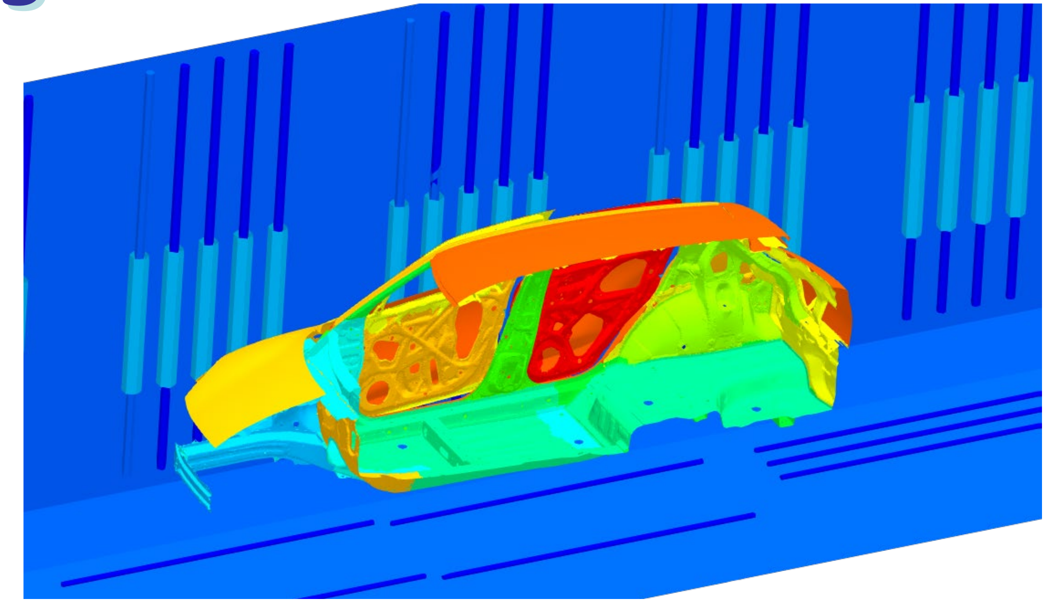
Accurate electrodeposition simulation of automobile bodies with edge-based smoothed finite element method using 4-node tetrahedral meshes



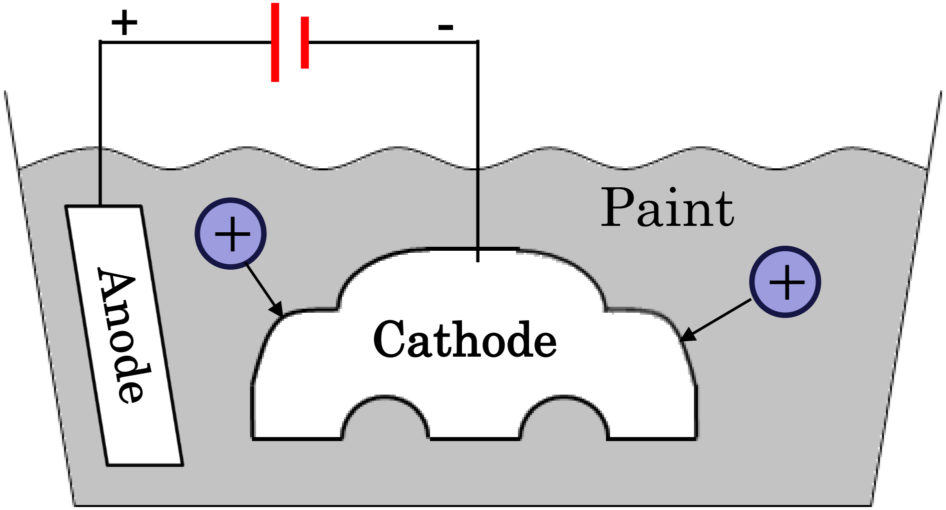
Yuki ONISHI

Tokyo Institute of Technology (Japan)

What is Electrodeposition (ED) ?

Outline

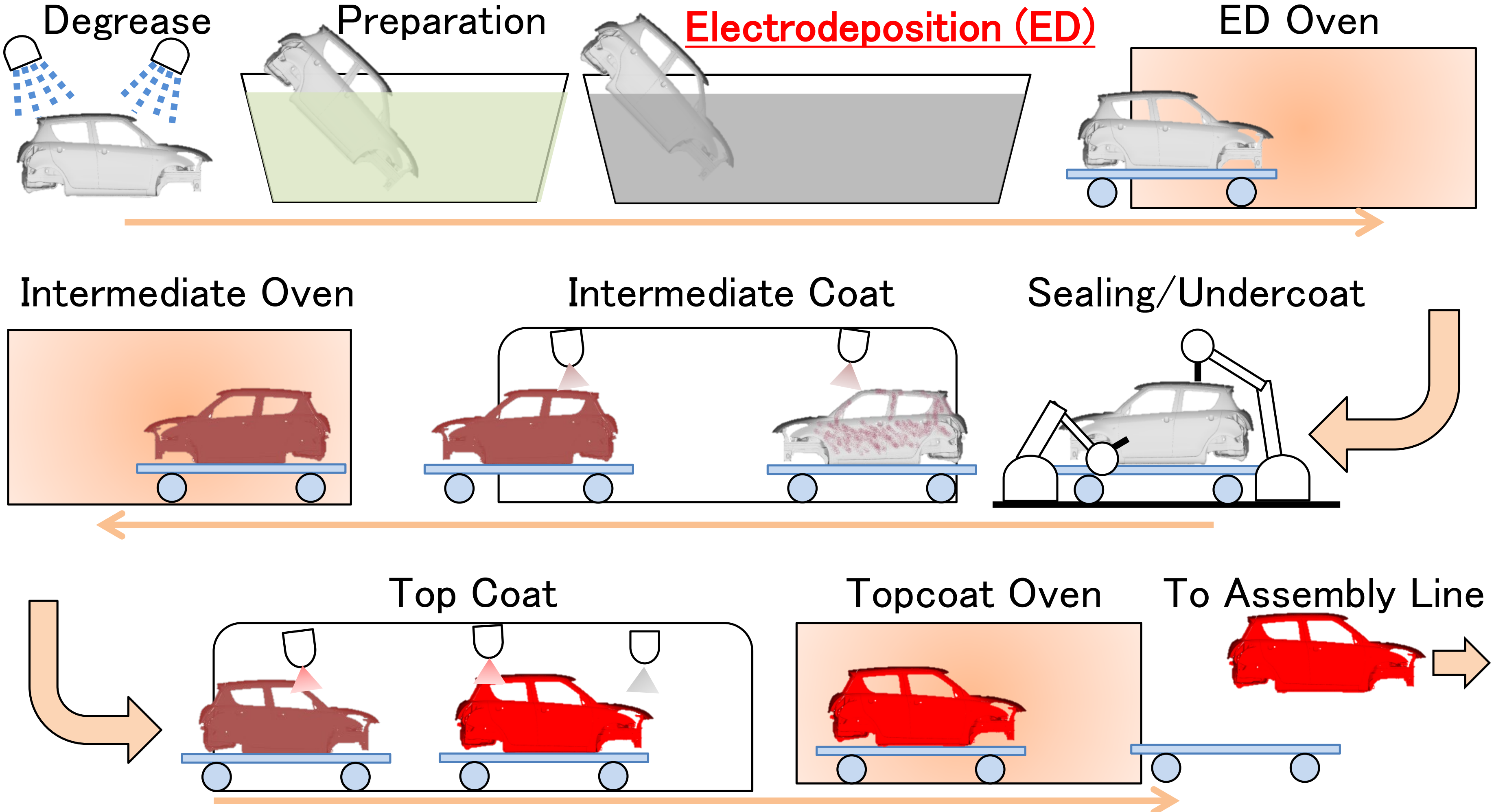

http://n-link.nissan.co.jp/



- Most widely-used **anti-rust basecoat** methods for various metal products including carbodies.
- Depositing coating film by applying **direct electric current** in a paint pool.
- Relatively good at depositing a **uniform film** on bodies in complex shape.

What is Electrodeposition (ED) ?

Overview of the Carbody Paint Shop

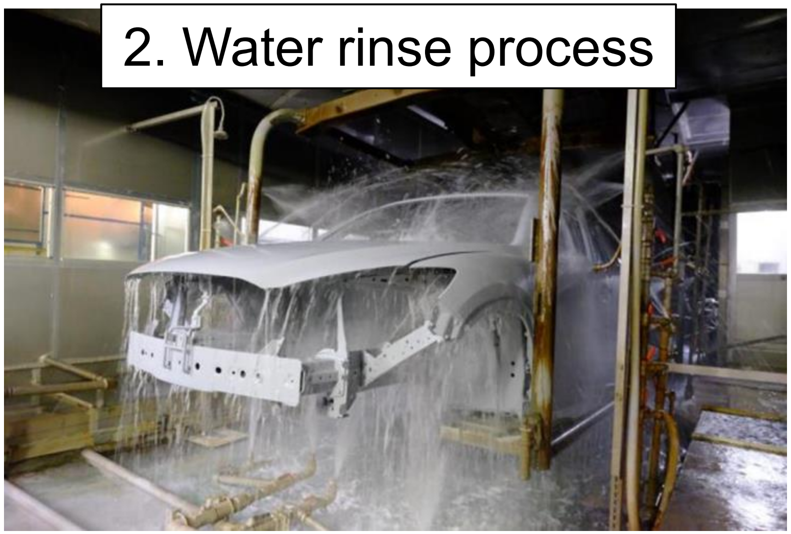


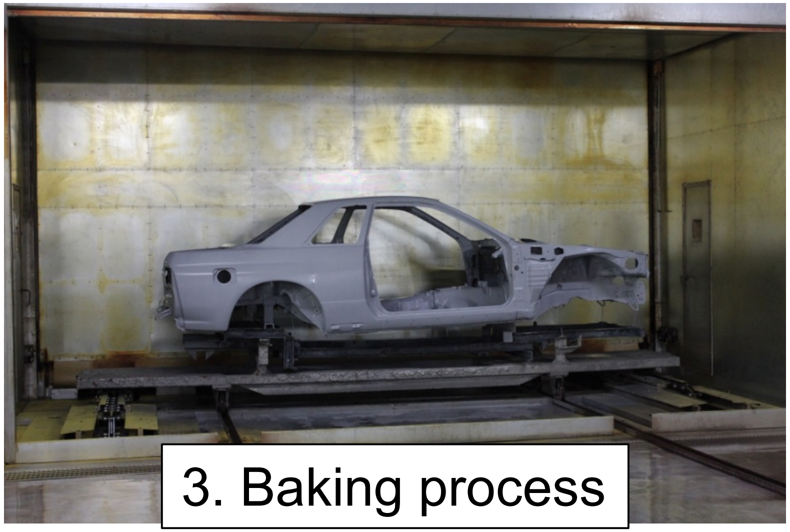
ED is responsible for the basecoat of a carbody.

What is Electrodeposition (ED) ?

Photos of ED Process Line


1. Deposition process


2. Water rinse process


3. Baking process

We focus on this process.

Importance of ED for Safety

Quiz. Is this car safe as when it was new?

https://wd40.pk/

- No, because **corrosion** on the carbody severely damaged its structural health. (At worst, the engine may fall off.)
- As corrosion progresses, the design strength/stiffness cannot be guaranteed, although safety tests (such as crash tests) are usually conducted using new cars without corrosion.

ED coating is important for automotive safety.

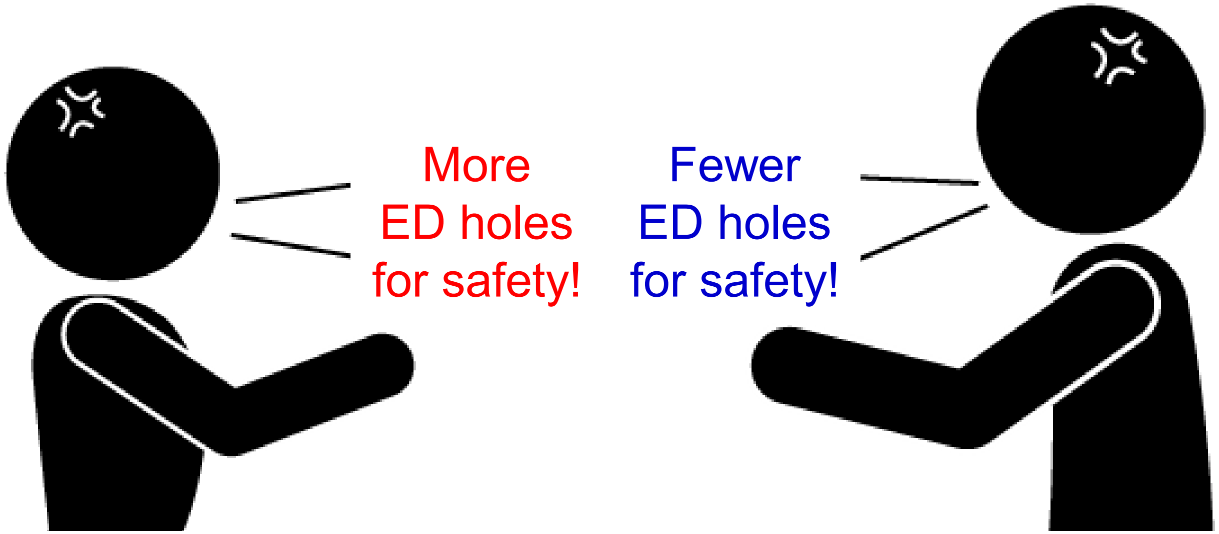
Difficulty in ED for Carbodies


https://www.goauto.ca/

- Undercarriages are exposed to severe corrosive environments, especially due to seawater or snow-melting chemicals.
- Some undercarriage parts (such as a **side sill**) have **bag-like complex structures** with many ED holes.
- It is not easy to deposit a required minimal thick film at the innermost faces of a bag-like structure, even for ED.
- Carbody design must consider the difficulty in ED process.

Need for ED Simulation

- In a car company, this kind of battle may happens.



Note that this is a fiction.

Corrosion Section

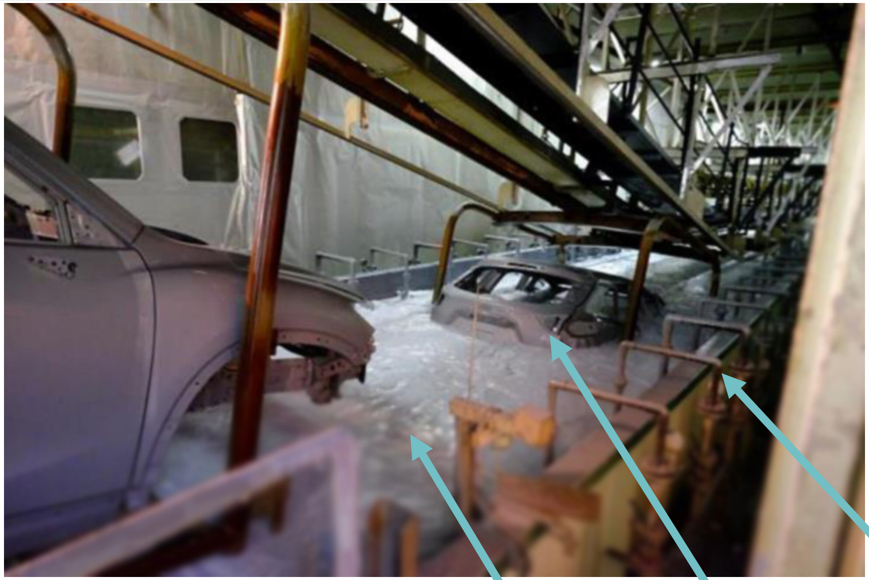
Strength/Stiffness Section

- To resolve this battle,

ED simulations are important to optimal car design as well as crash simulations for automotive safety.

What is ED Simulation?

Actual ED Line



1. Paint Pool
2. Carbody with Motion
3. Electrodes (Anodes)

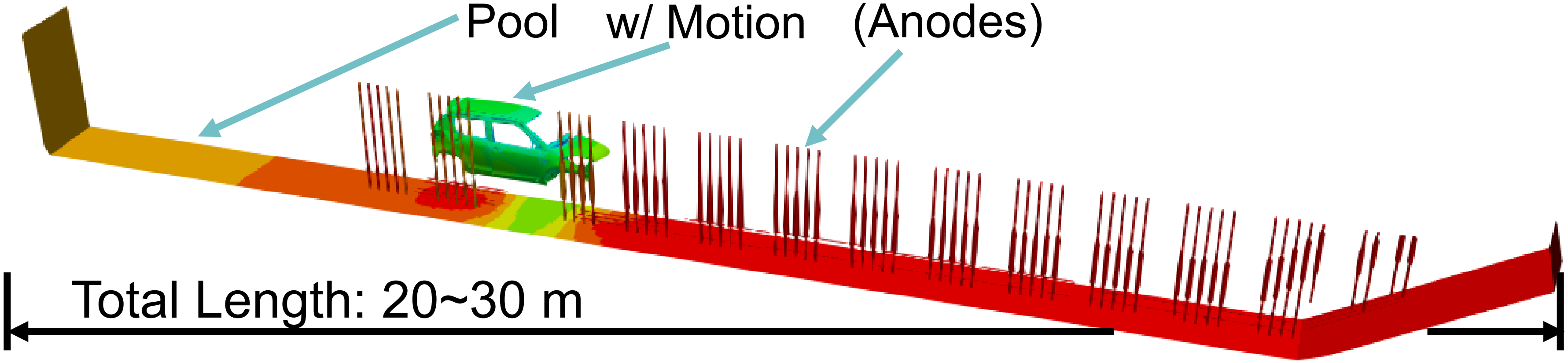
are reproduced in a computer.

ED Simulation

Paint Pool

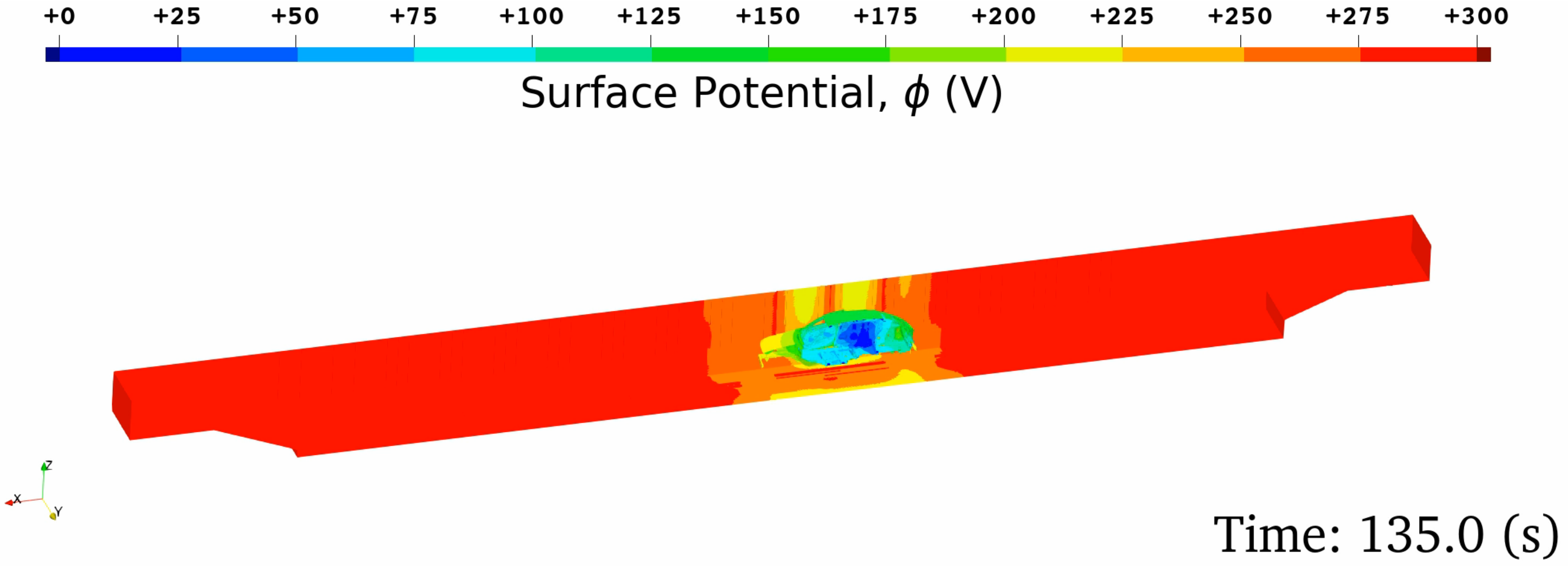
Carbody w/ Motion

Electrodes (Anodes)



Total Length: 20~30 m

What is ED Simulation?

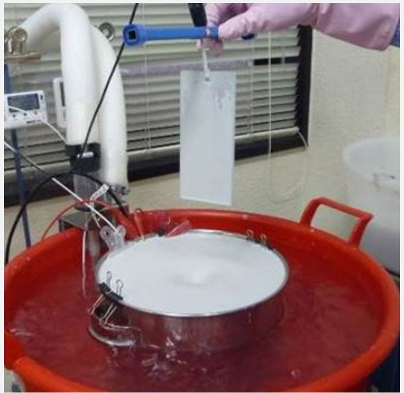


- **Governing Equation:**
  Electrostatic Laplace equation ($\nabla^2\phi = 0$) in the paint pool domain.
- **Boundary Condition (BC):**
  1. Insulation BC,
  2. Anodic (electrode surface) BC,
  3. Cathodic (carbody surface) BC:
     Film resistance/growth constitutive model.
- **Outputs:**
  1. Surface potential,
  2. Current density,
  3. Coated film thickness.

