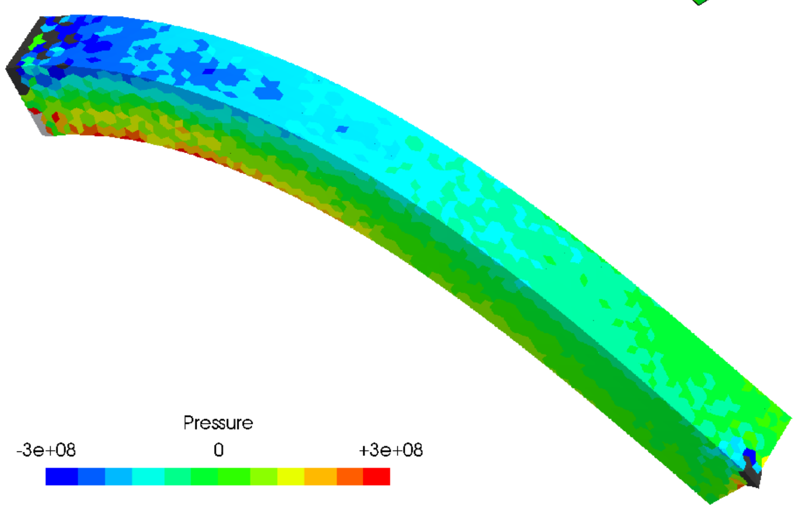
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





#1: Bending of a Cantilever

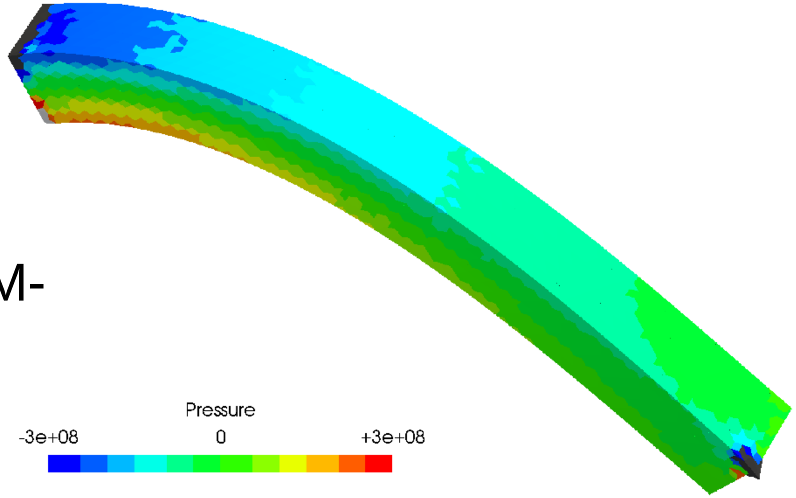
Pressure Distributions

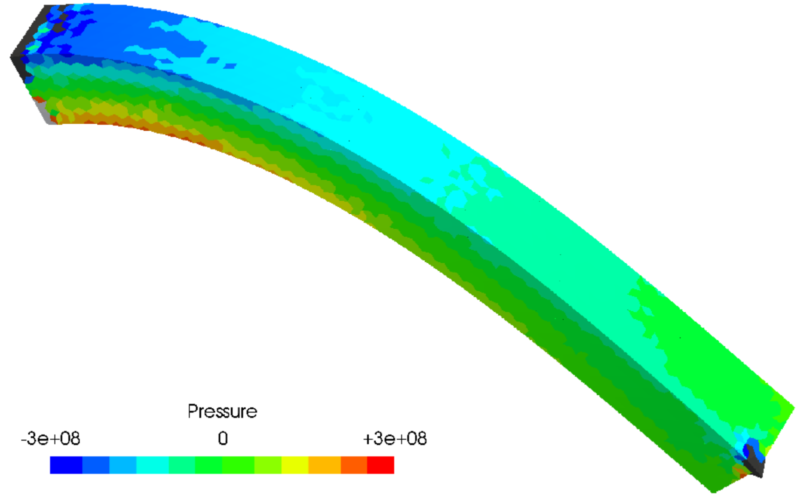
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