Framework of ED Simulation

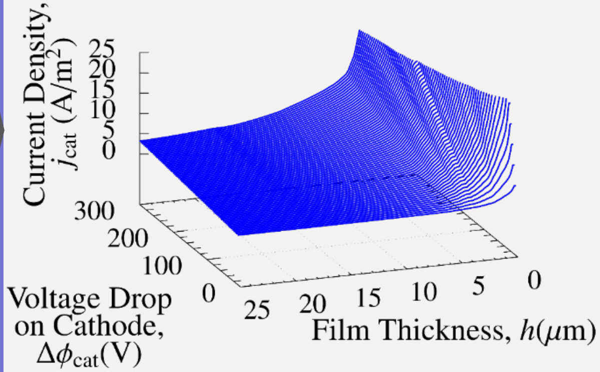
1. Basic Tests

(one-plate tests)



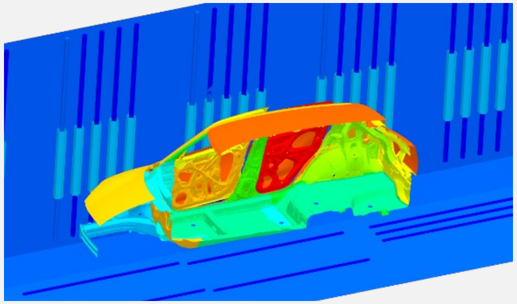
2. Model Identification

(film resistance/growth)



3. Calculation

(FEM etc.)



Framework for Solid Mechanics Simulation

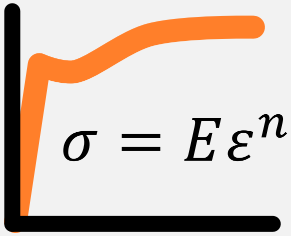
1. Basic Tests

(Tensile test)

https://www.aandd.co.jp/adhome/products/test/rth_rti.html

2. Model Identification

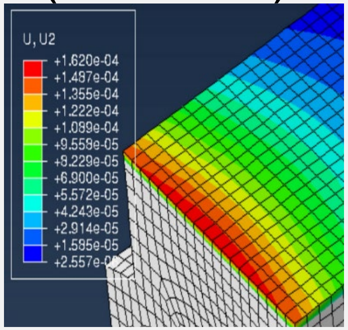
(stress/strain curve)



$$\sigma = E\varepsilon^n$$

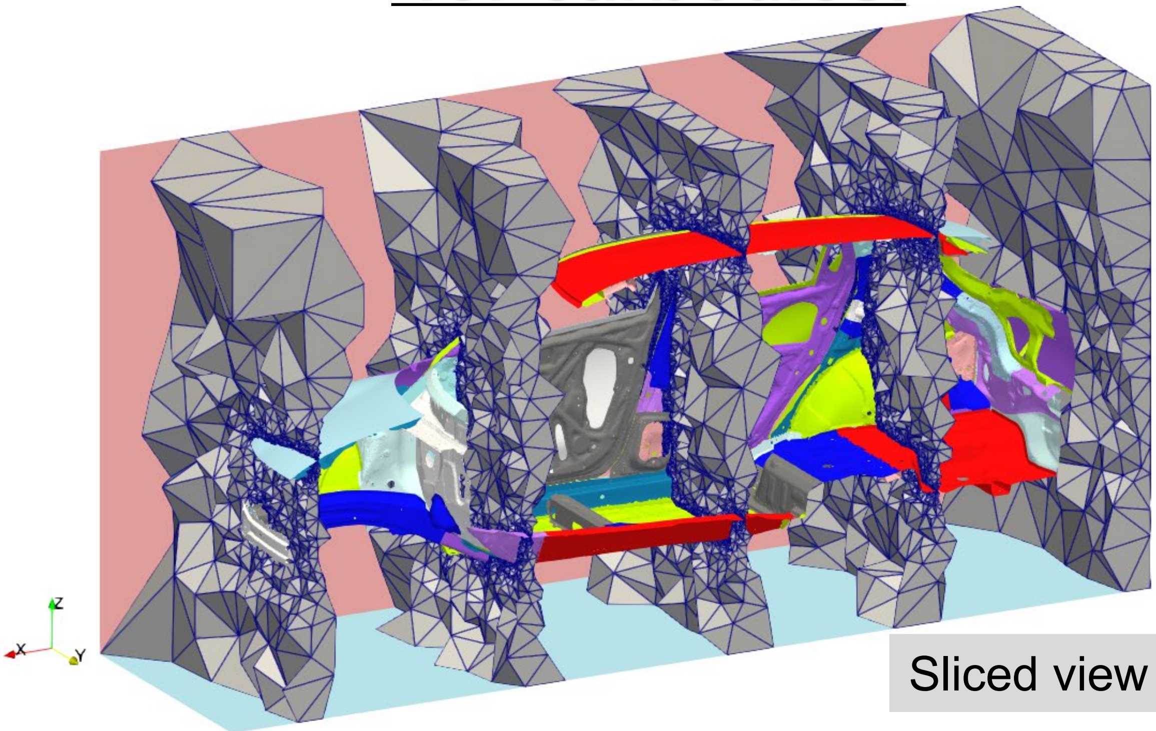
3. Calculation

(FEM etc.)



Meshing Issue 1

Issue 1: Impossible to make a good HEX mesh for carbodies.

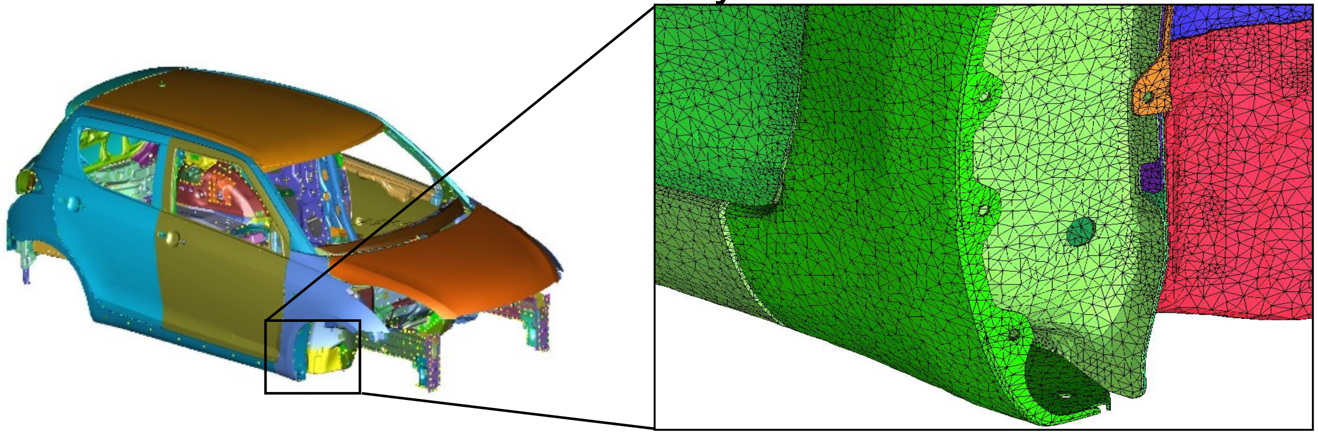


Sliced view of a carbody mesh

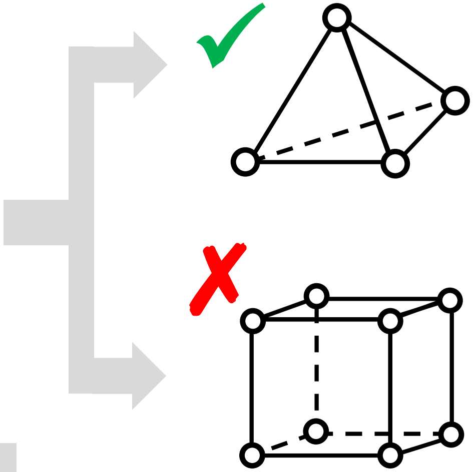
- An ED simulation requires a mesh for the space around the carbody like CFD.
- In contrast to CFD, an ED mesh includes the room space and many narrow spaces among plates (such as side sills).

Meshing Issue 1 (cont.)

Only the surface mesh is shown.



Zoom-in view of a carbody mesh



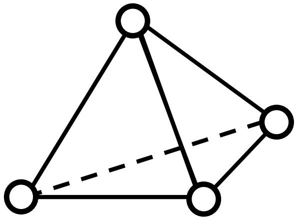
- The shape of a carbody is too complex to be discretized into a good HEX mesh.
- The cutcell or snappy HEX meshing is basically not suitable for the geometry with many holes.
  (∵ Massive increase in DOF, Linear mesh convergence rate, Presence of hanging nodes or polyhedral cells, Inapplicable to solid dynamics, etc.)

TET meshes are preferable in ED simulation.

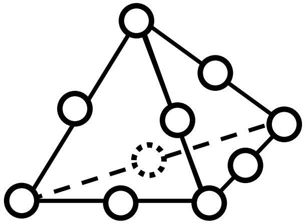
Meshing Issue 2

Issue 2: Both the standard 4-node and 10-node tetrahedral elements are inconvenient.

- 4-node TET (**T4**) has poor accuracy with only a linear mesh convergence rate.
  ⟹ FEM-T4 and FVM-T4 require very fine meshes to obtain accurate results.

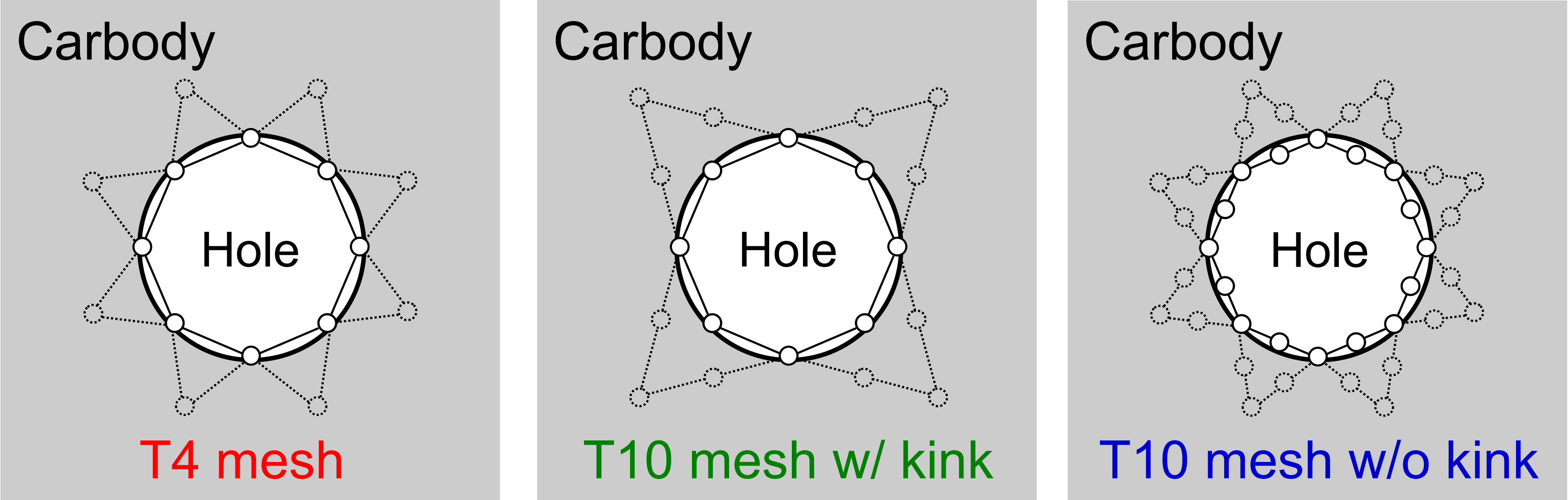


- 10-node TET (**T10**) has good accuracy with a quadratic mesh convergence rate; however, T10 mesh requires massively large DOF to represent complex shapes without any kink of element shapes.



Meshing Issue 2 (cont.)

⟹ If there is a small **hole** on a carbody, the surface mesh around the hole looks like…



✗ T10 w/ kink leads to severe accuracy loss.

✗ T10 w/o kink leads to a massive increase in DOF.

The standard T4 and T10 elements are both inconvenient for carbodies to achieve accurate simulation with minimal DOF.

Motivation

By the way, …

- The smoothed finite element method (S-FEM) has become popular in recent years as a next-generation high-performance FEM.
- Especially, the edge-based S-FEM using T4 mesh (ES-FEM-T4) is known to achieve a **superlinear mesh convergence rate even with T4 meshes**.

Therefore, we expect that…

ES-FEM-T4 could be a solution for the meshing issues to achieve fast and accurate ED simulation.

Objective

Development of ED simulator using ES-FEM-T4 for practical (fast & accurate) ED simulations.

Table of body contents:

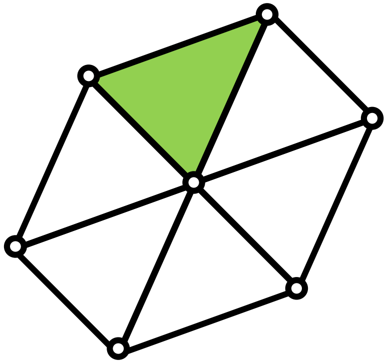
1. Formulation of ES-FEM-T4 in ED Simulation
2. Benchmark Analyses
3. Validation Analyses (for the ED Constitutive Model)
4. Summary

Formulation of ES-FEM-T4 in ED Simulation

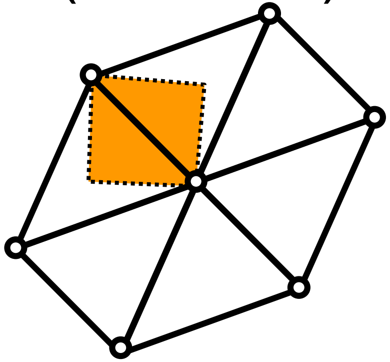
What is S-FEM?

- The ***Smoothed* finite element method (S-FEM)** is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs, depending on the **domains of strain smoothing**.

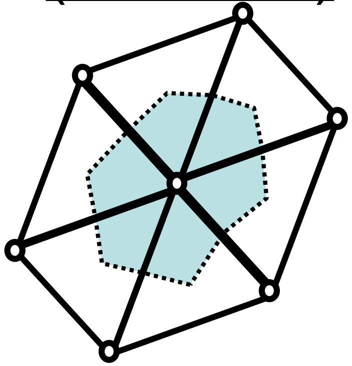
For example in 2D triangular (T3) mesh:



Standard FEM



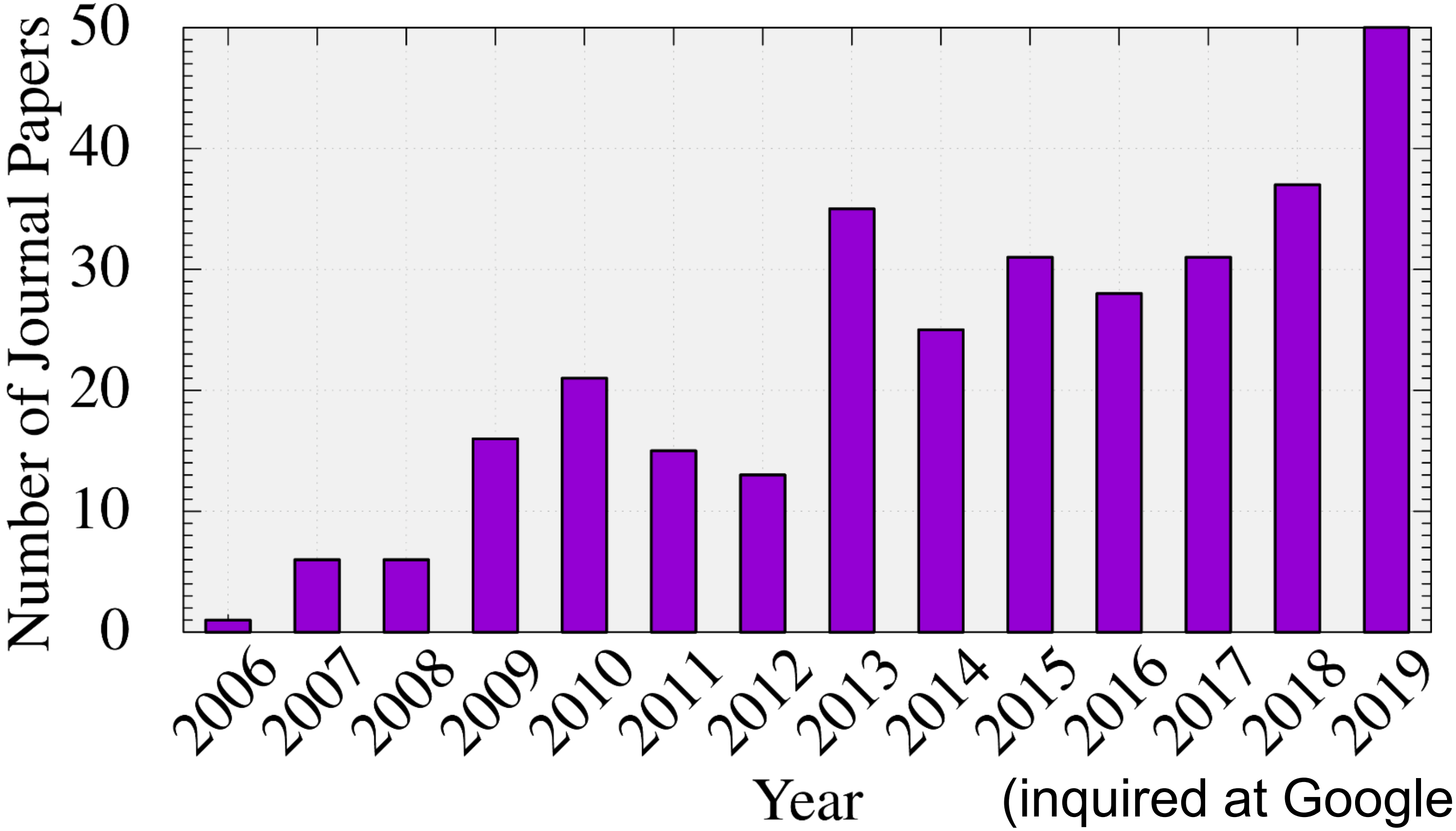
Edge-based S-FEM (ES-FEM)



Node-based S-FEM (NS-FEM)

How popular is S-FEM?

Number of journal papers written **in English** whose **title** contains “**smoothed finite element**”:



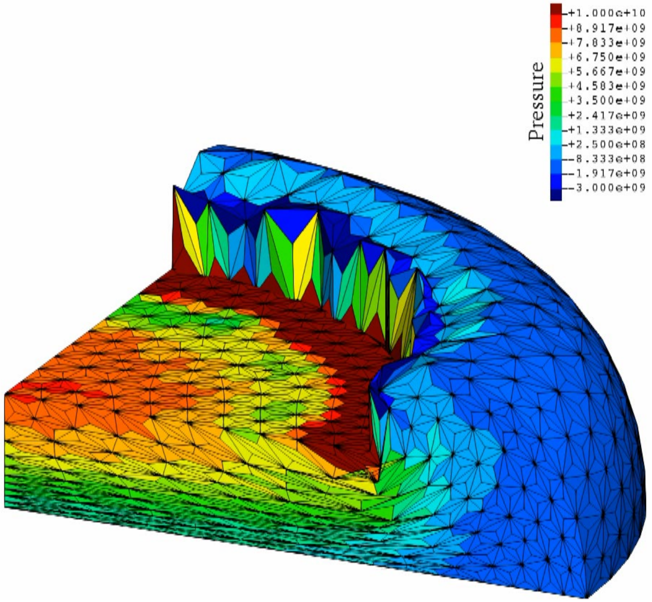
(inquired at Google Scholar)

The attraction of S-FEM is expanding continuously.

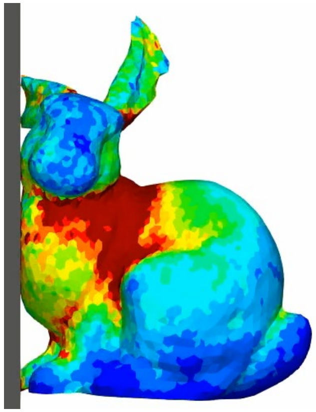
Applications of S-FEMs in Our Lab

- Solid mechanics

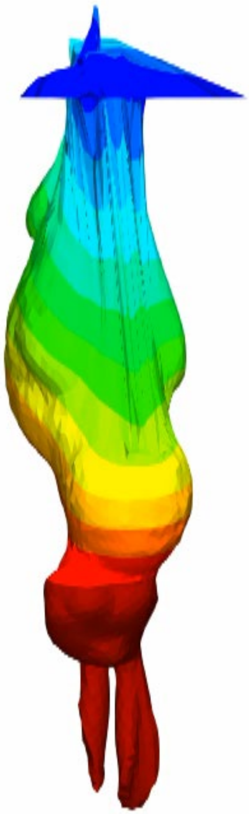
Static Implicit



Dynamic Explicit

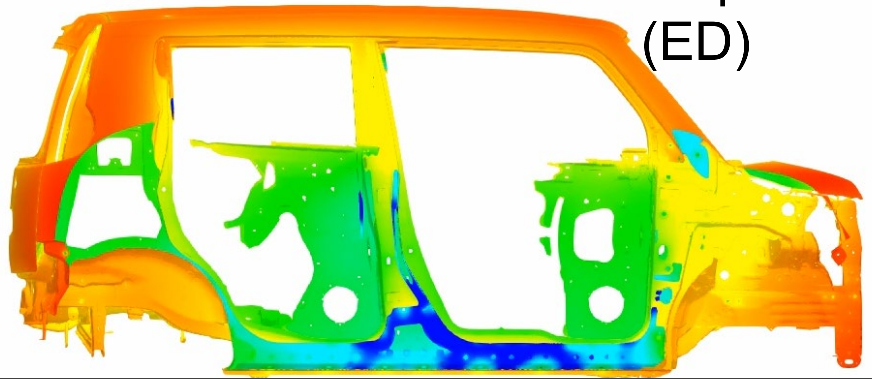


Viscous Implicit



- Laplace Field

Electrodeposition (ED)

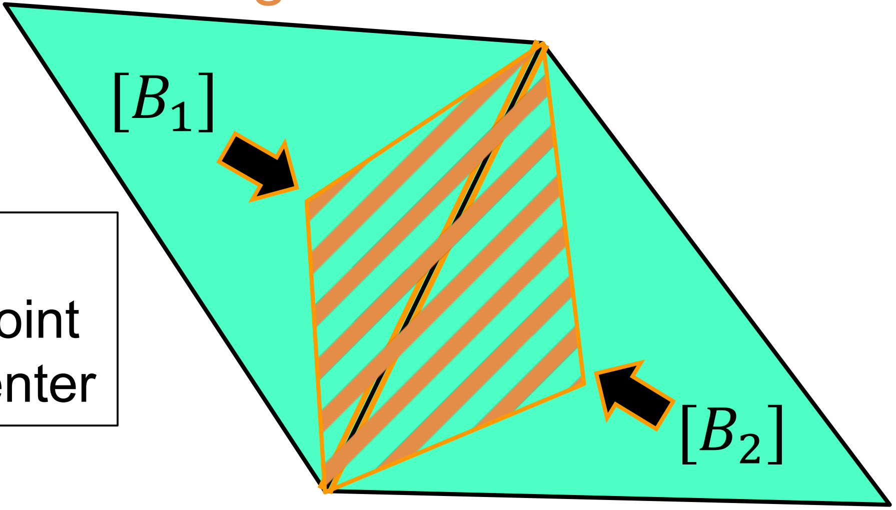


Brief Formulation of ES-FEM

Let us consider two 3-node triangular (T3) elements in 2D.

- Calculate $[B](= \mathrm{d}\boldsymbol{N}/\mathrm{d}\boldsymbol{x})$ at each element as usual.
- Distribute $[B]$ to the connecting edge with an area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate current density $(\boldsymbol{J})$ and nodal internal current $\{i^{\mathrm{int}}\}$ in each edge smoothing domain.

As if putting
an integration point
on each edge center



$[{}^{\mathrm{Edge}}B]$

↳ ${}^{\mathrm{Edge}}\boldsymbol{J}, \{{}^{\mathrm{Edge}}i^{\mathrm{int}}\}$ etc.

Mathematical Formulation of ES-FEM

- ${}^{\mathrm{Edge}}V_k$: Volume of the edge smoothing domain of Edge k,

$$ {}^{\mathrm{Edge}}V_k = \sum_{e \in {}^{\mathrm{Edge}}\boldsymbol{E}_k} {}^{\mathrm{Elem}}V_e / 6 \, . $$

- ${}^{\mathrm{Edge}}\boldsymbol{E}_k$ is the set elements connected to Edge k,
- ${}^{\mathrm{Elem}}V_e$ is the volume of Element e,
- “6” denotes the number of edges of a tetrahedron.

- $[{}^{\mathrm{Edge}}B_k]$: B-matrix of Edge k,

$$ [{}^{\mathrm{Edge}}B_k] = \frac{1}{{}^{\mathrm{Edge}}V_k} \sum_{e \in {}^{\mathrm{Edge}}\boldsymbol{E}_k} \left([{}^{\mathrm{Elem}}B_e] \, {}^{\mathrm{Elem}}V_e / 6 \right) . $$

- $[{}^{\mathrm{Elem}}B_e]$ is the B-matrix of Element e.

Mathematical Formulation of ES-FEM

- $\{{}^{\mathrm{Edge}}J_k\}$: Current density vector of Edge k,

$$\{{}^{\mathrm{Edge}}J_k\} = -\kappa\,[{}^{\mathrm{Edge}}B_k]\{{}^{\mathrm{Edge}}\phi_k\}\ .$$

- κ is the electric conductivity (constant),
- $\{{}^{\mathrm{Edge}}\phi_k\}$ is the nodal potential vector related to Edge k,

- $\{{}^{\mathrm{Edge}}i_k^{\mathrm{int}}\}$: Contribution of Edge k for the internal current vector,

$$\{{}^{\mathrm{Edge}}i_k^{\mathrm{int}}\} = -\,[{}^{\mathrm{Edge}}B_k]^{\mathrm{T}}\,\{{}^{\mathrm{Edge}}J_k\}\ {}^{\mathrm{Edge}}V_k\ .$$

- $\{i^{\mathrm{int}}\}$: The total internal current vector,

$$\{i^{\mathrm{int}}\} = \sum_{k\in\boldsymbol{G}}\{{}^{\mathrm{Edge}}i_k^{\mathrm{int}}\}.$$

- $\boldsymbol{G}$ is the set of all edges in the FE mesh.

That's all. The formulation is quite simple!

Characteristics of ES-FEM-T4

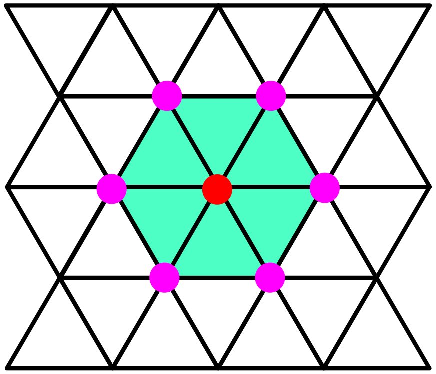
Advantage

- Superlinear mesh convergence (as fast as 2nd-order elems.).
- Same input file as FEM-T4.
- No increase in DOF (nodal potentials only; ∴ easy to code).

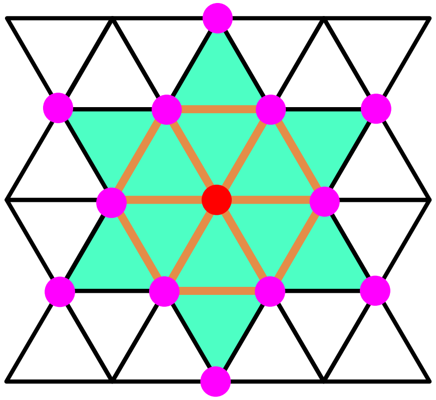
Disadvantage

- Longer assembling time of $[K]$ (~x2 of FEM-T4 w/ the same mesh).
- Wider bandwidth in $[K]$ (~x3 of FEM-T4 w/ the same mesh).
- No longer an independent T4 element.

A node is referred by 6 elements, ⇒ 7 nodes.



FEM-T3 (Bandwidth: 7)



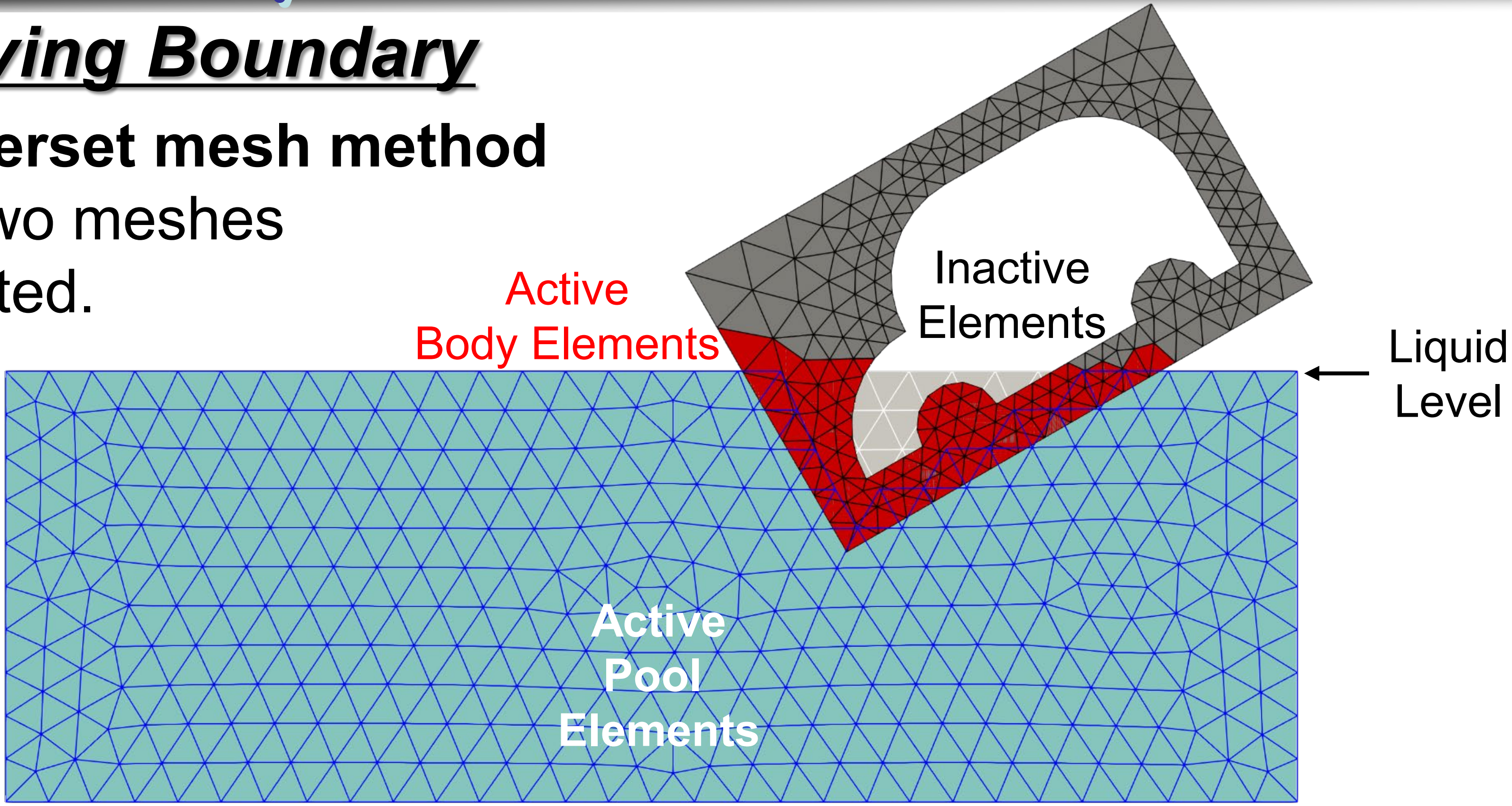
ES-FEM-T3 (Bandwidth: 13)

A node is referred by 12 edges, ⇒ 12 elements, ⇒ 13 nodes.

Other 3 Key Features in ED Formulation

1. Moving Boundary

The **overset mesh method** using two meshes is adopted.



- Each interfacial node of the active pool elements is tied in the active body elements with the multi-point constraint (**MPC**).
- The classical **method of Lagrange multiplier** is used to satisfy the MPCs.

Other 3 Key Features in ED Formulation

2. Iterative Matrix Solver

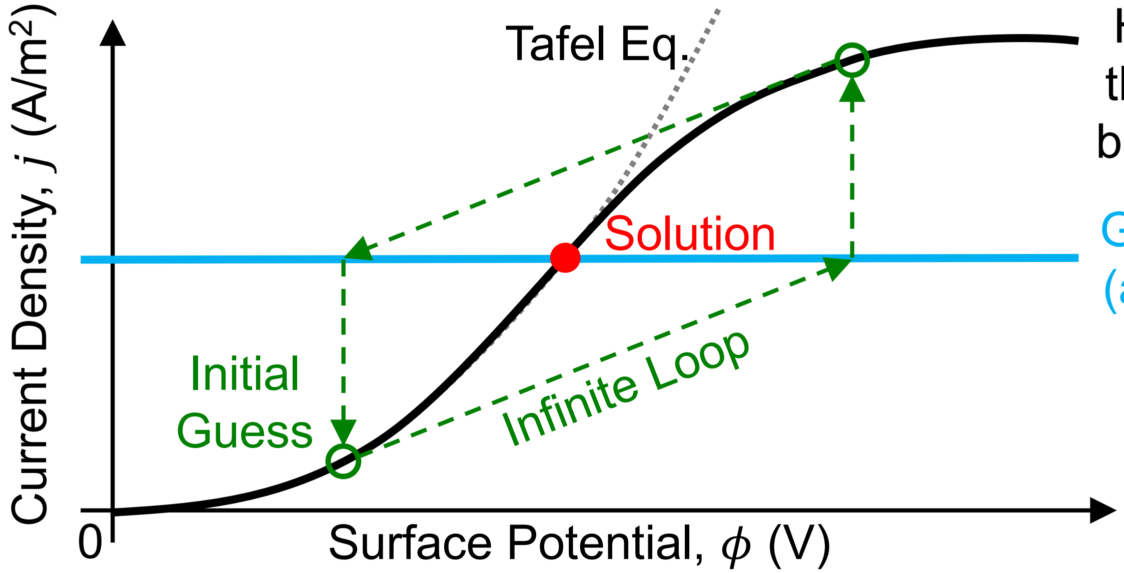
The minimum residual method (**MINRES**)
with the point Jacobi preconditioner
is adopted.

- Electrostatic Laplace equation forms a symmetric matrix.
- Matrix is indefinite due to the method of Lagrange multiplier used by the overset mesh method etc..
- CG cannot solve symmetric indefinite systems without static condensation.
- In contrast, **MINRES can solve symmetric indefinite systems without static condensation.**
  (Why is MINRES without static condensation not so popular?)

Other 3 Key Features in ED Formulation

3. Treatment for Strong Nonlinear

The polarization curve for BC has an “S” shape.



$2H_2O + 2e^- \rightarrow 2OH^- + \mathbf{H_2} \uparrow$

H_2 bubbles cover up the cathode face and becomes an insulator.

Given Anode BC (as a constant j)

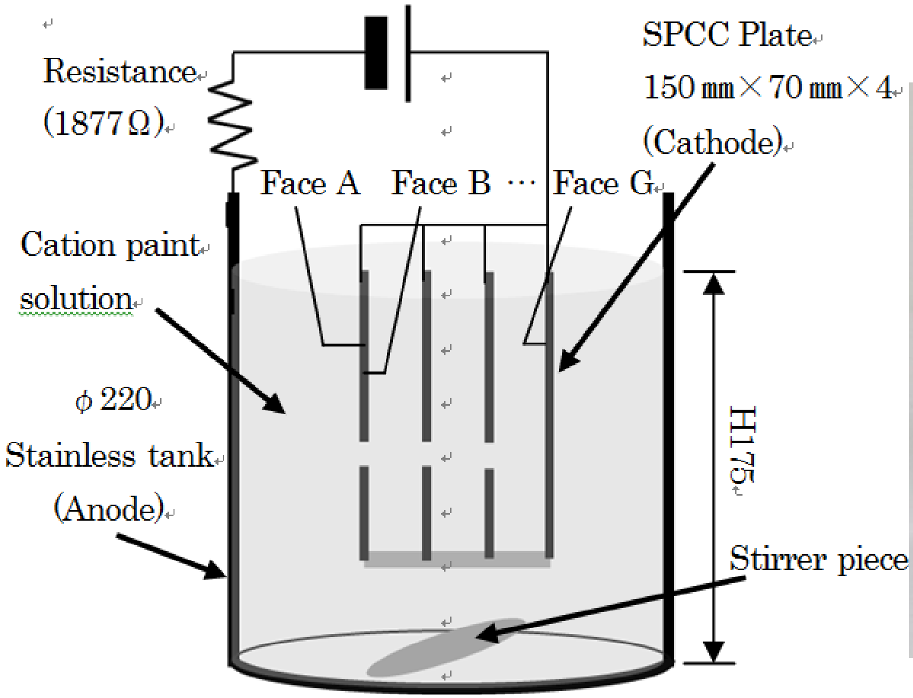
- The polarization curve (Robin BC) is ideally described with the Tafel eq..
- In reality, H_2 bubbles generated on cathode block the current, forming an **“S” shaped polarization curve**.
- An “S” shape may cause infinite loops in the Newton-Raphson method.
- The ED solver needs to introduce special treatments, such as cutback of $\delta\phi$; yet, the loop count becomes a lot (~8 times) anyway.

Benchmark Analyses

4-Plate BOX Simulation

Outline

- 4 Plates forms 3 bags.
- 3rd bag is difficult to be deposited.



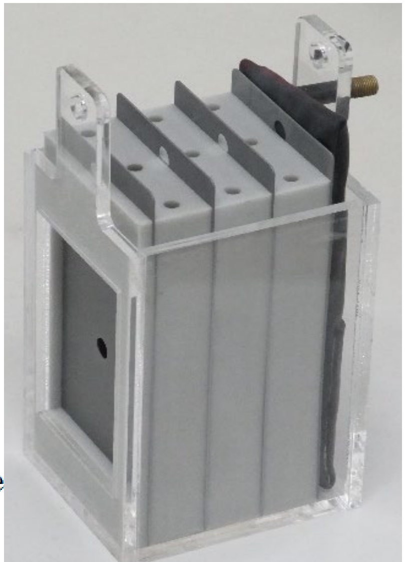
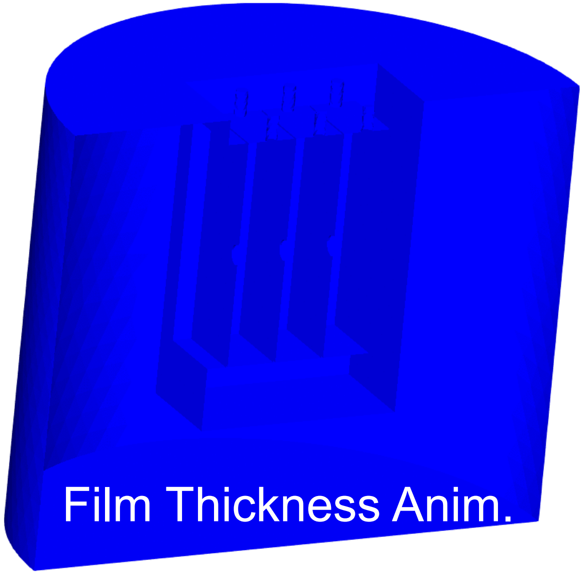


Photo of a BOX

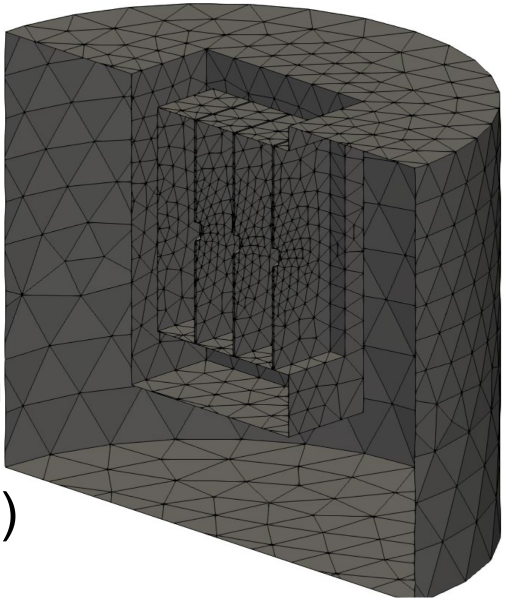


- ■ Imitating a bag-like structure such as a side sill in a carbody.
- ■ Film thickness on the **innermost surface** (G-Face) is the most important so as to guarantee corrosion protection.
- ■ The film thickness is evaluated with **4 different meshes for mesh validation using FEM-T4 and ES-FEM-T4.**

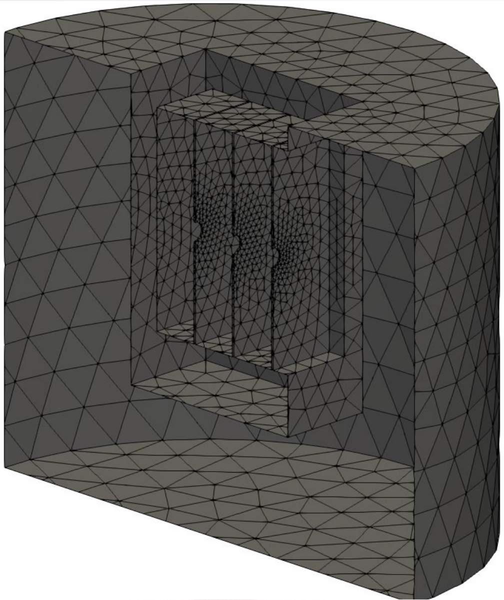
4-Plate BOX Simulation

Overview of Meshes

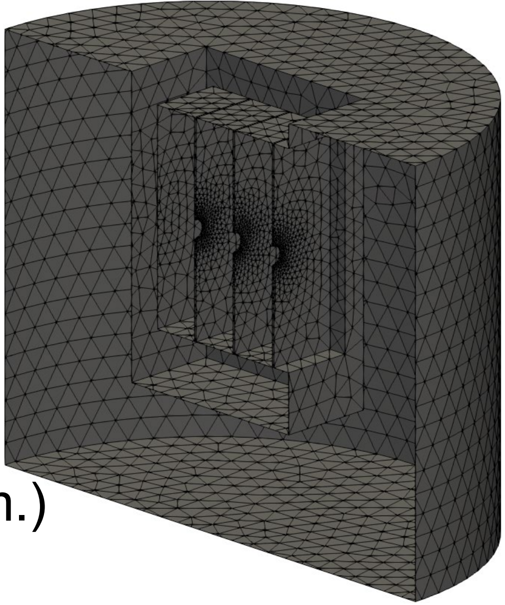
Only the surface meshes are shown.