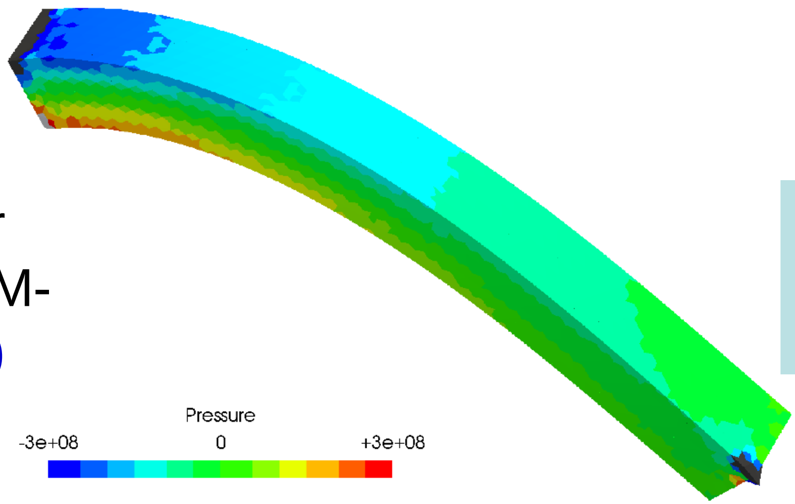
$\nu^{\text{ini}} = 0.499$

F-bar ES-FEM-T4(2)

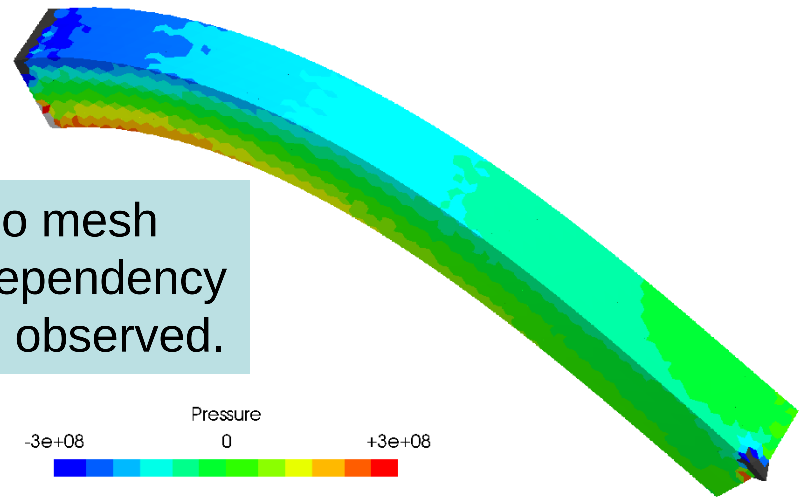




F-bar ES-FEM-T4(3)

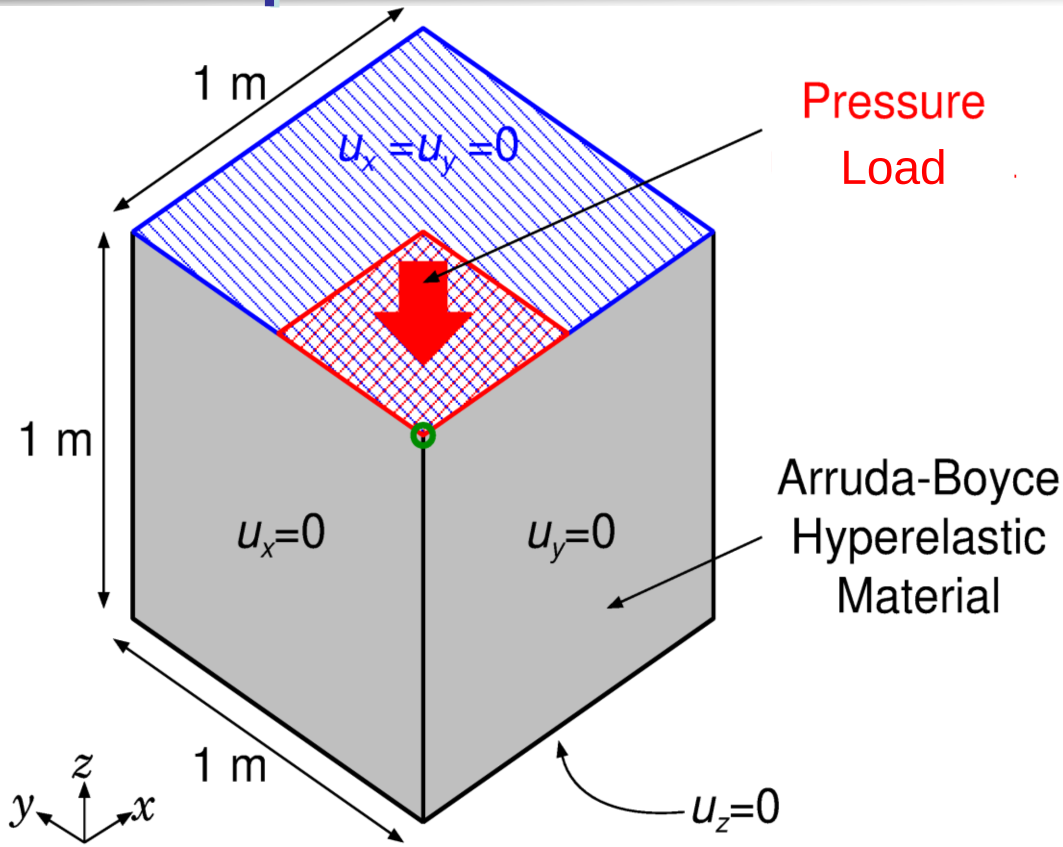


No mesh dependency is observed.