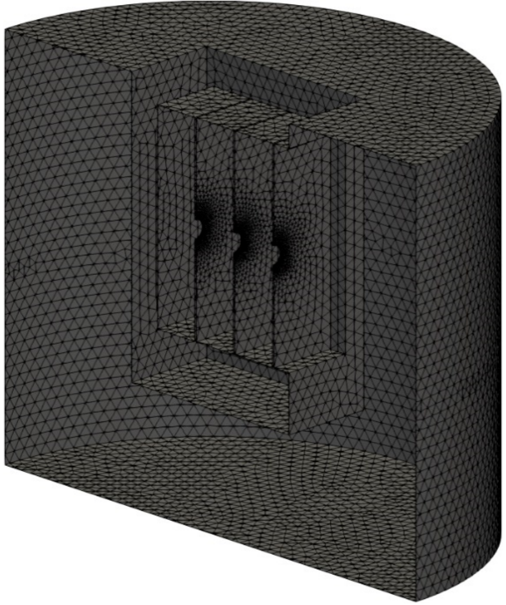
3.2 mm Mesh
Seed Size
(31k T4 elem.)



1.6 mm Mesh
Seed Size
(65k T4 elem.)



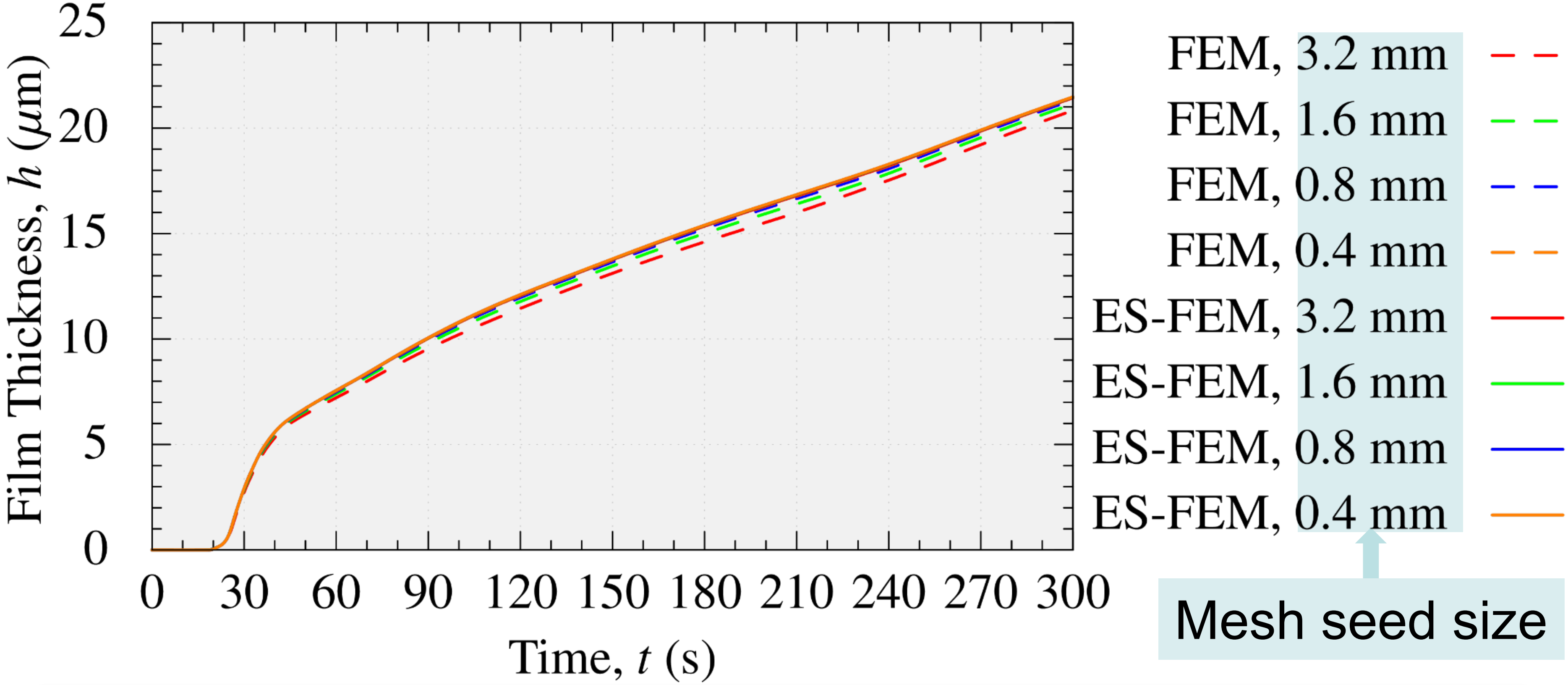
0.8 mm Mesh
Seed Size
(169k T4 elem.)



0.4 mm Mesh
Seed Size
(716k T4 elem.)

4-Plate BOX Simulation

Film Thickness on A-Face (outermost surface)

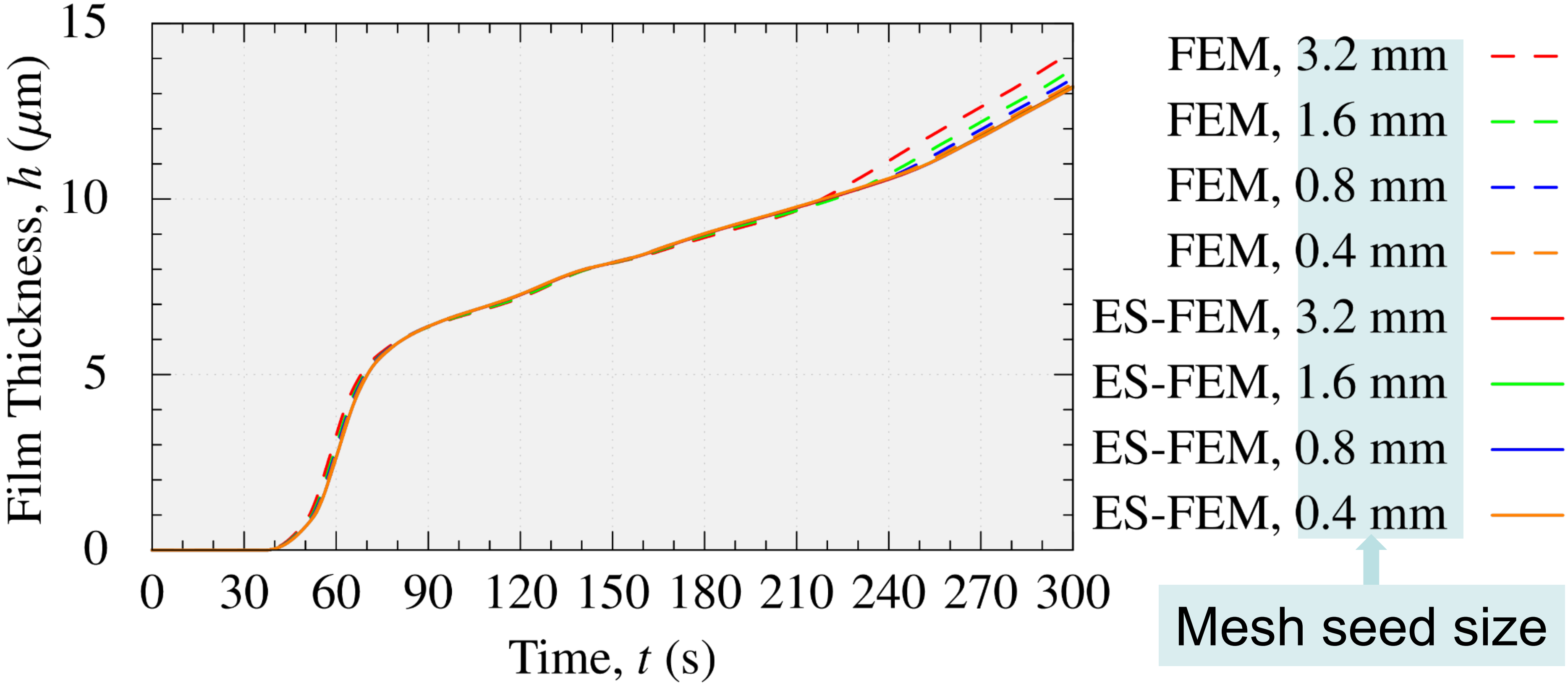


Mesh seed size

FEM results (dashed lines) have ***tiny*** errors due to mesh coarseness.

4-Plate BOX Simulation

Film Thickness on C-Face (surface in the 1st bag)

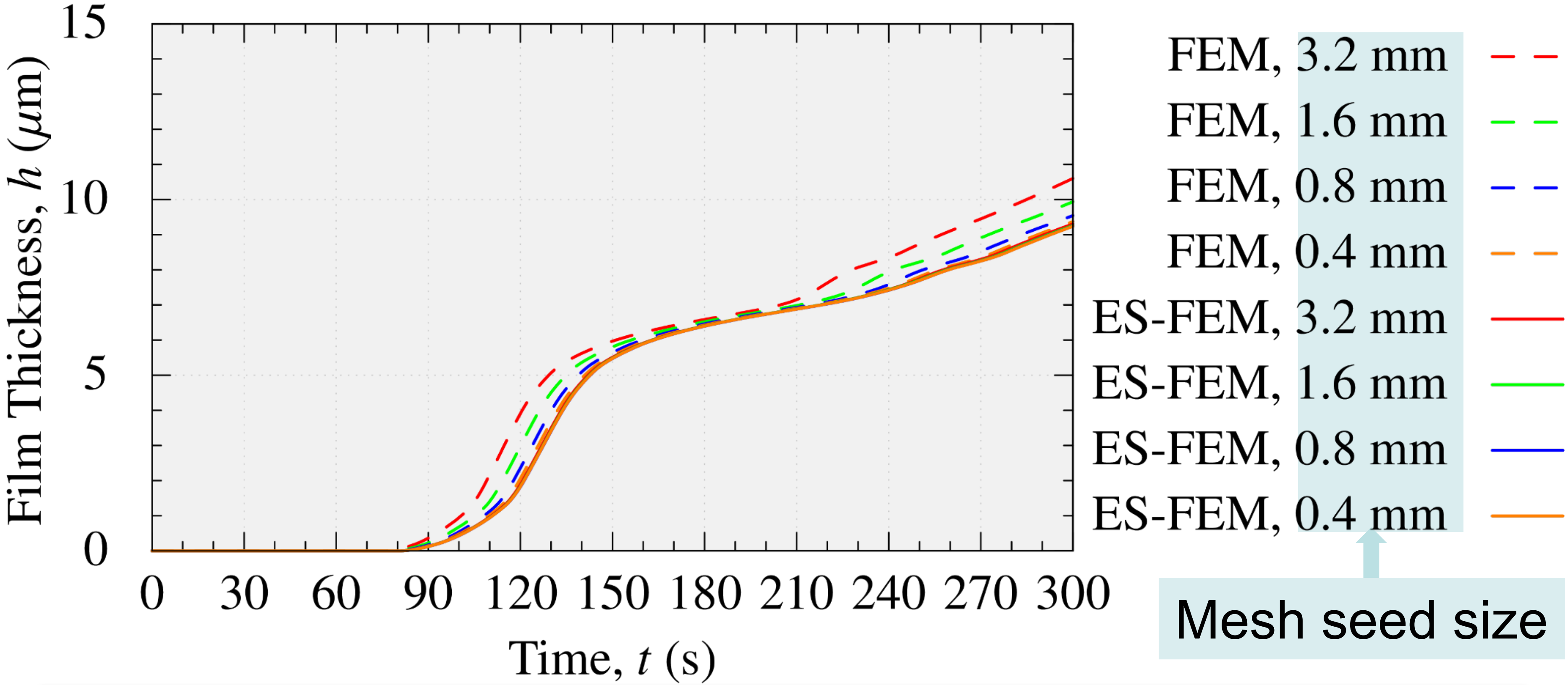


Mesh seed size

FEM results (dashed lines) have ***small*** errors
due to mesh coarseness.

4-Plate BOX Simulation

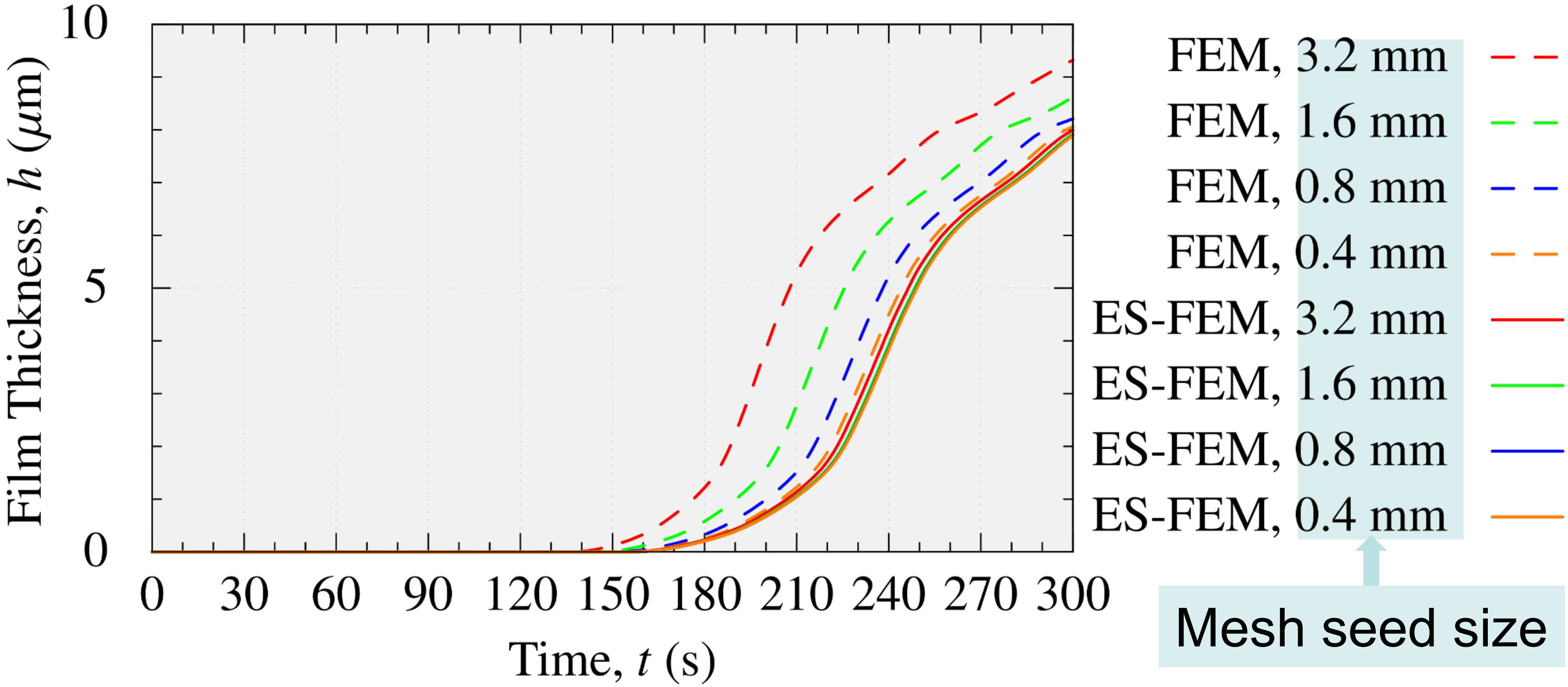
Film Thickness on E-Face (surface in the 2nd bag)



FEM results (dashed lines) have ***medium*** errors due to mesh coarseness.

4-Plate BOX Simulation

Film Thickness on G-Face (innermost surface)

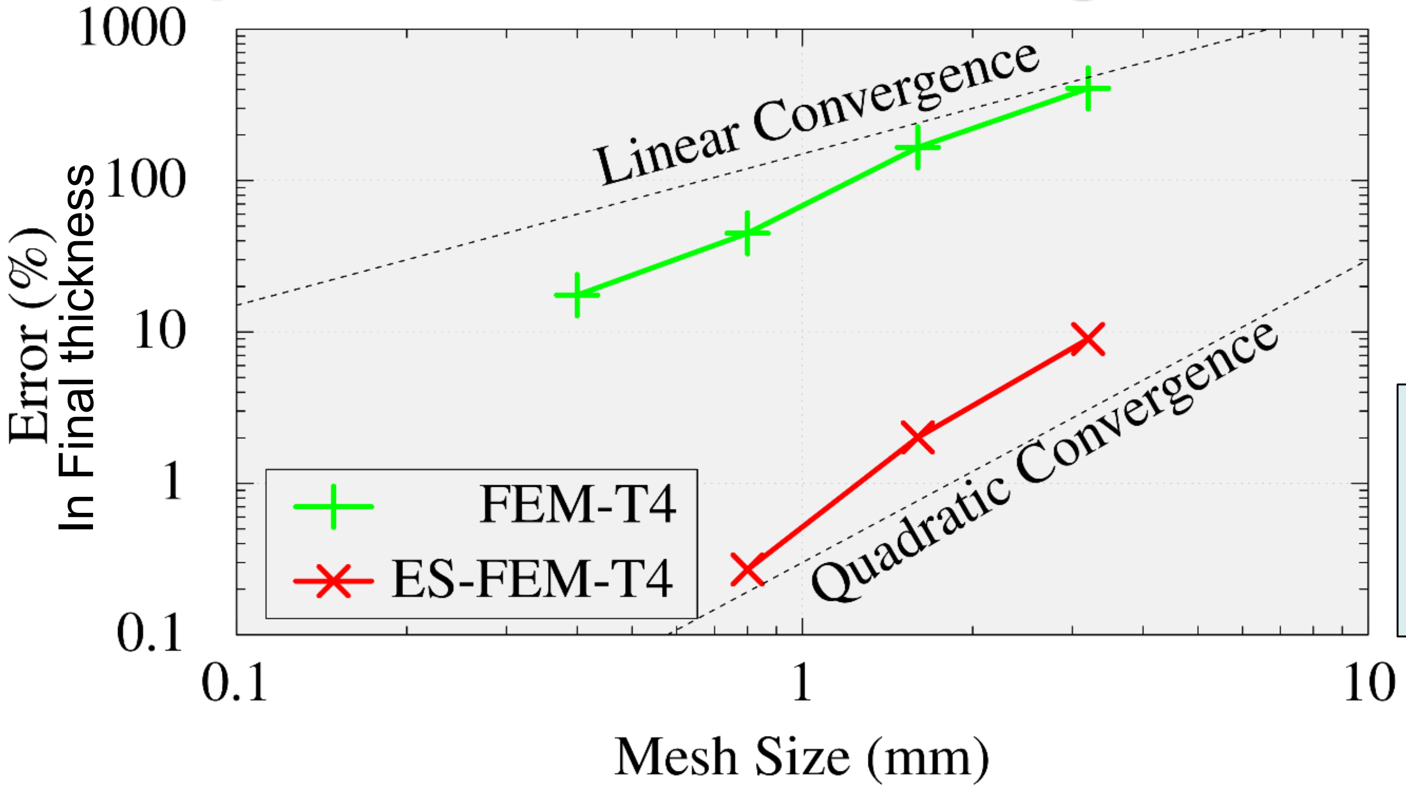


Mesh seed size

FEM results (dashed lines) have ***large*** errors due to mesh coarseness.
Meanwhile, ES-FEM (solid lines) results have no such errors.

4-Plate BOX Simulation

Comparison of Mesh Convergence Rate on G-Face



The result of ES-FEM with the finest mesh (0.4 mm) is used as the reference.

- FEM-T4 shows a linear convergence.
- ES-FEM-T4 shows a quadratic convergence.

ES-FEM-T4 has much better mesh convergence rate than FEM-T4.

4-Plate BOX Simulation

Comparison of Calculation Time

on a PC (only 1 CPU: Intel i9-9960X)

Mesh Size	FEM-T4	ES-FEM-T4
3.2 mm	7 s	10 s
1.6 mm	8 s	14 s
0.8 mm	12 s	26 s
0.4 mm	41 s	125 s

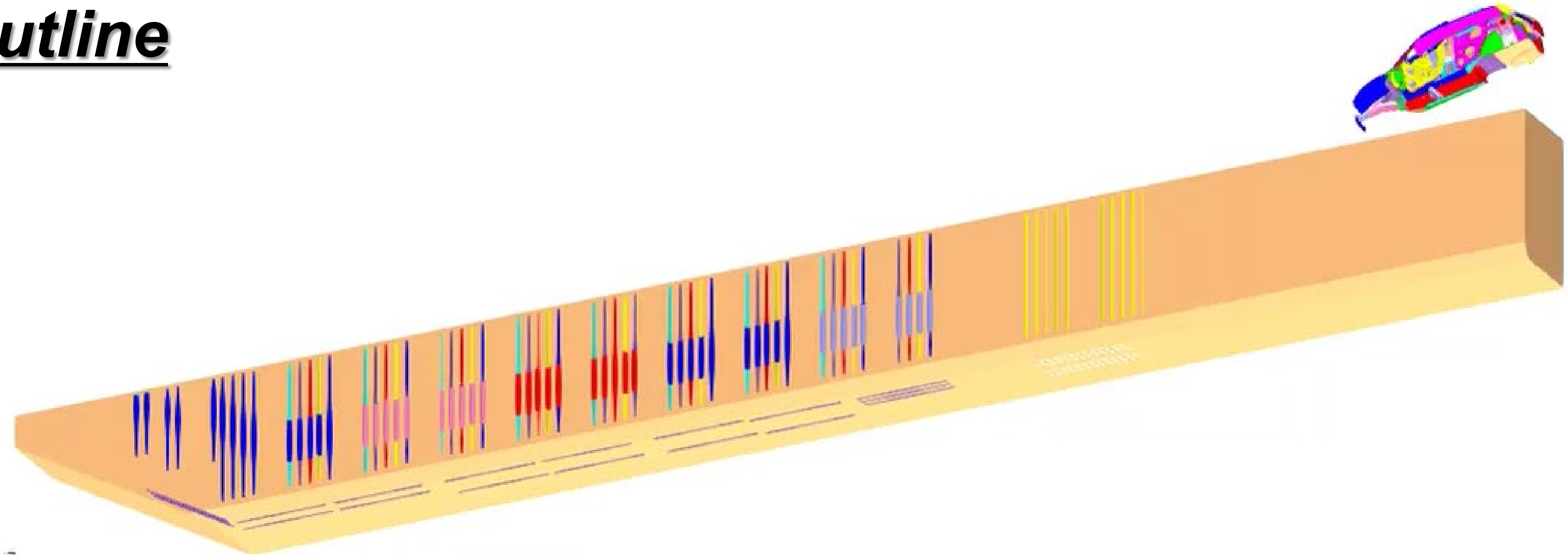
Comparable Accuracy

- With the same mesh, ES-FEM is slower than FEM by x2.
- For the same accuracy, ES-FEM is faster than FEM by x4.

ES-FEM-T4 is supremely efficient
in comparison to FEM-T4.

Actual Line Simulation

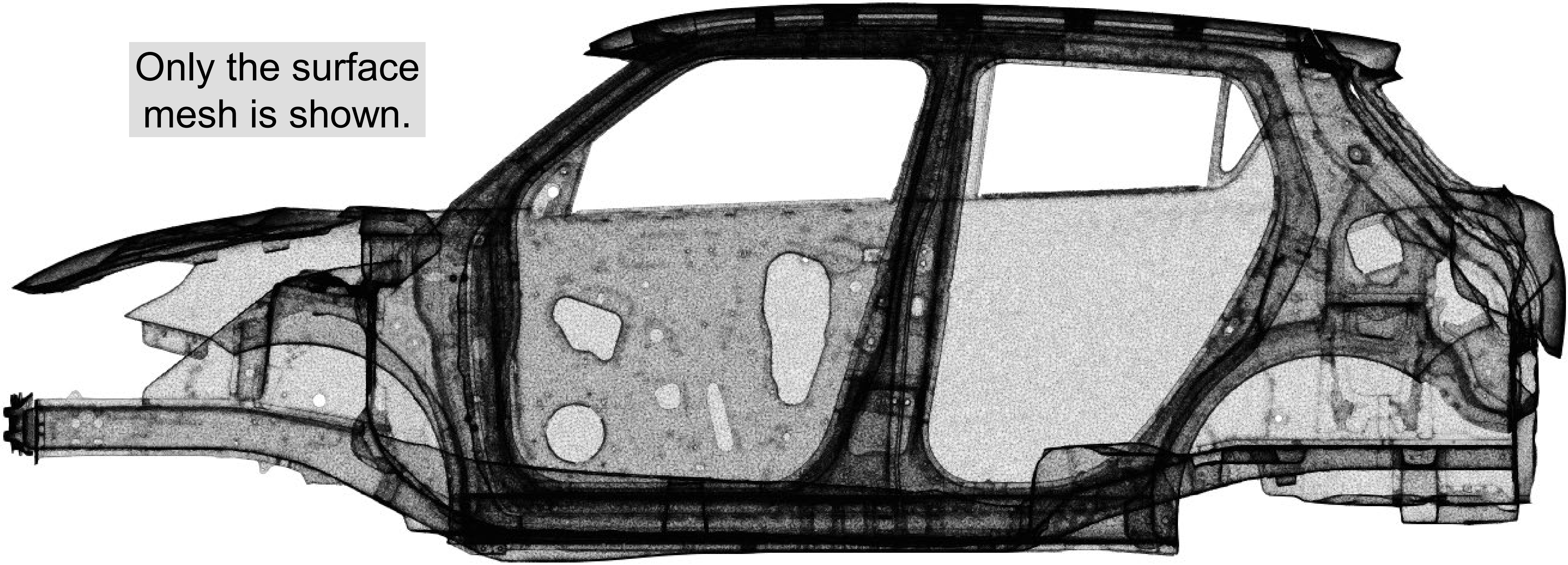
Outline



- **Half-body** analysis (only right-hand side).
- Entire line shape, carbody motion, and electrode conditions are faithfully reproduced.
- 1000 timesteps in 300 s (i.e., average $\Delta t = 0.3$ s).
- The film thickness is evaluated with **3 different meshes for mesh validation using FEM-T4 and ES-FEM-T4.**

Actual Line Simulation

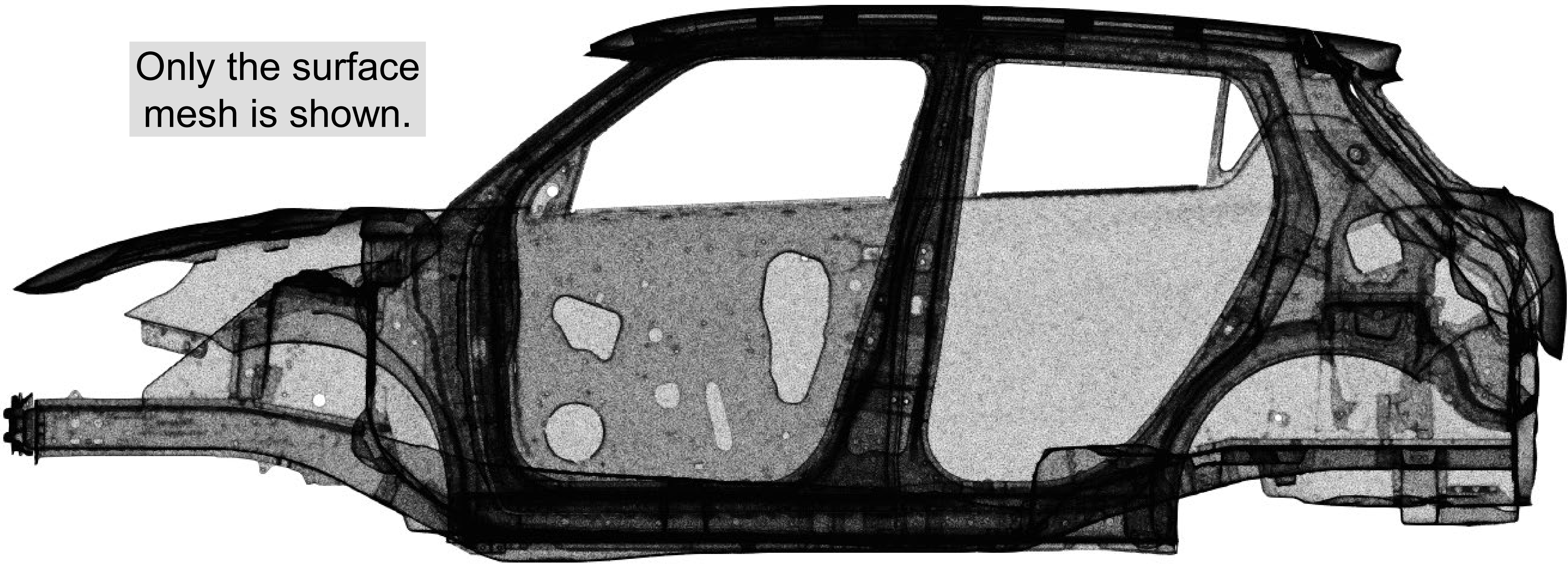
Overview of Surface Mesh of 10M Element Mesh


Only the surface mesh is shown.

- There are many ED holes around narrow spaces among plates.

Actual Line Simulation

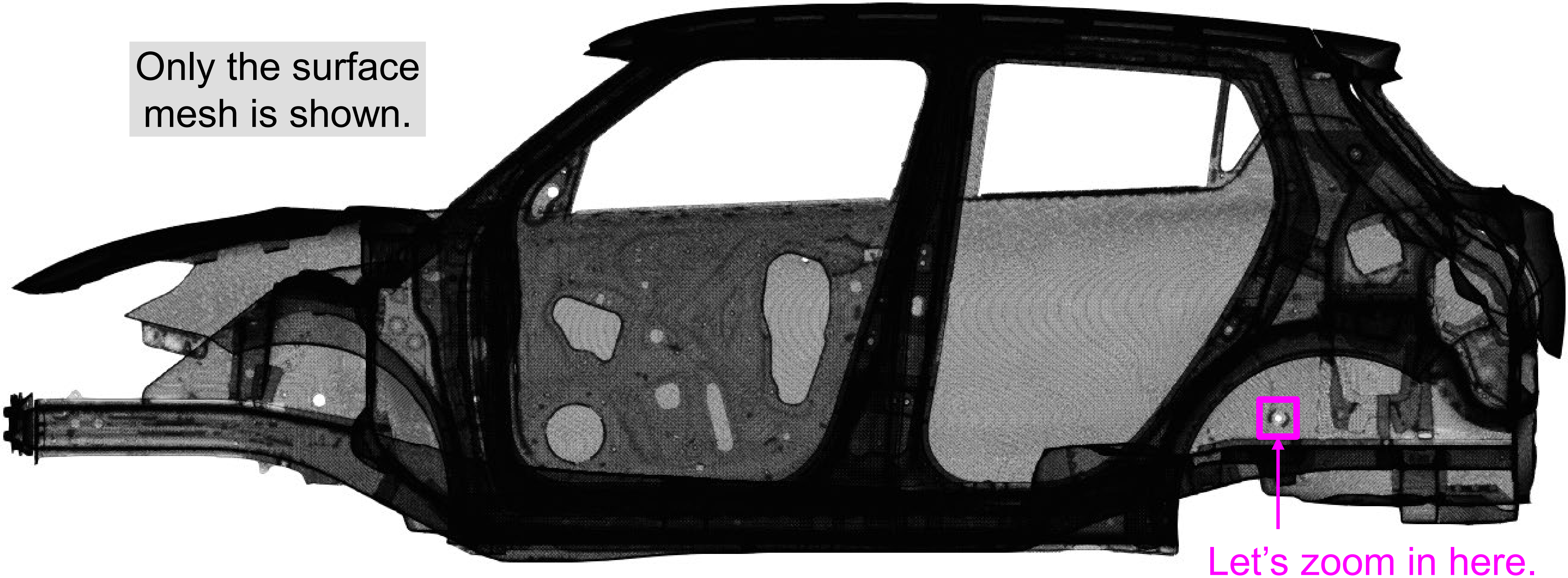
Overview of Surface Mesh of 16M Element Mesh


Only the surface mesh is shown.

- There are many ED holes around narrow spaces among plates.

Actual Line Simulation

Overview of Surface Mesh of 51M Element Mesh



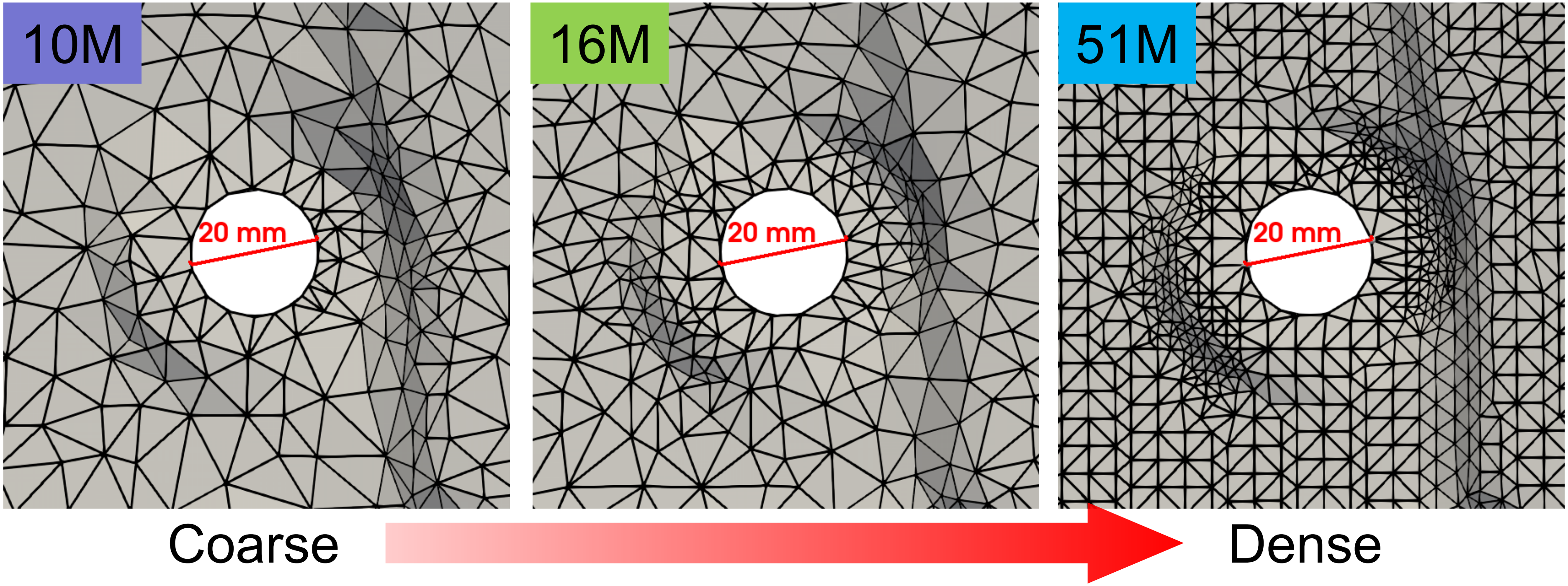
Only the surface mesh is shown.

Let’s zoom in here.

- There are many ED holes around narrow spaces among plates.
- The difference in the mesh can be seen clearly by zooming in around a hole.

Actual Line Simulation

Zoom in View around a Hole on Carbody

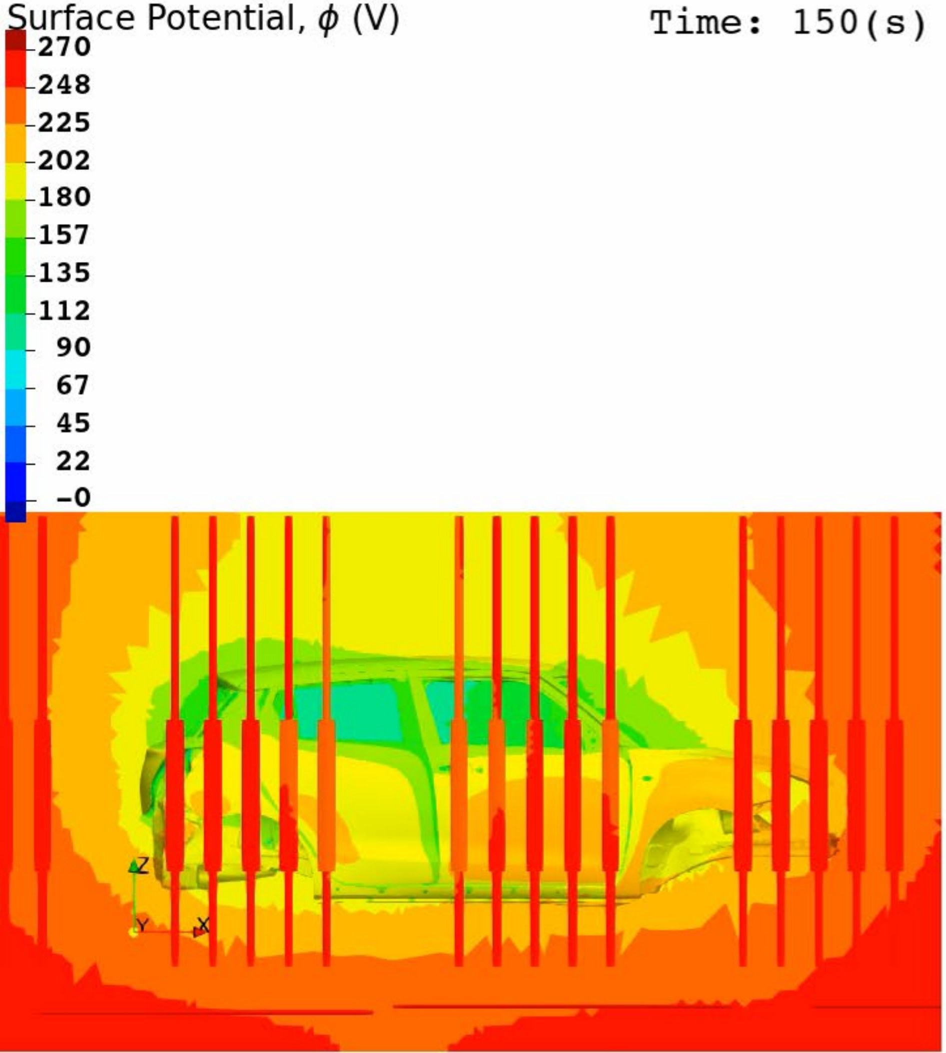


- There are many ED holes around narrow spaces among plates.
- The difference in the mesh can be seen clearly by zooming in around a hole.

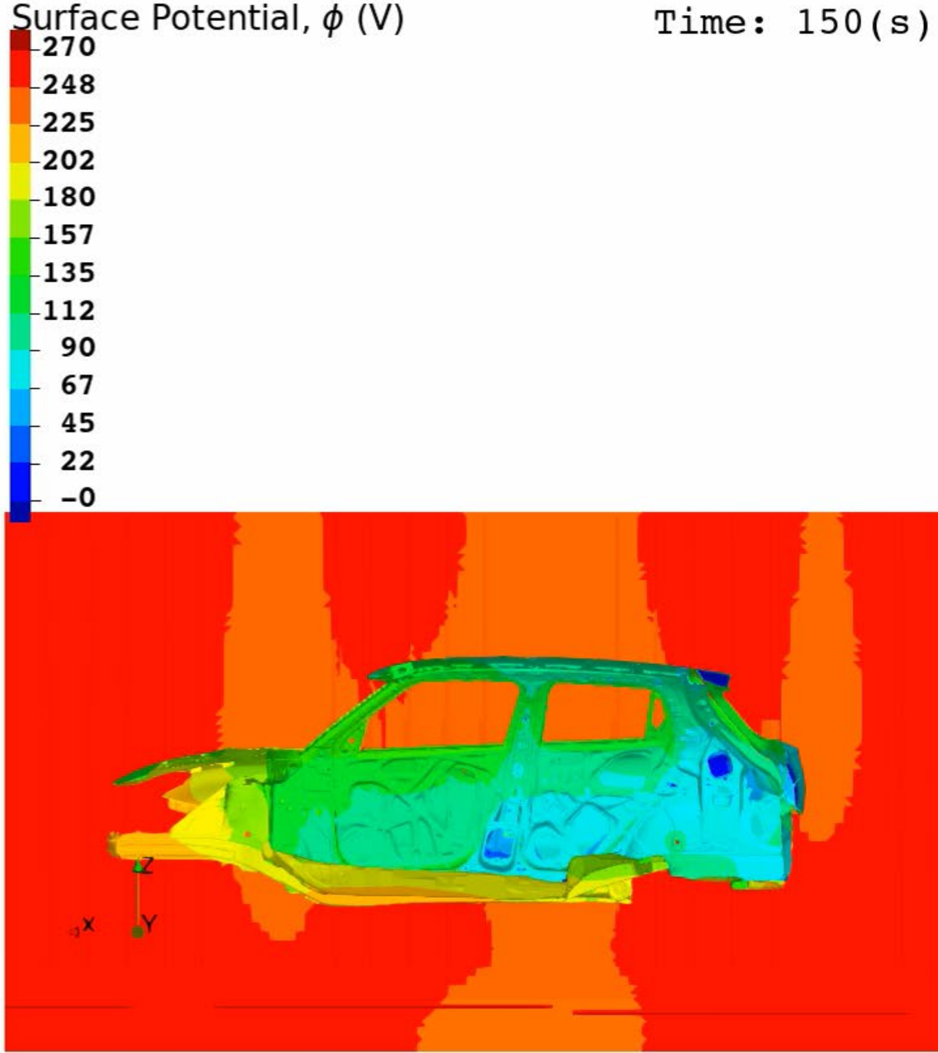
Actual Line Simulation

Reference Solution of ϕ (ES-FEM with 51M Elems.)