#2: Compression of a Block

Outline



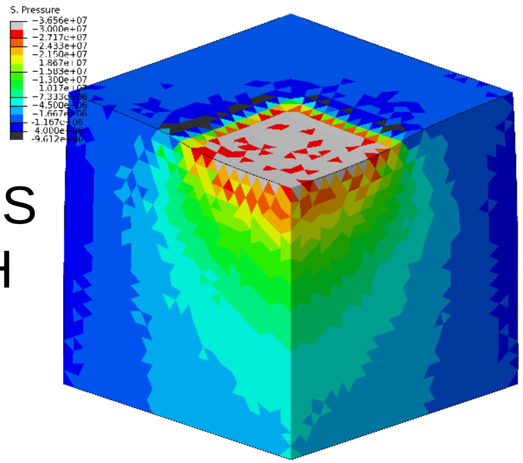
- Arruda-Boyce hyperelastic material ($\nu_{\text{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured tetra mesh.

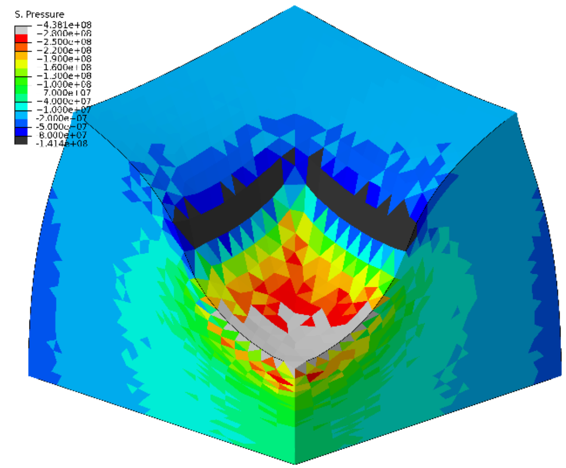
#2: Compression of a Block

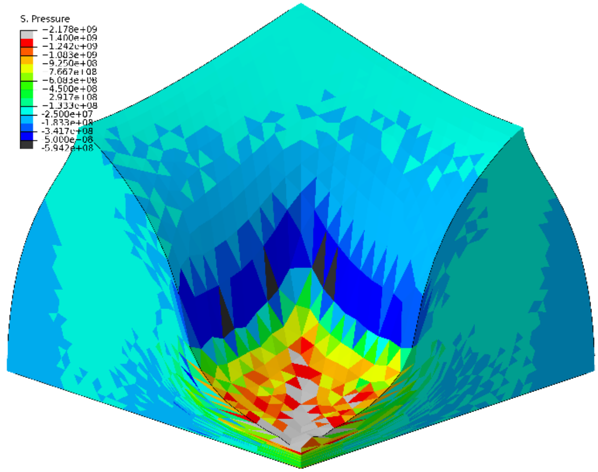
Pressure Distribution

Early stage | Middle stage | Later stage