Outer View



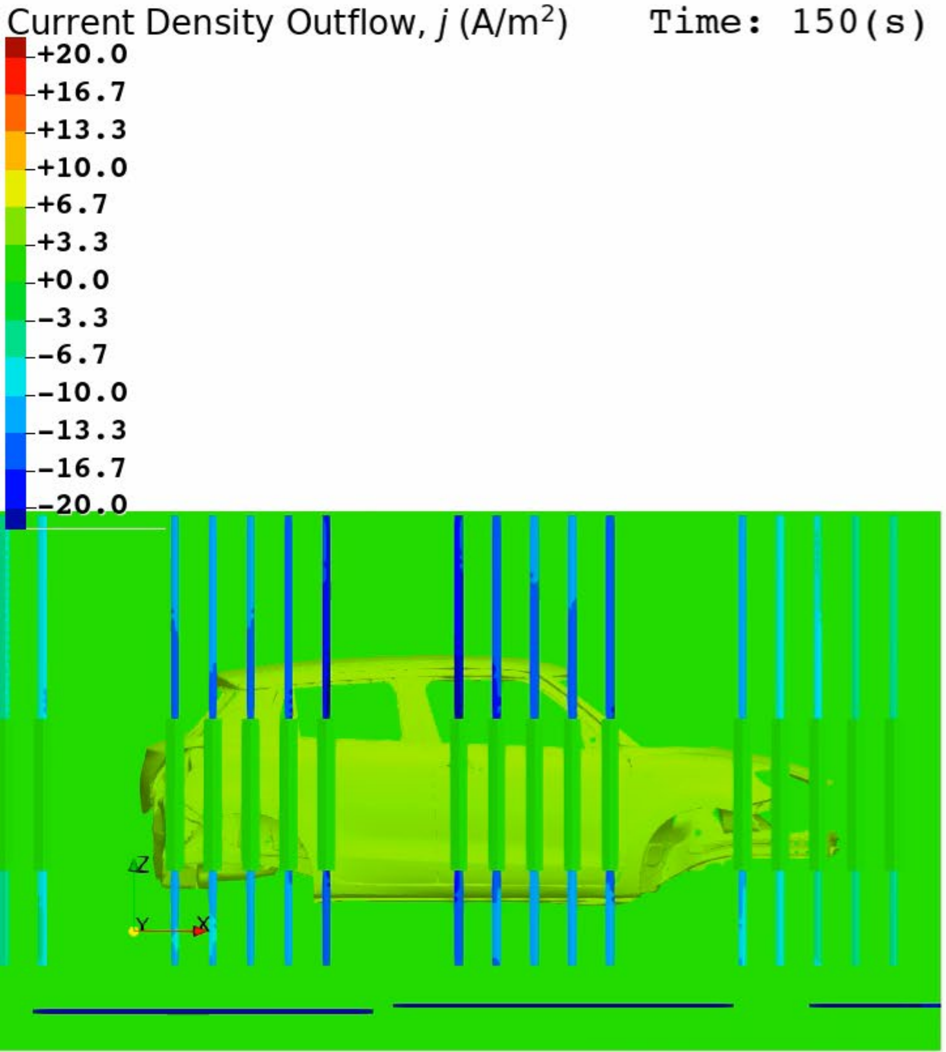
Inner View



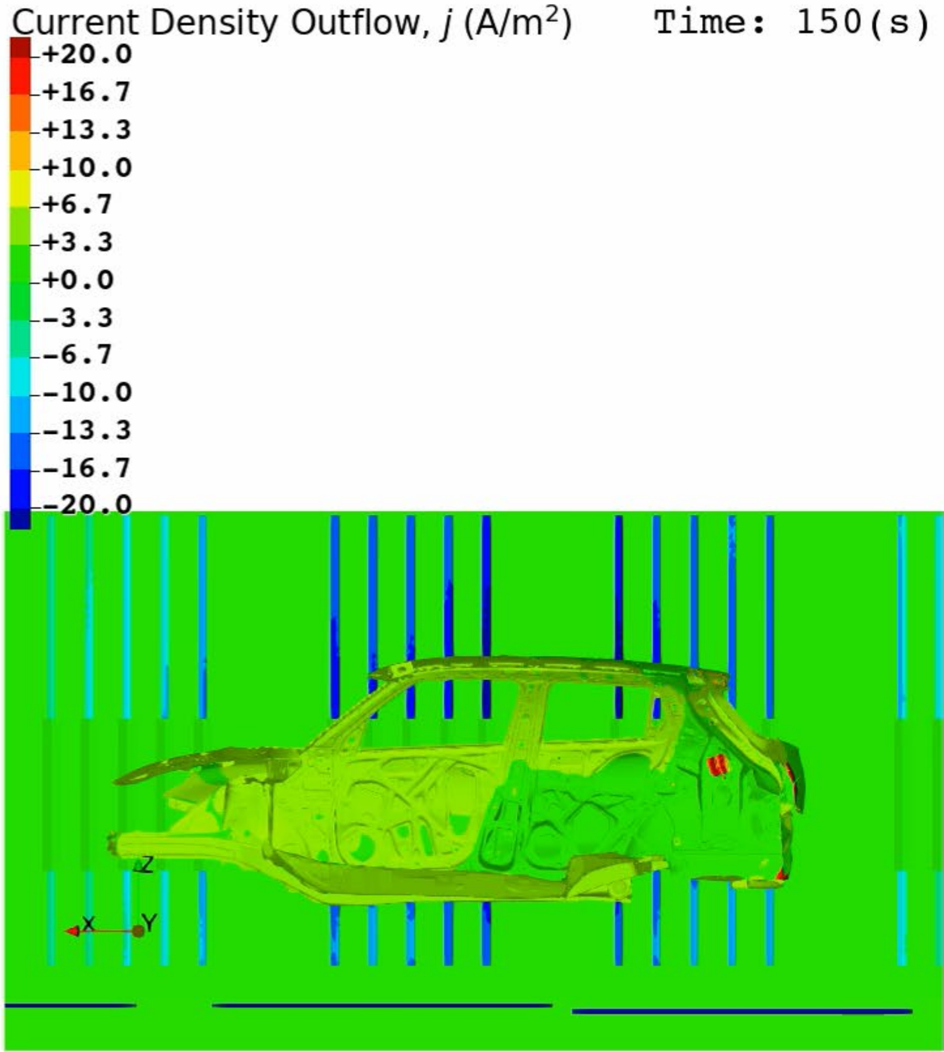
Actual Line Simulation

Reference Solution of j (ES-FEM with 51M Elems.)

Outer View



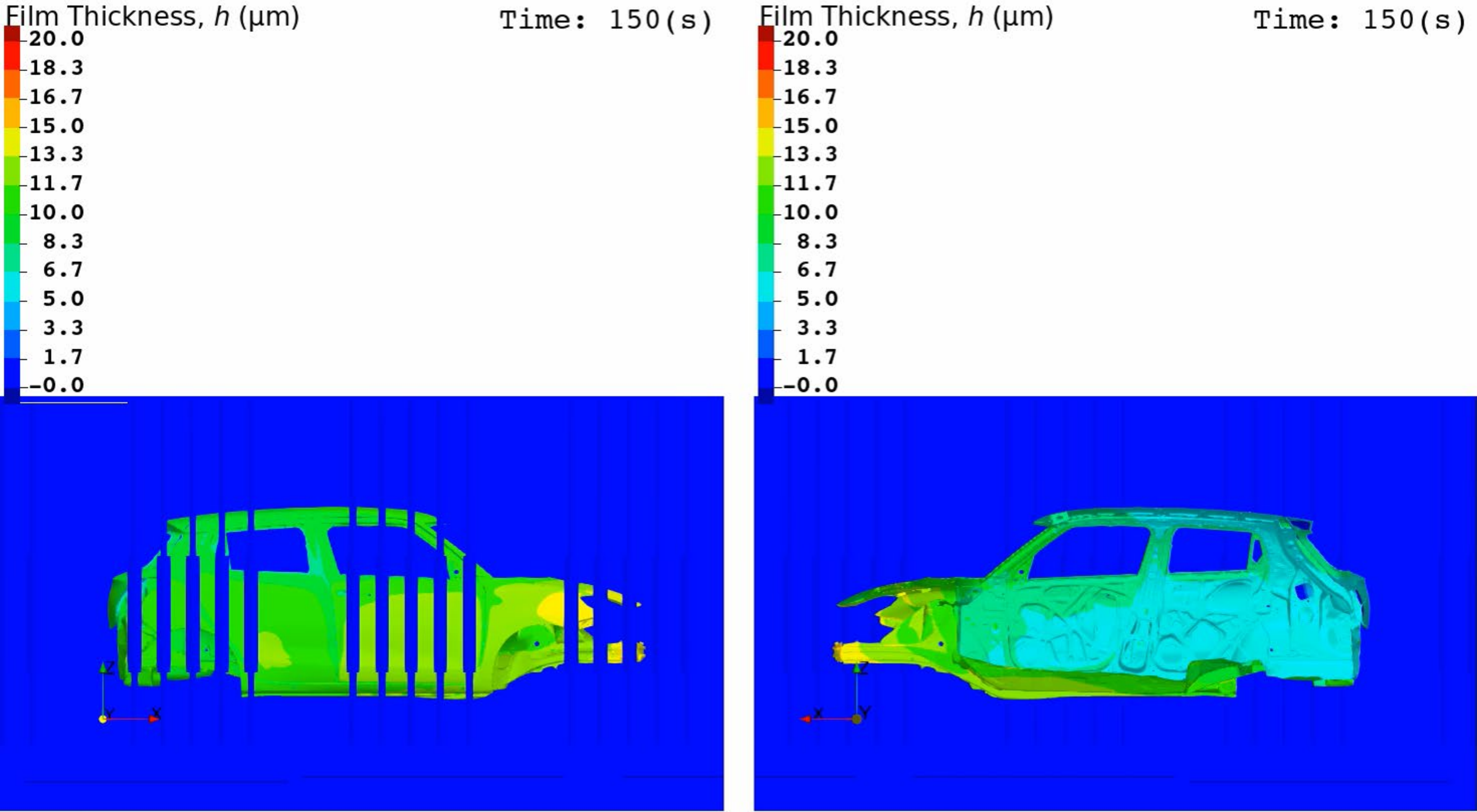
Inner View



Actual Line Simulation

Reference Solution of h (ES-FEM with 51M Elems.)

Outer View **Inner View**

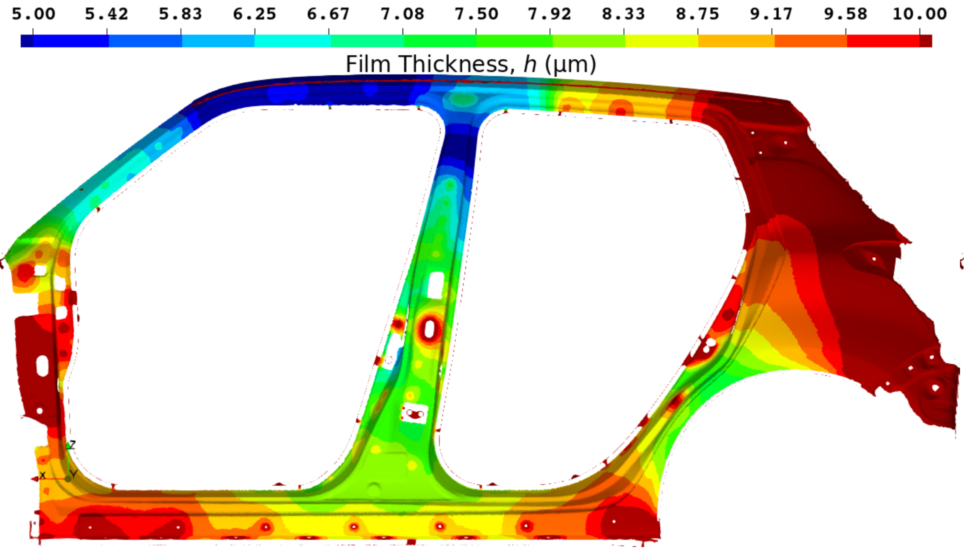


Actual Line Simulation

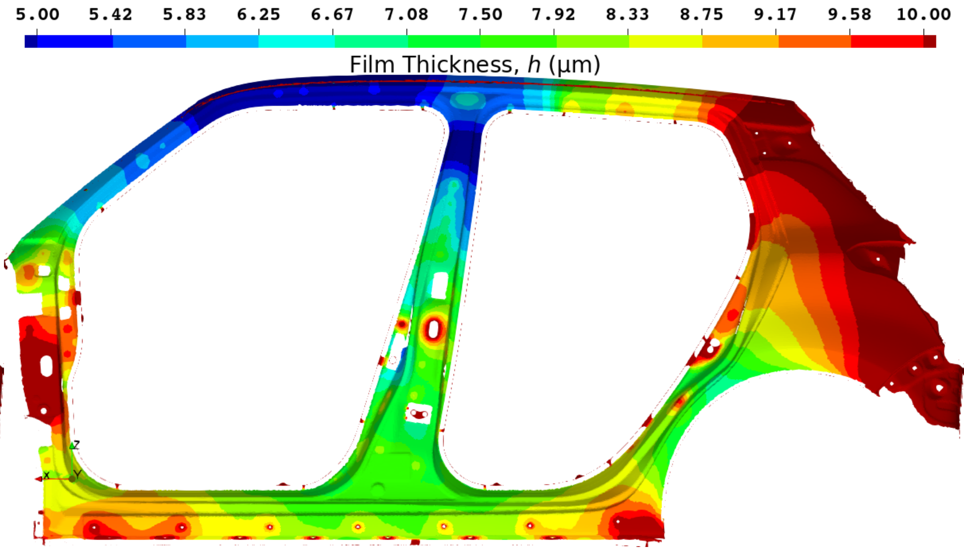
Film Thickness Distribution with 51M Elem. Mesh

(Clipped View on Side Sill)

FEM-T4



ES-FEM-T4



Reference Solution

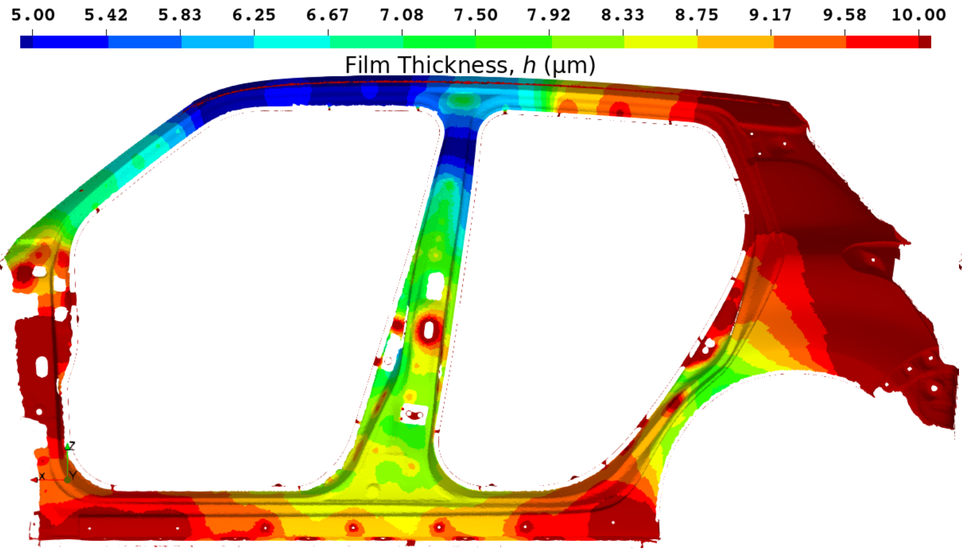
- FEM shows *a little thicker* result.
  (The center of the side sill is Yellow.)
- The ES-FEM result is regarded as a reference solution.
  (The center of the side sill is Green.)

Actual Line Simulation

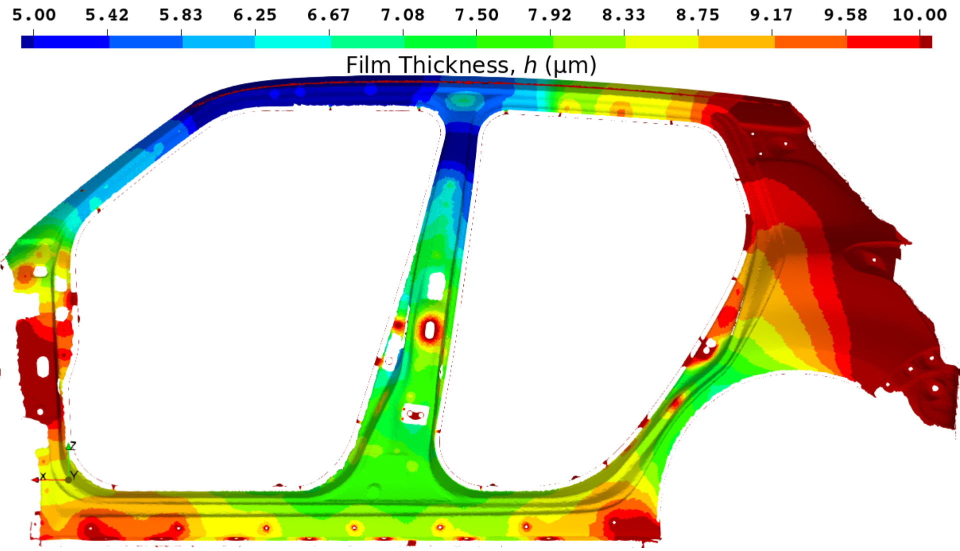
Film Thickness Distribution with 16M Elem. Mesh

(Clipped View on Side Sill)

FEM-T4



ES-FEM-T4



- FEM shows *a much thicker* result.
  (The center of the side sill is Orange.)
- ES-FEM shows a similar result.
  (The center of the side sill is Green.)

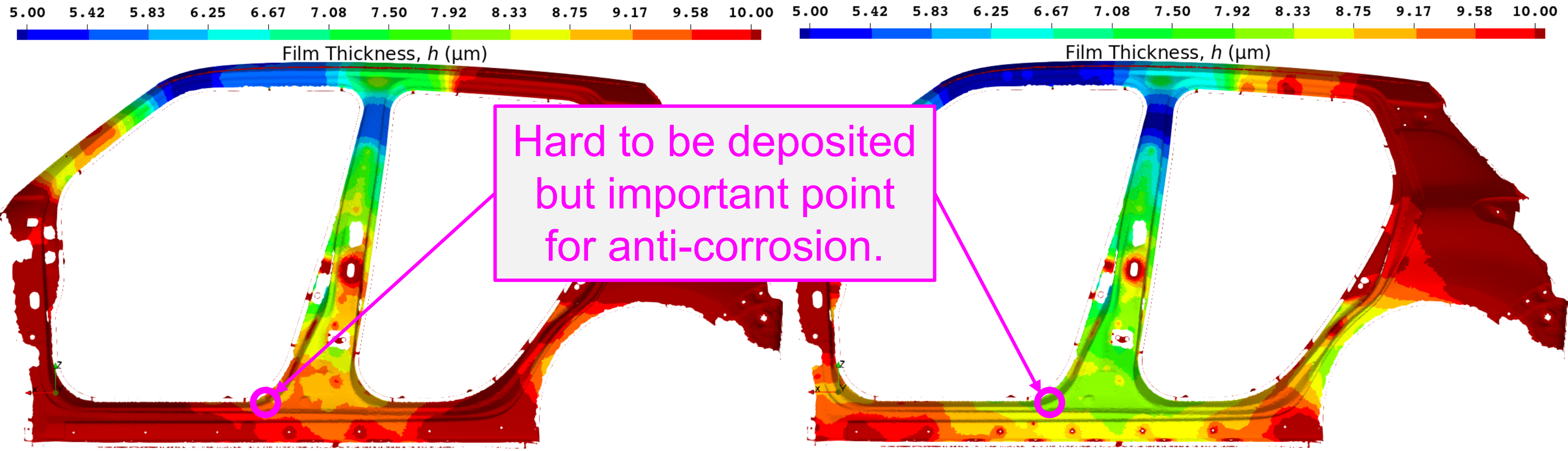
Actual Line Simulation

Film Thickness Distribution with 10M Elem. Mesh

(Clipped View on Side Sill)

FEM-T4

ES-FEM-T4



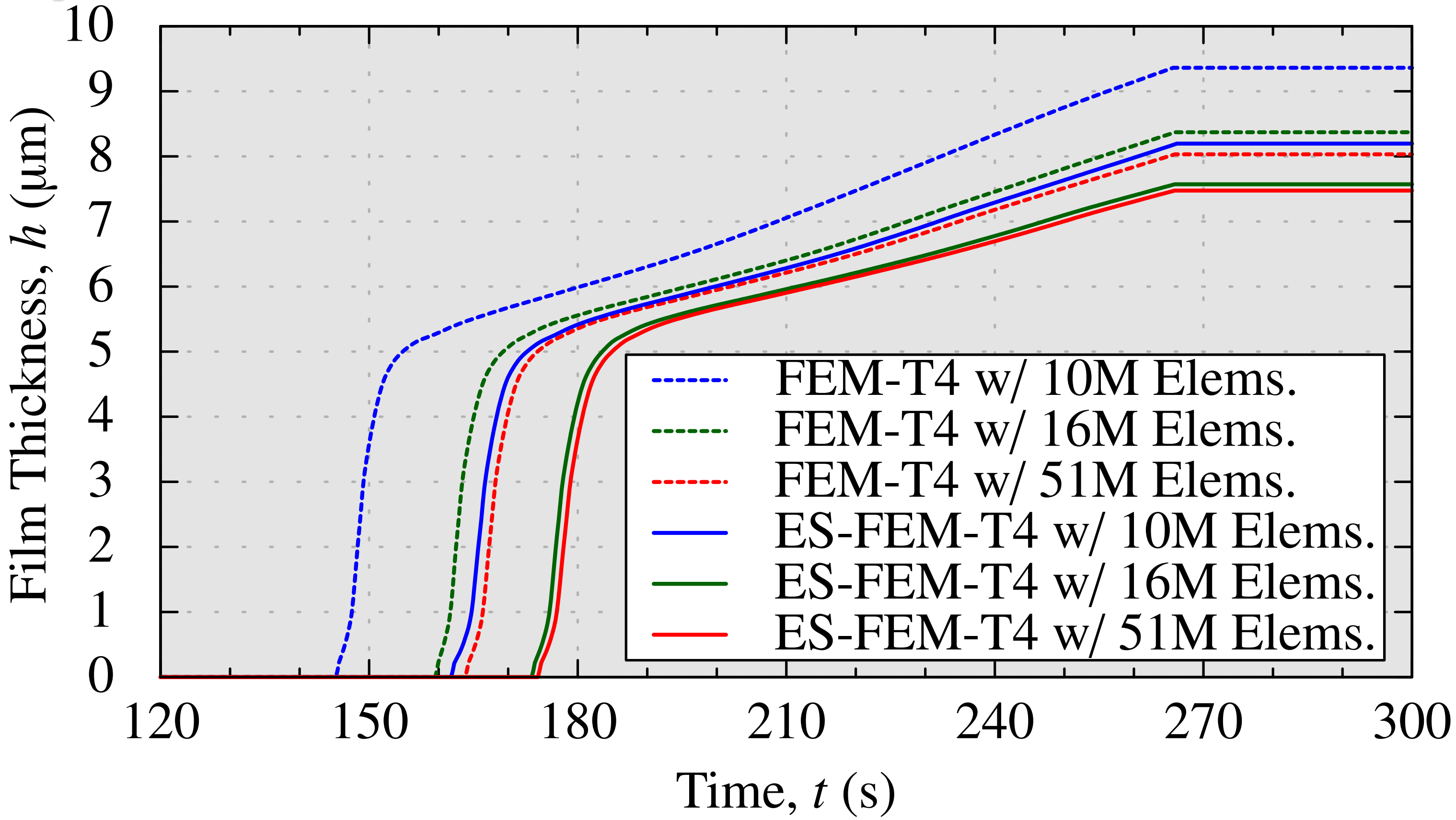
Hard to be deposited but important point for anti-corrosion.