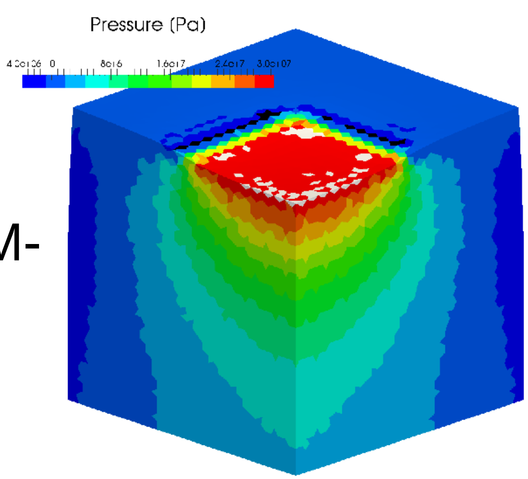
ABAQUS C3D4H

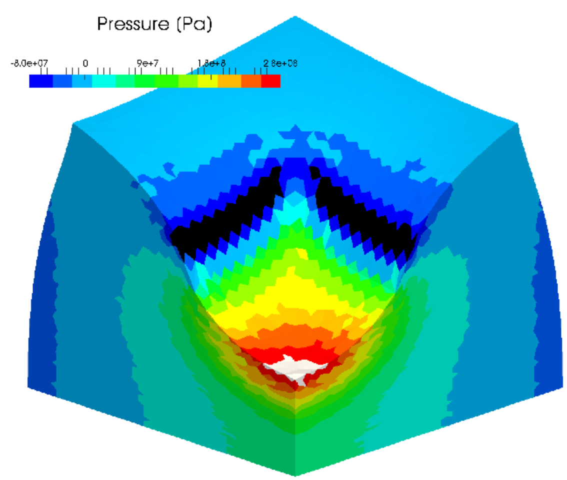


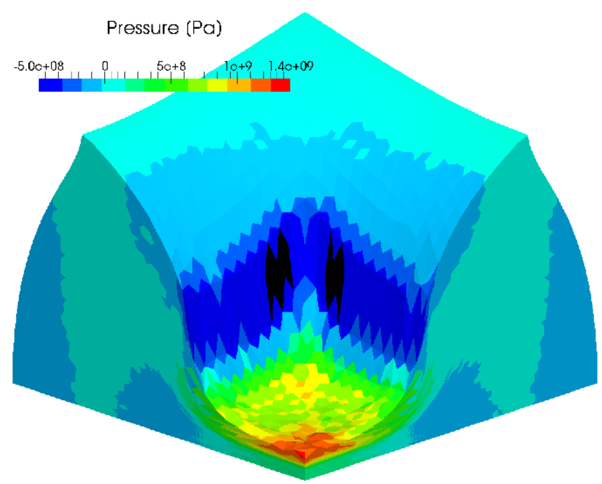




F-bar ES-FEM-T4(2)





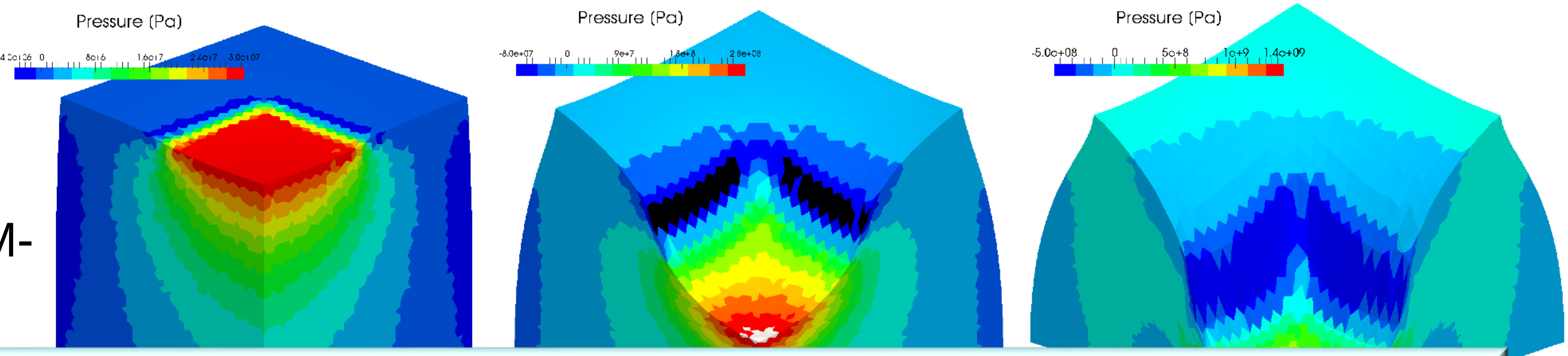


#2: Compression of a Block

Pressure Distribution

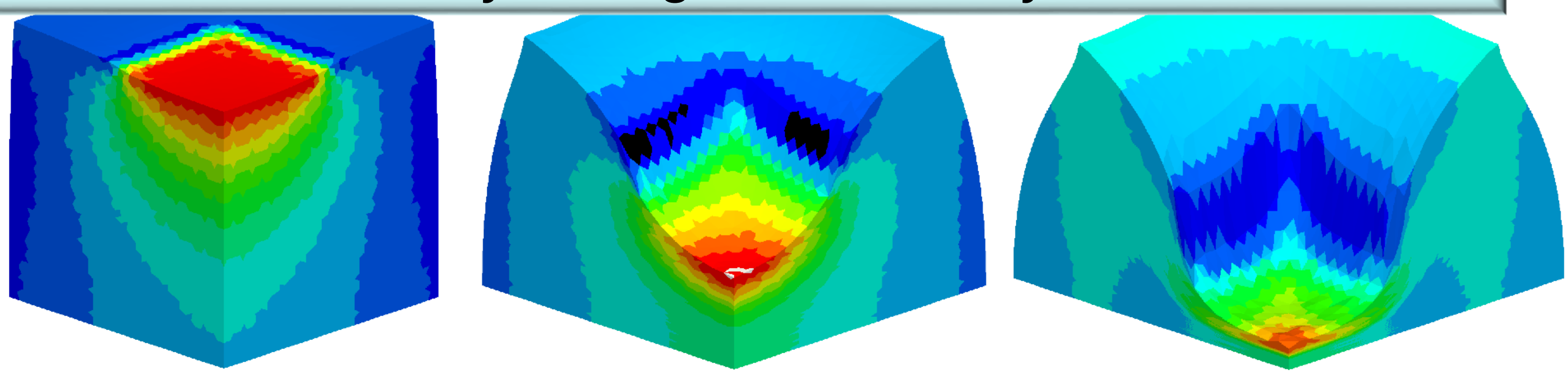
Early stage | Middle stage | Later stage