- FEM shows ***a massively thicker*** result. (The center of the side sill is Red.)
- ES-FEM shows a little thicker result. (The center of the side sill is Yellow.)

Let's compare the time history of film thickness at a certain point.

Actual Line Simulation

Comparison of Time-histories of Film Thickness



- FEM-T4 with 51M elems. and ES-FEM-T4 with 10M elems. has almost comparable accuracy.
- ES-FEM-T4 almost gets mesh converges with 16M elems. .

Actual Line Simulation

Calculation Time

On a cluster (TSUBAME3.0: Intel Xeon E5-2680 v4, using 64 CPUs)

# of Elements	FEM-T4	ES-FEM-T4
10M	1.6 h	1.9 h
16M	2.3 h	3.4 h
51M	6.0 h	8.5 h

Comparable Accuracy

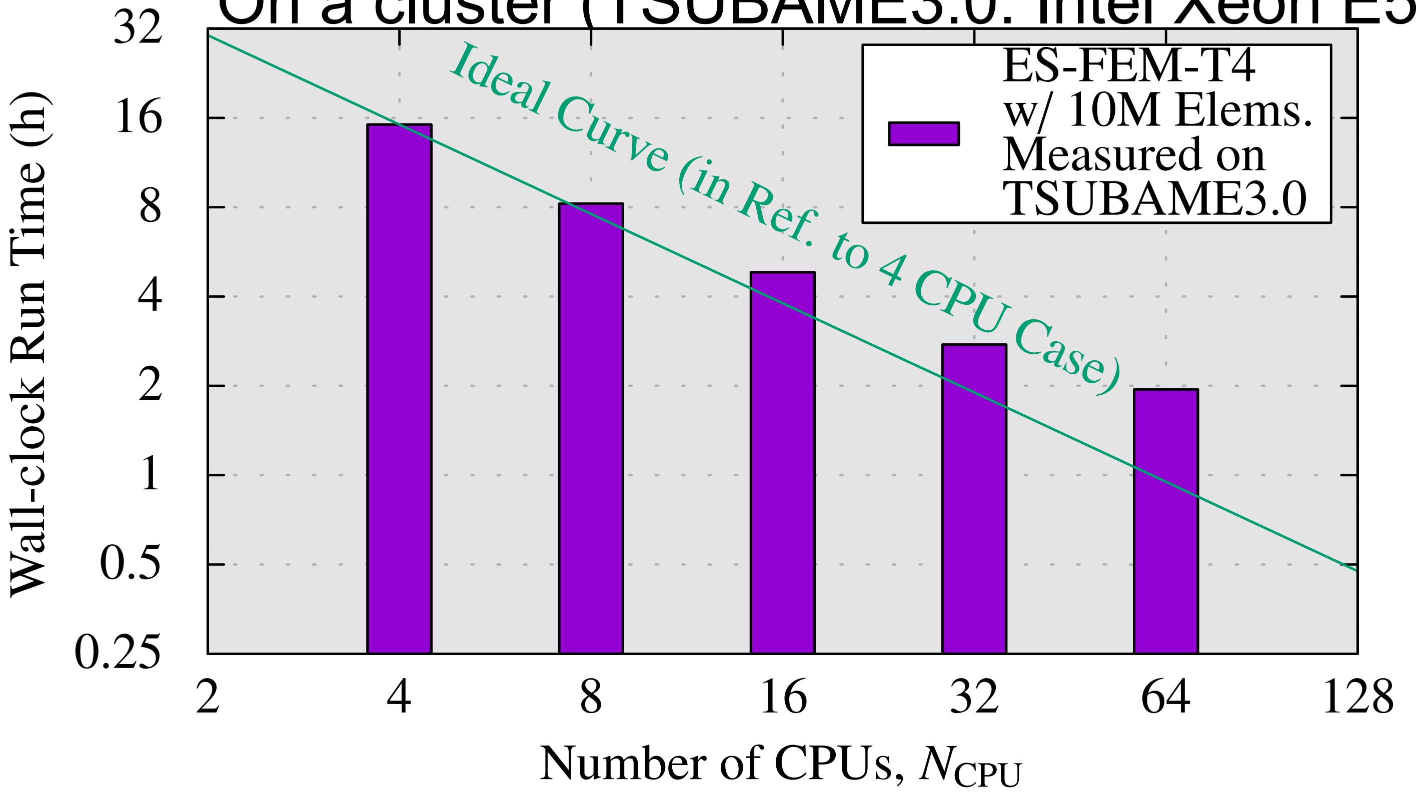
- With the same mesh, ES-FEM is slower than FEM by x1.5.
- For the same accuracy, ES-FEM is faster than FEM by x3.

For accurate ED simulations,
ES-FEM-T4 is much better than FEM-T4.

Actual line Simulation

Strong Scaling Test (with 10M Elem. Mesh)

On a cluster (TSUBAME3.0: Intel Xeon E5-2680 v4)



Our ES-FEM code scales to some extent up to 64 CPUs at least.

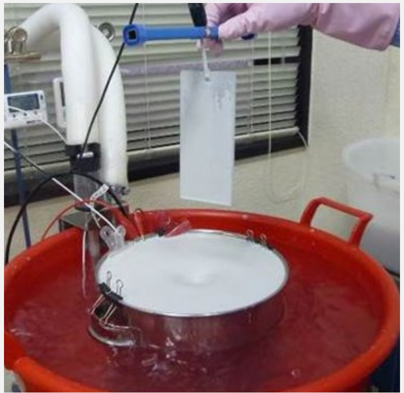
∵ Some tasks, including MPCs for the moving boundary, are not yet fully parallelized (our future work).

Validation Analyses
(for the ED Constitutive Model)

Framework of ED Simulation

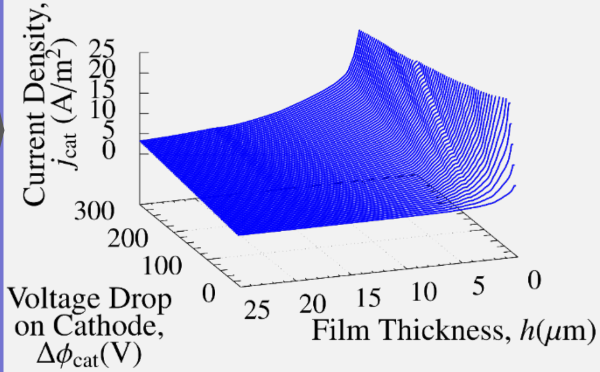
1. Basic Tests

(one-plate tests)



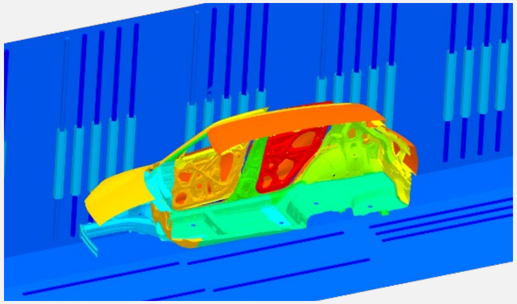
2. Model Identification

(film resistance/growth)



3. Calculation

(FEM etc.)



Framework for Solid Mechanics Simulation

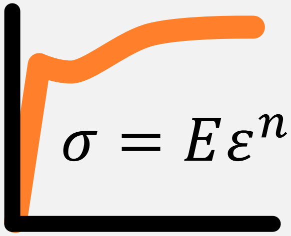
1. Basic Tests

(Tensile test)

https://www.aandd.co.jp/adhome/products/test/rth_rti.html

2. Model Identification

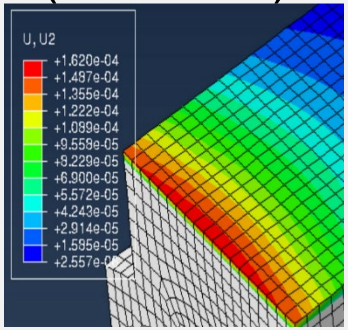
(stress/strain curve)



$$\sigma = E\varepsilon^n$$

3. Calculation

(FEM etc.)



Two Complexities in ED Phenomena

There are two nonlinearities in ED phenomena, thus our ED boundary model consists of 2 sub-models:

1. Film resistance model

   Film resistance R is NOT linear to film thickness h:

   $$R \neq \alpha h$$

   R: resistance, α: const., h: film thickness.

2. Film growth model

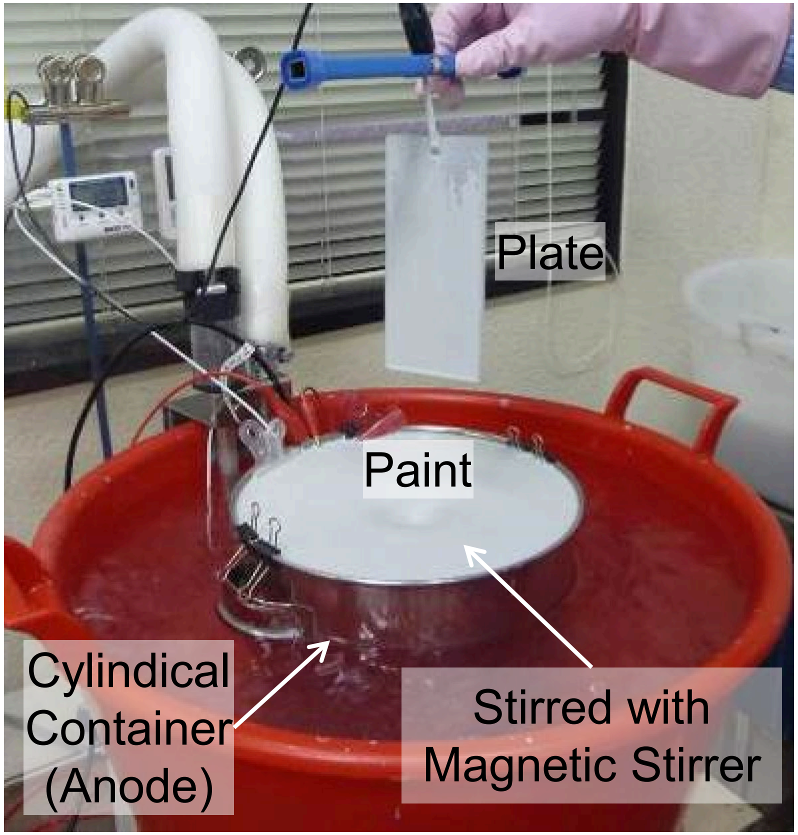
   Film growth rate $\dot{h}$ is NOT linear to current density j:

   $$\dot{h} \neq \beta j$$

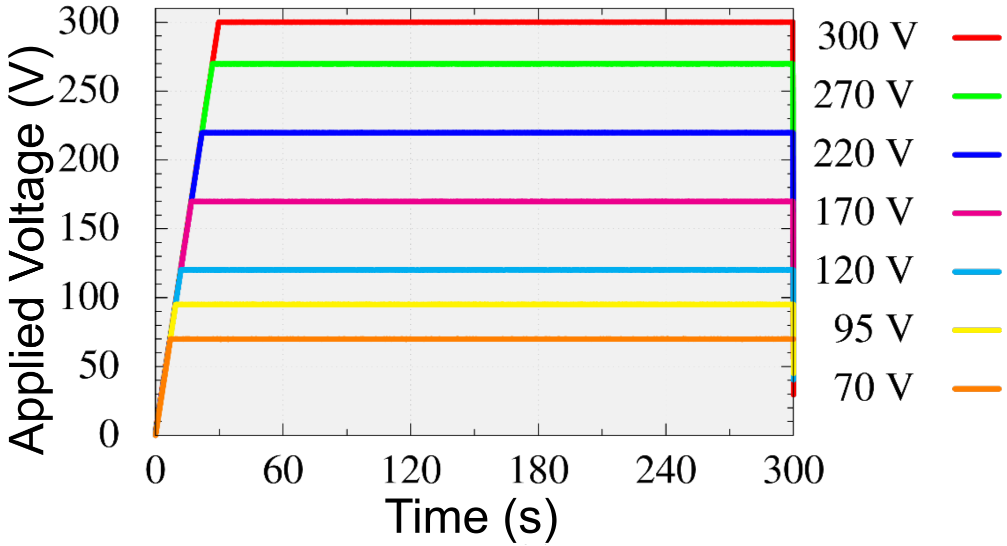
   $\dot{h}$: film growth rate, β: const., j: current density.

We need to conduct many lab tests
to reveal these nonlinear behavior.

Outline of the One-Plate Test



Input	Applied Voltage, Deposition Time, Stirring Speed (on/off).
Output	Time history of Current, Film thickness after Deposition.



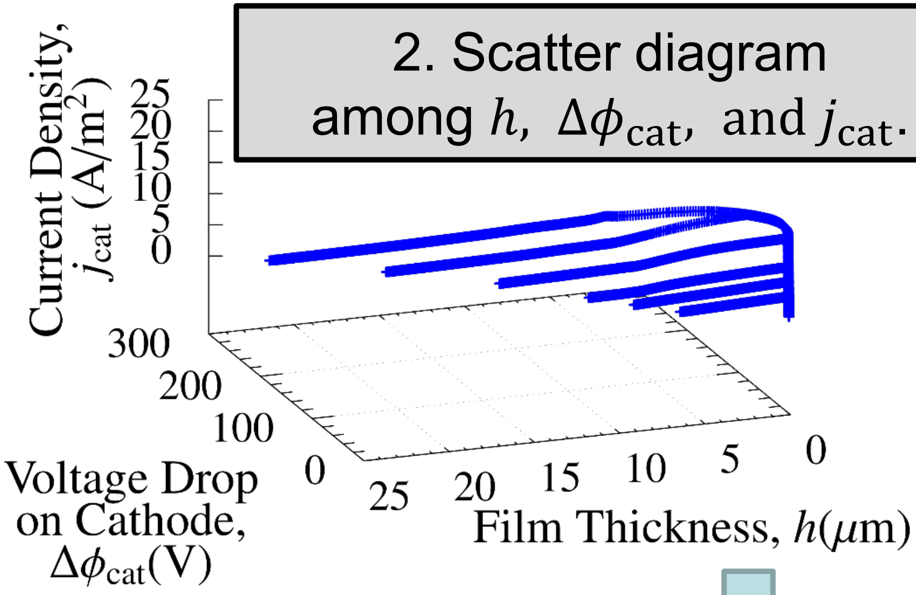
- One plate is dipped in paint contained in a cylindrical anode.
- Many tests are conducted with various applied voltages, deposition times, and stirring speed: about 150 tests in total. (∵ In-situ measurement of film thickness is impossible.)

Procedure to Identify Film Resistance Model

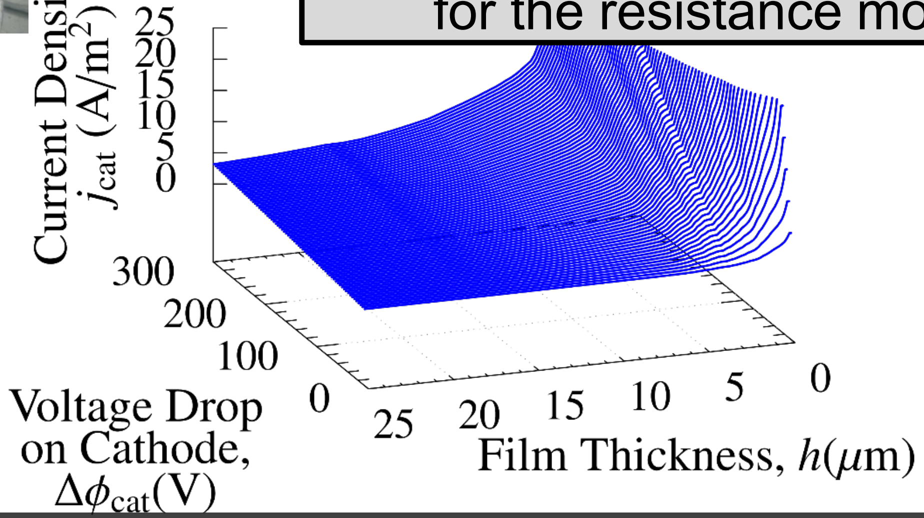
1. One-plate tests.

Plot

2. Scatter diagram among h, $\Delta\phi_{\mathrm{cat}}$, and j_{cat}.



3. Approximated surface for the resistance model.



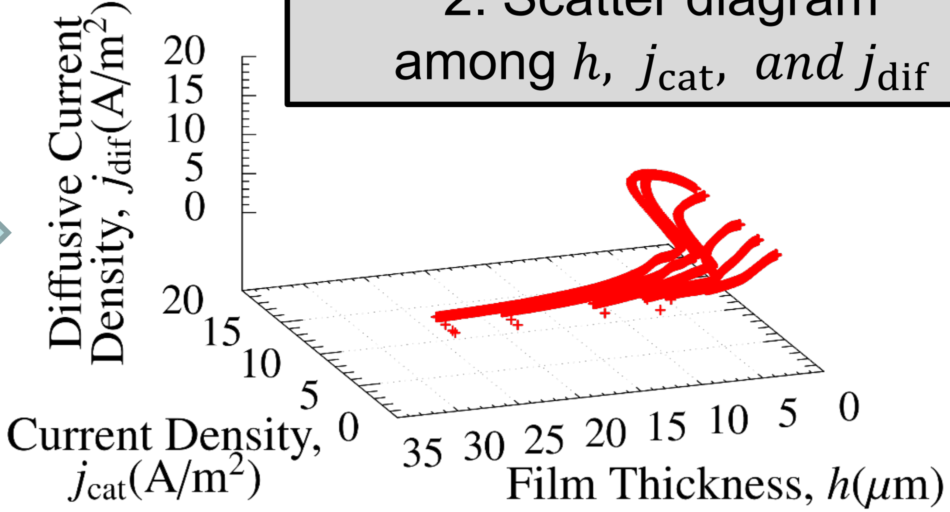
Data Fitting

Procedure to Identify Film Growth Model

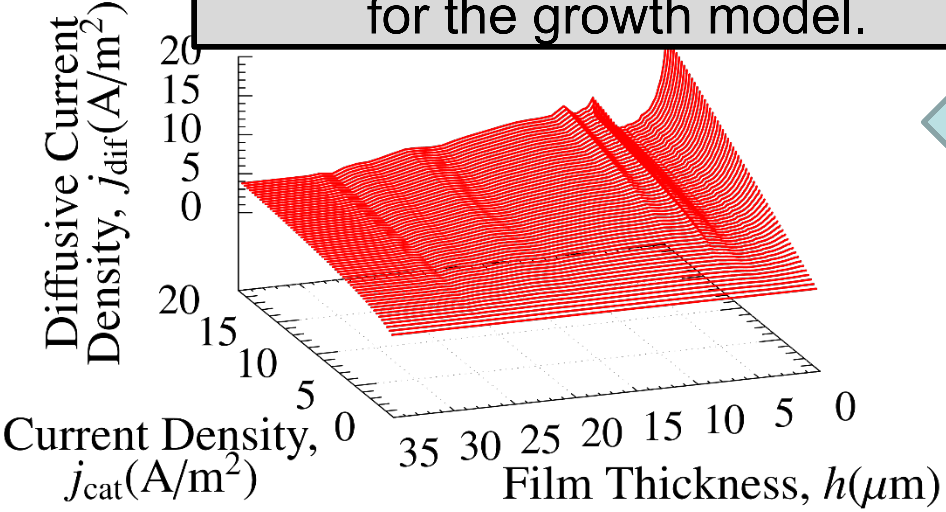
1. One-plate tests.