F-bar ES-FEM-T4(3)



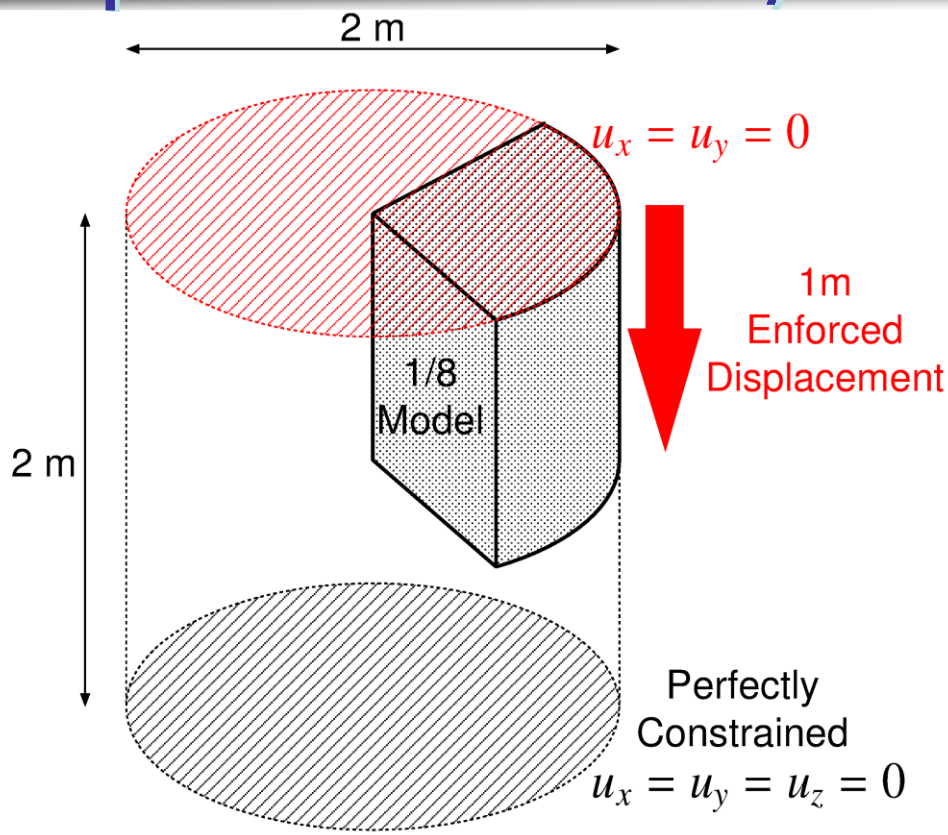
In case the Poisson's ratio is 0.499,
F-barES-FEM-T4(2) or later can suppress pressure oscillation successfully in large strain analysis.

F-bar ES-FEM-T4(4)



#3: Compression of 1/8 Cylinder

Outline

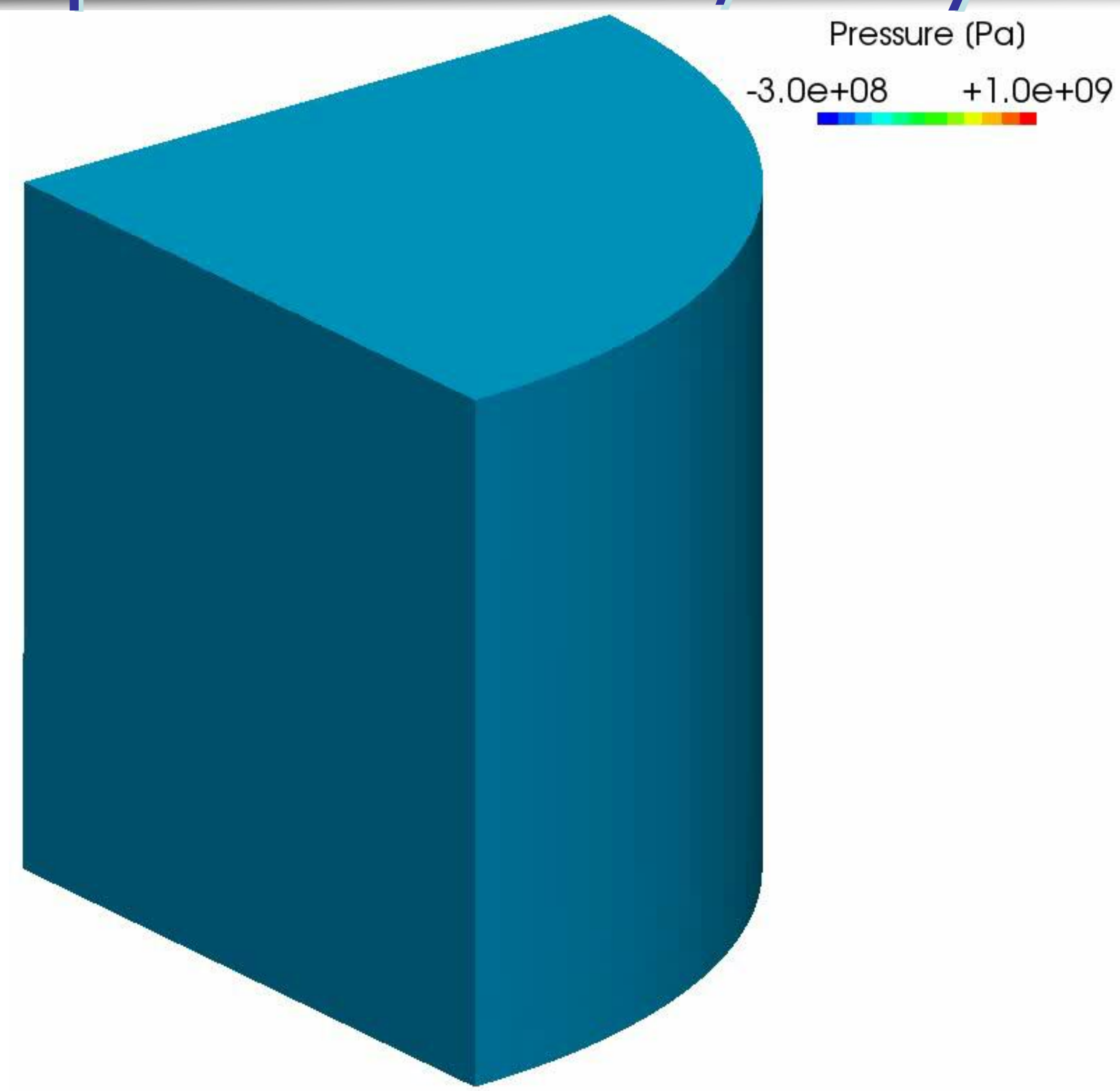


- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Enforced displacement is applied to the top surface.
- Compared to ABAQUS C3D4H with the same unstructured tetra mesh.

#3: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

50% nominal compression

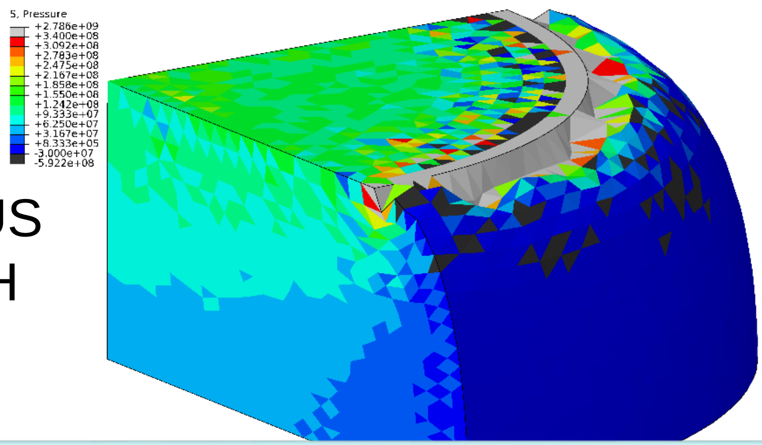


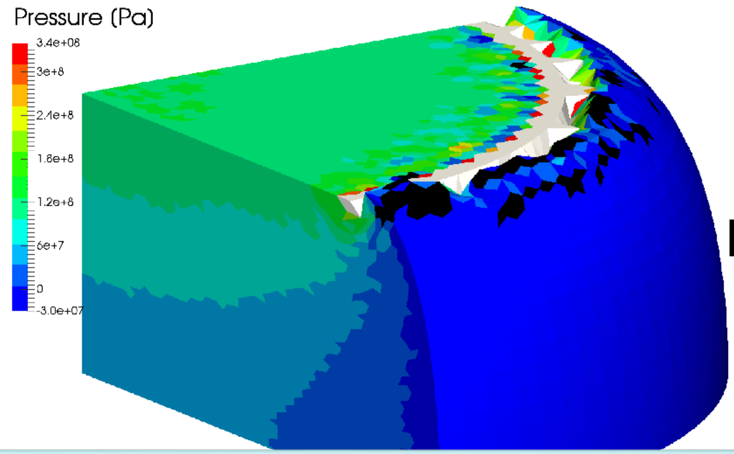
Almost smooth pressure distribution is obtained except just around the rim.

#3: Compression of 1/8 Cylinder

Pressure Distribution

ABAQUS C3D4H

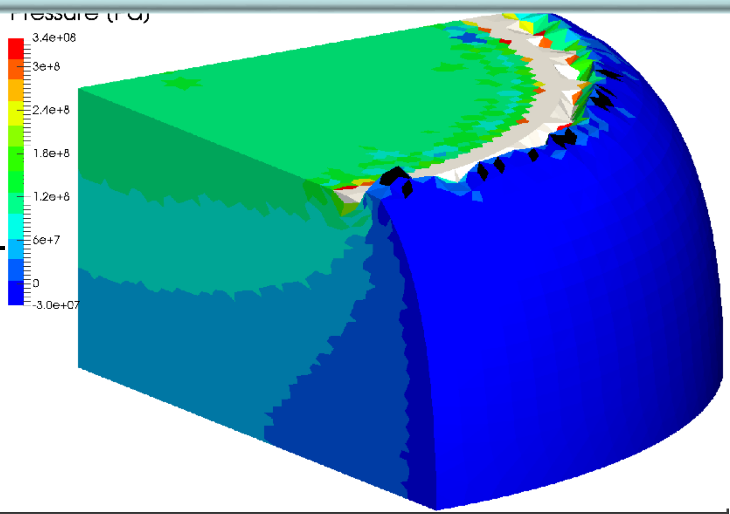


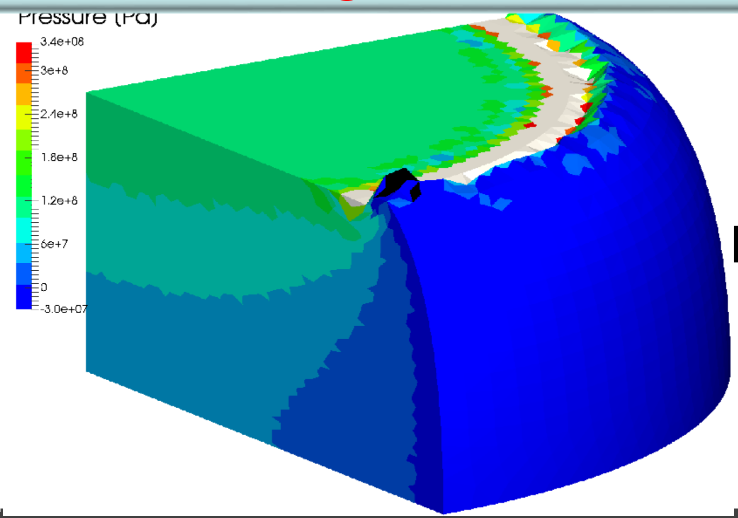


F-bar ES-FEM-T4(2)

F-barES-FEM-T4 with a sufficient cyclic smoothing also resolves the corner locking issue.

F-bar ES-FEM-T4(3)





F-bar ES-FEM-T4(4)

Characteristics of F-barES-FEM-T4

Benefits

- ✓ Locking-free with tetra meshes.
- ✓ No increase in DOF;
  No need of static condensation;
  Easy extension to dynamic explicit analysis.
- ✓ No difficulty in contact analysis.
- ✓ Adjustable smoothing level by changing the number of cyclic smoothings (c).
- ✓ Suppression of pressure oscillation in nearly incompressible materials.

In point of accuracy, F-barES-FEM-T4 is excellent!!

Characteristics of F-barES-FEM-T4

Drawbacks

- ✗ Blur of high-frequency pressure distribution.
  (The impact of blur seem to be not significant.)
- ✗ Increase in bandwidth of the exact tangent stiffness $[K]$.
  In case of standard unstructured T4 meshes,

Method	Approx. Bandwidth	Approx. Ratio
Standard FEM-T4	40	1
F-barES-FEM(1)	390	x10
F-barES-FEM(2)	860	x20
F-barES-FEM(3)	1580	x40
F-barES-FEM(4)	2600	x65

In point of speed, F-barES-FEM-T4 needs some improvements.
e.g.) finding a good sparse approximation of $[K]$
for iterative matrix solvers.

Summary

Summary

- A new FE formulation named "F-barES-FEM-T4" is proposed.
- F-barES-FEM-T4 combines the F-bar method and ES-FEM-T4.
- Owing to the cyclic smoothing, F-barES-FEM-T4 is locking-free and also pressure oscillation-free with no increase in DOF.
- Only one drawback of F-barES-FEM-T4 is the decrease of calculation speed due to the increase in bandwidth of $[K]$, which is our future work to solve.

Thank you for your kind attention!

Appendix

Characteristics of FEM-T4s

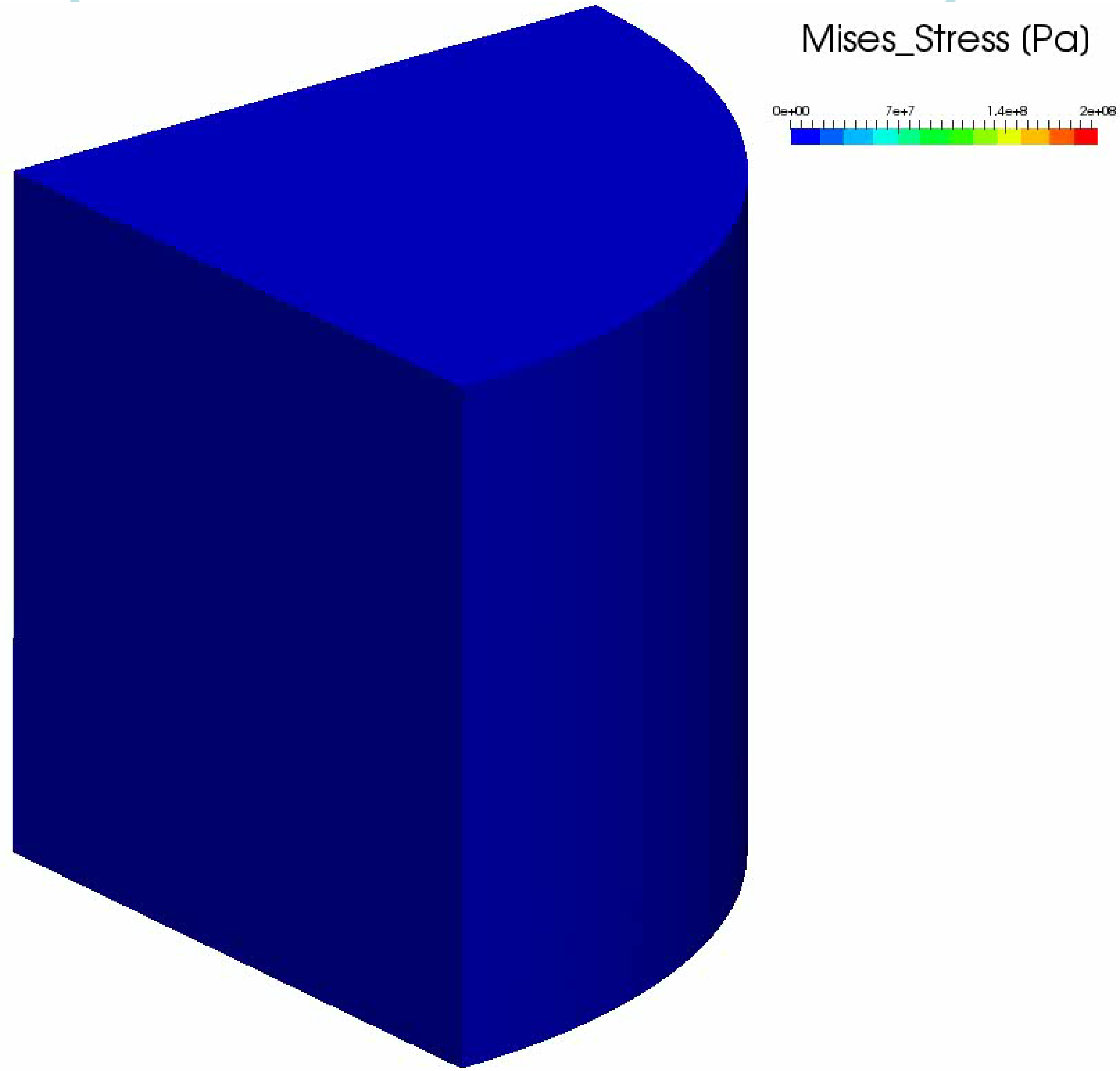
	Shear & Volumetric Locking	Zero-Energy Mode	Dev/Vol Coupled Material	Pressure Oscillation	Corner Locking	Severe Strain
Standard FEM-T4	✗	✓	✓	✗	✗	✓
ABAQUS C3D4H	✓	✓	✓	✗	✗	✓
Selective S-FEM-T4	✓	✓	✗	✗	✗	✓
bES-FEM-T4 hES-FEM-T4	✓	✓	✓	✗	✗	✗
F-bar ES-FEM-T4	✓	✓	✓	✓*	✓*	✓

*) when the num. of cyclic smoothings is sufficiently large.

#3: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

50% nominal compression



Smooth Mises stress distribution is obtained except just around the rim.