Plot

2. Scatter diagram among h, j_{cat}, and j_{dif}



3. Approximated surface for the growth model.

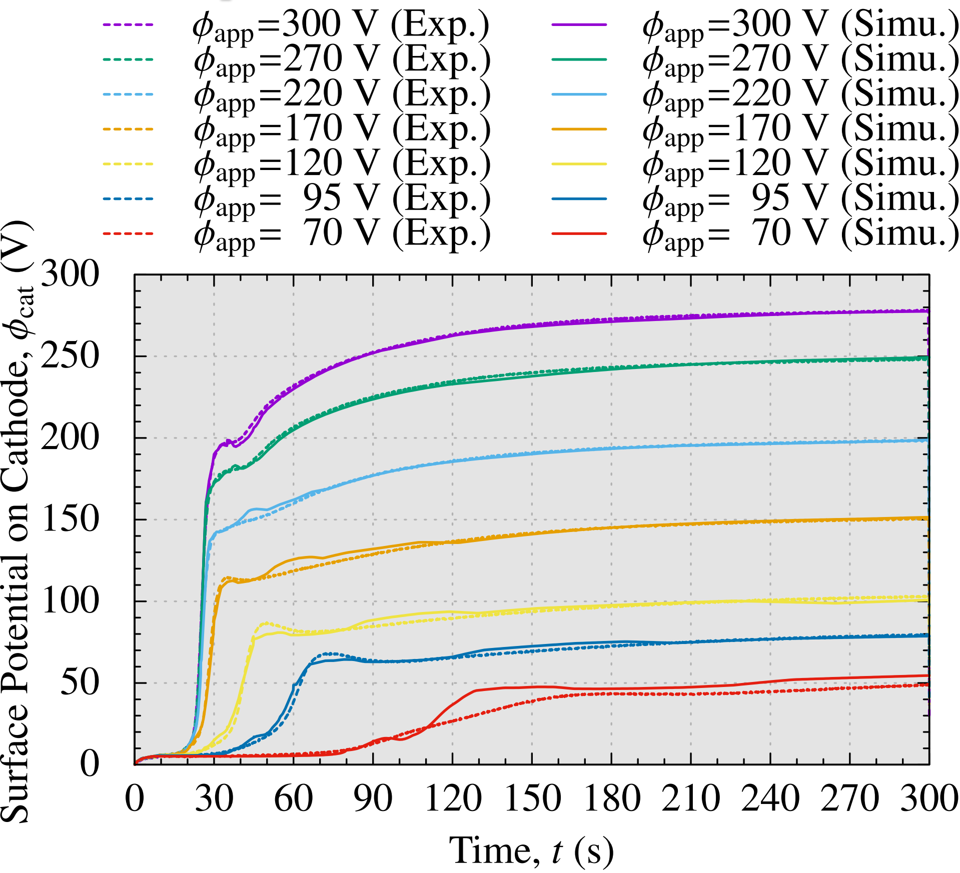


Data Fitting

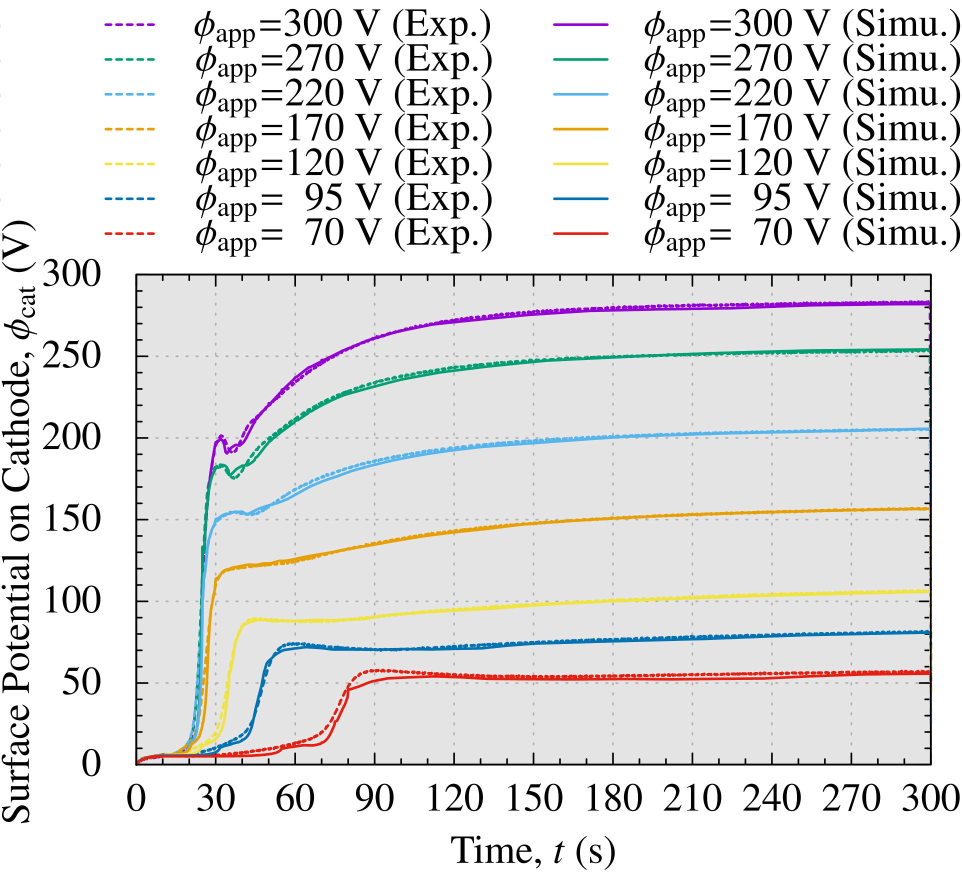
Diffusive current density j_{dif} represents the amount of electricity consumed for the electrolysis of water (H_2O), not used for the deposition of the coating film.

One-Plate Test/Simulation

Comparison of Surface Potential Time Histories



Stirred (with Flow)

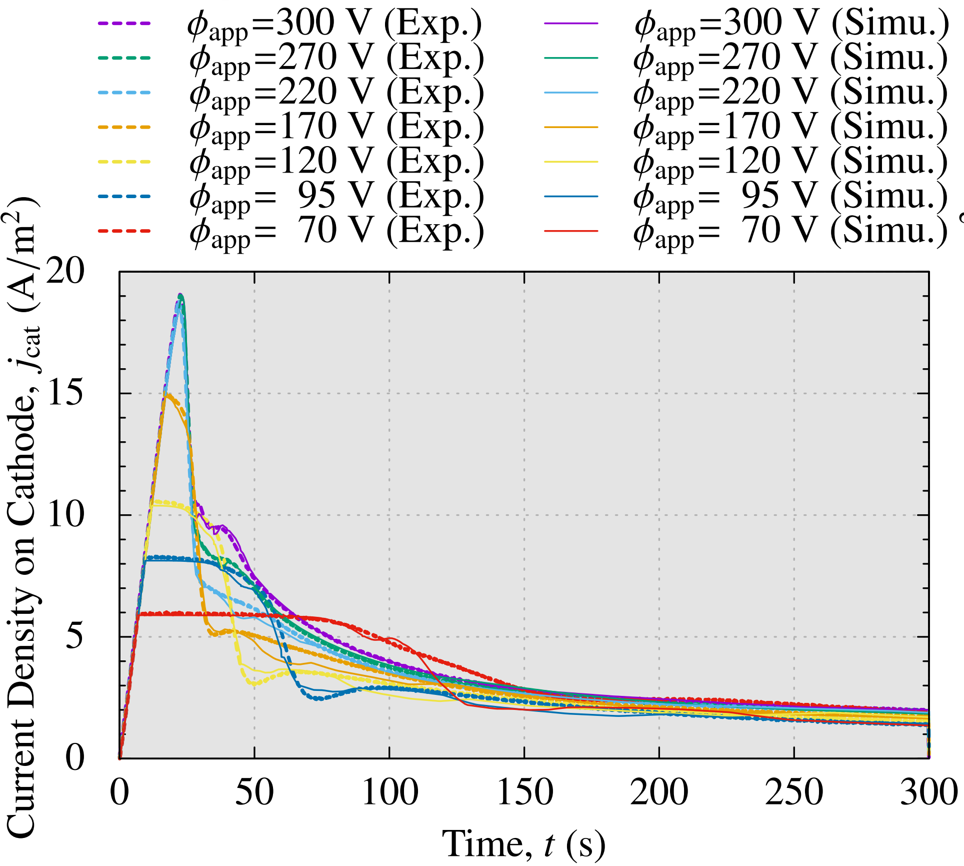


Unstirred (without Flow)

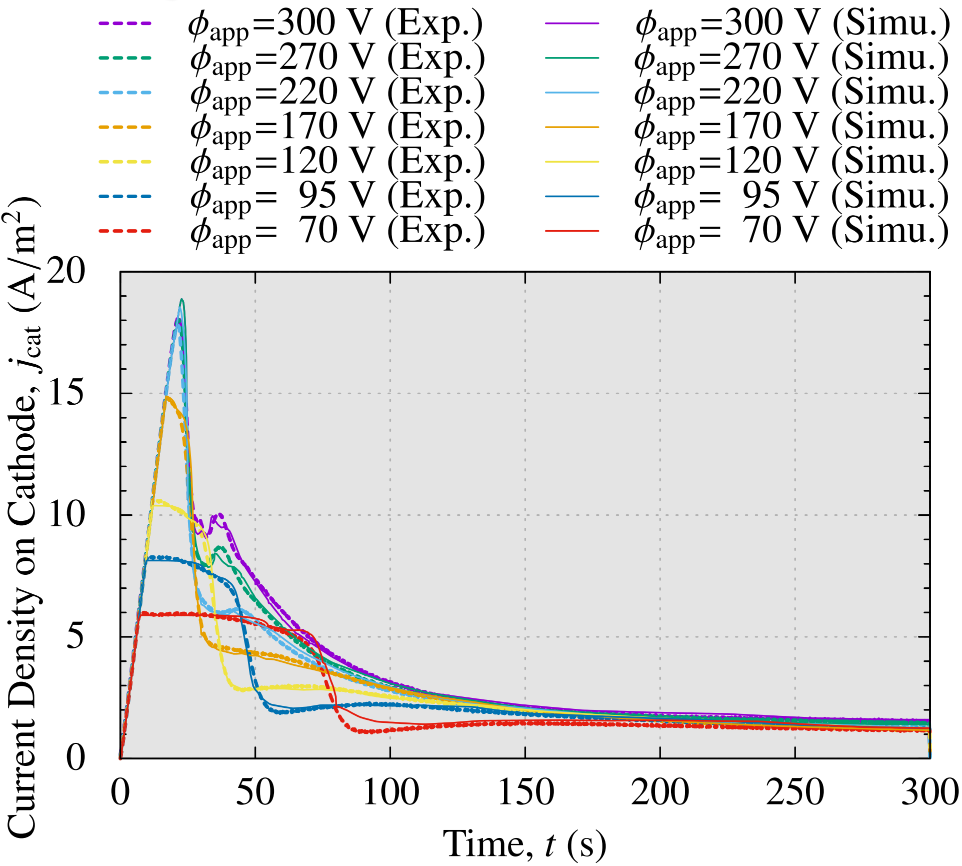
The simulation curves agree well with the experimental ones with small error in the results at lower voltages.

One-Plate Test/Simulation

Comparison of Current Density Time Histories



Stirred (with Flow)

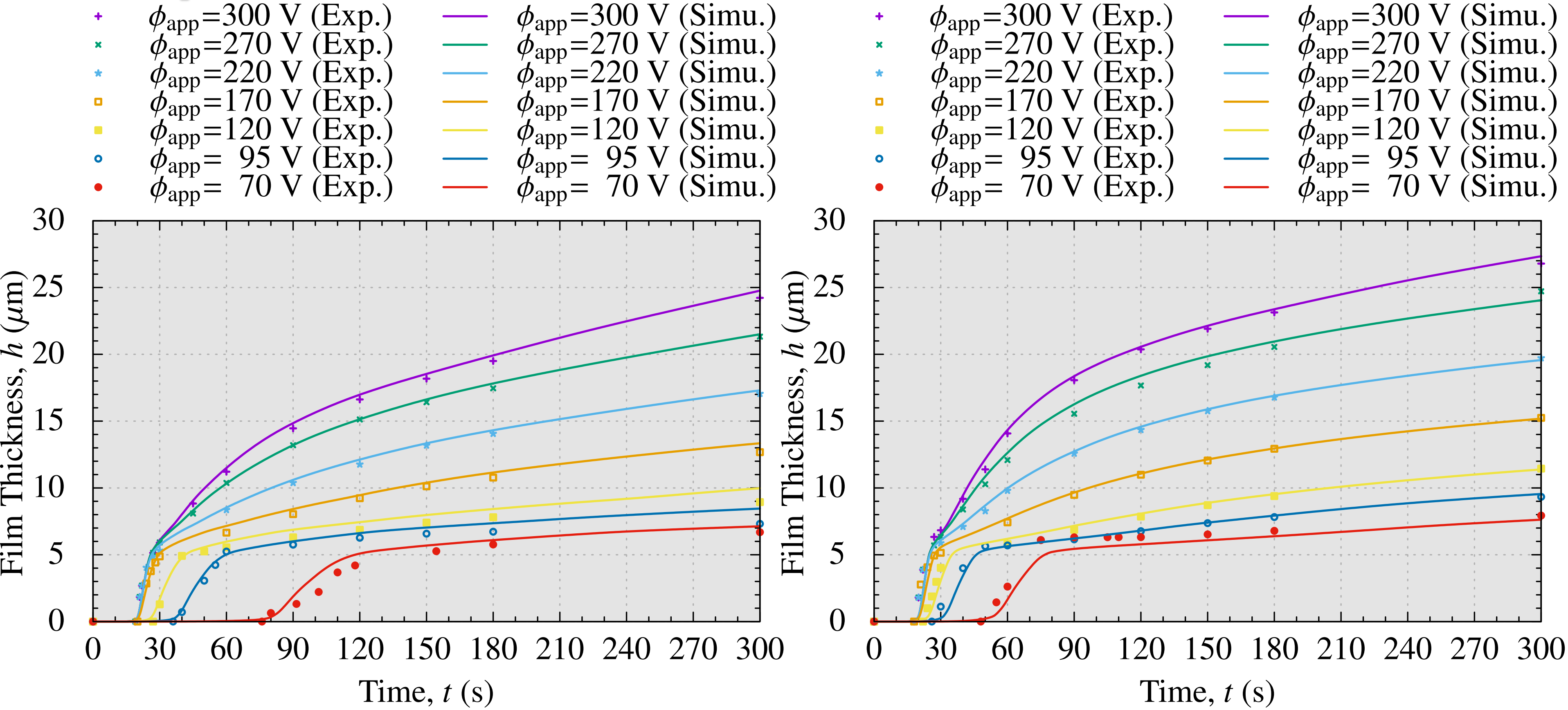


Unstirred (without Flow)

The simulation curves agree well with the experimental ones with small error in the results at lower voltages.

One-Plate Test/Simulation

Comparison of Film Thickness Time Histories



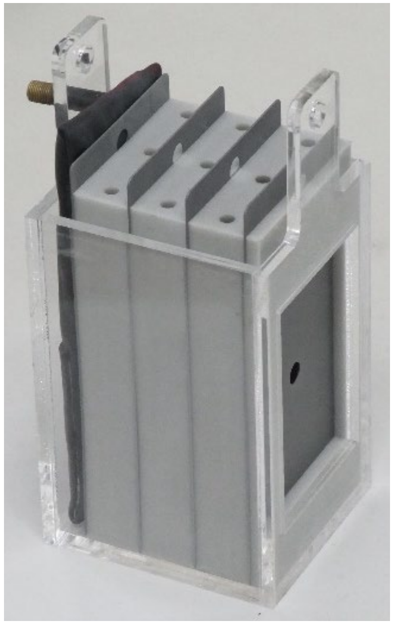
Stirred (with Flow)

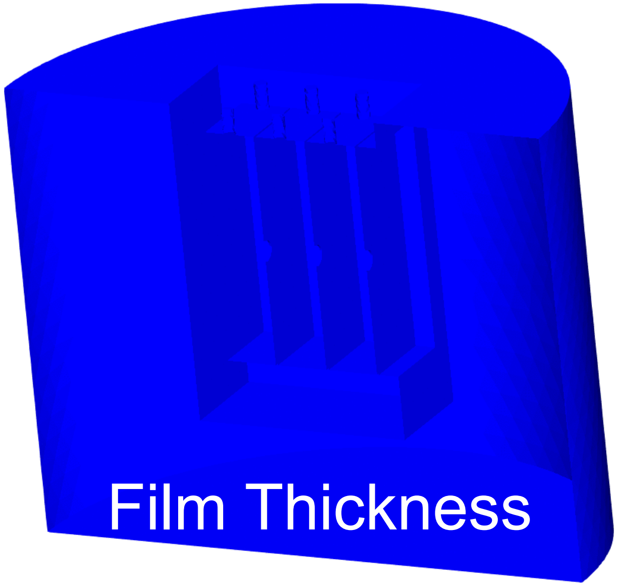
Unstirred (without Flow)

The simulation curves agree well with the experimental ones with small error in the results at lower voltages.

4-Plate BOX Test/Simulation

Outline



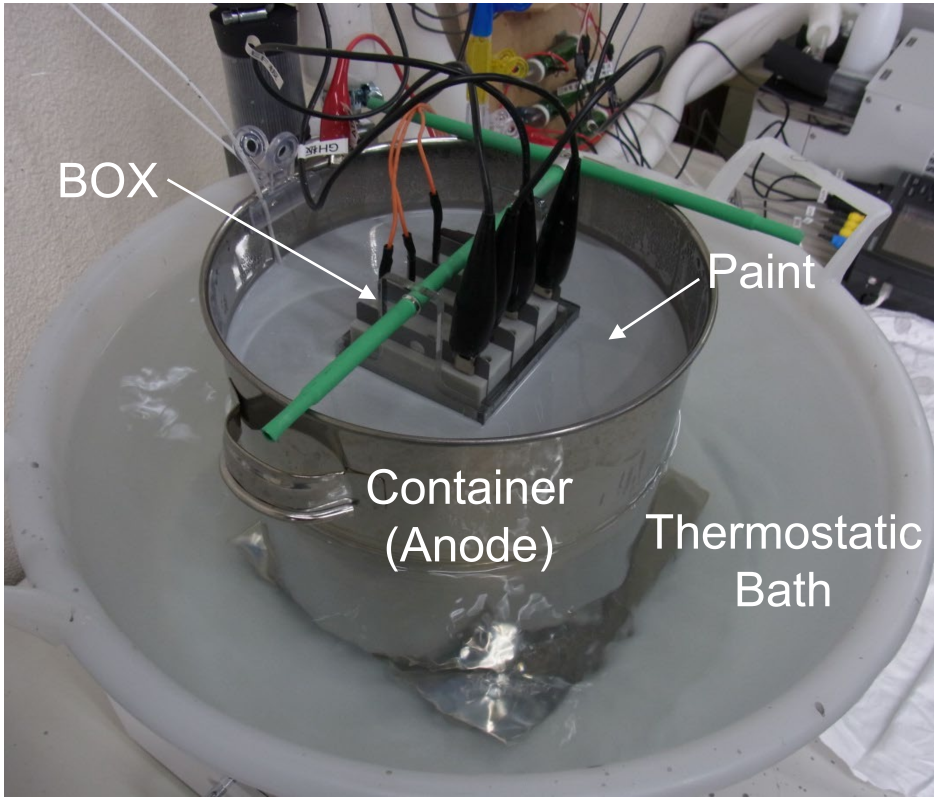


(http://plaza.rakuten.co.jp/heroc)

- Model for stirred BC is assigned to the outer surface, while that for unstirred BC is assigned to the other inner surfaces.
- **Accuracy of surface potential and final film thickness at the measurement points are evaluated** using ES-FEM-T4.

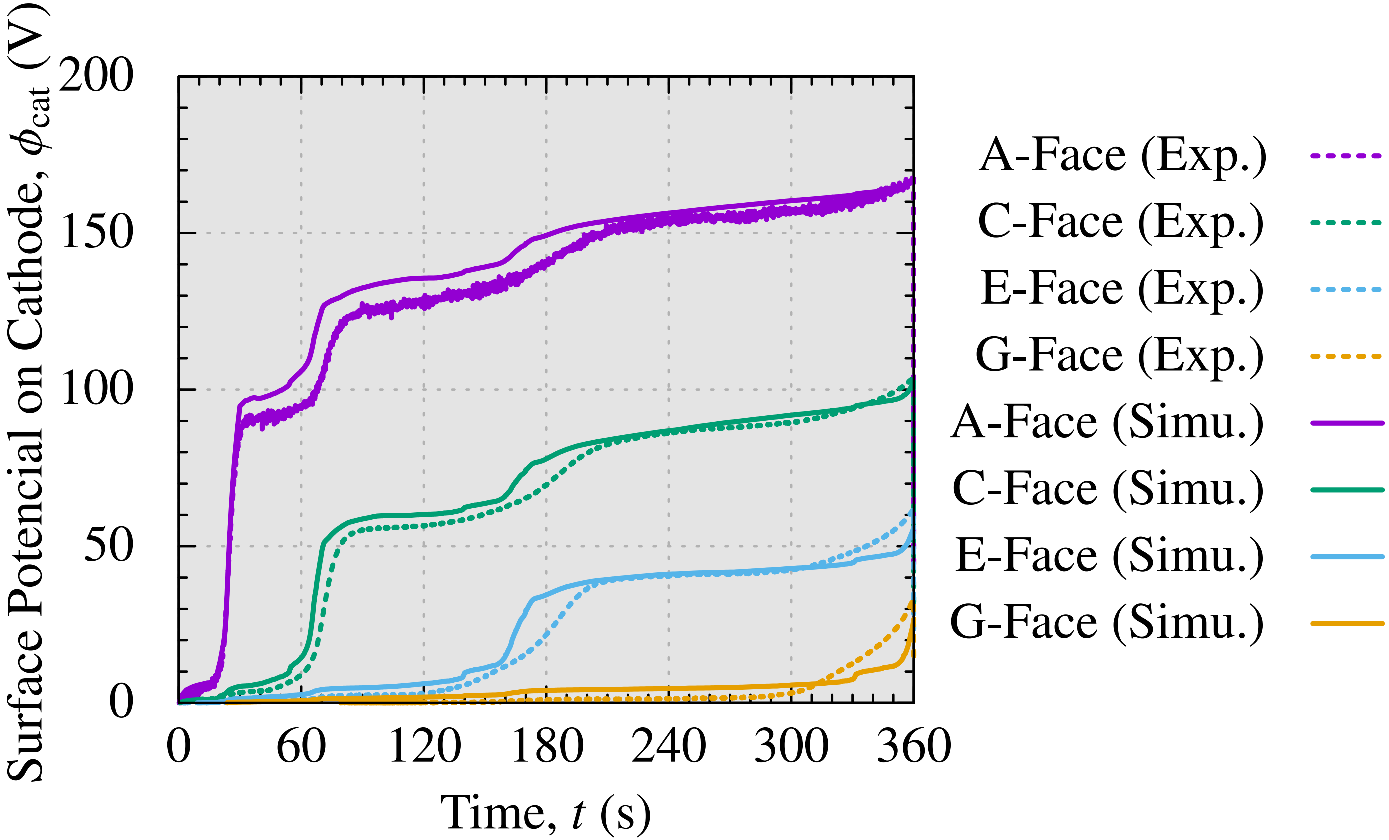
4-Plate BOX Simulation

Photo of the 4-Plate BOX Test



4-Plate BOX Test/Simulation

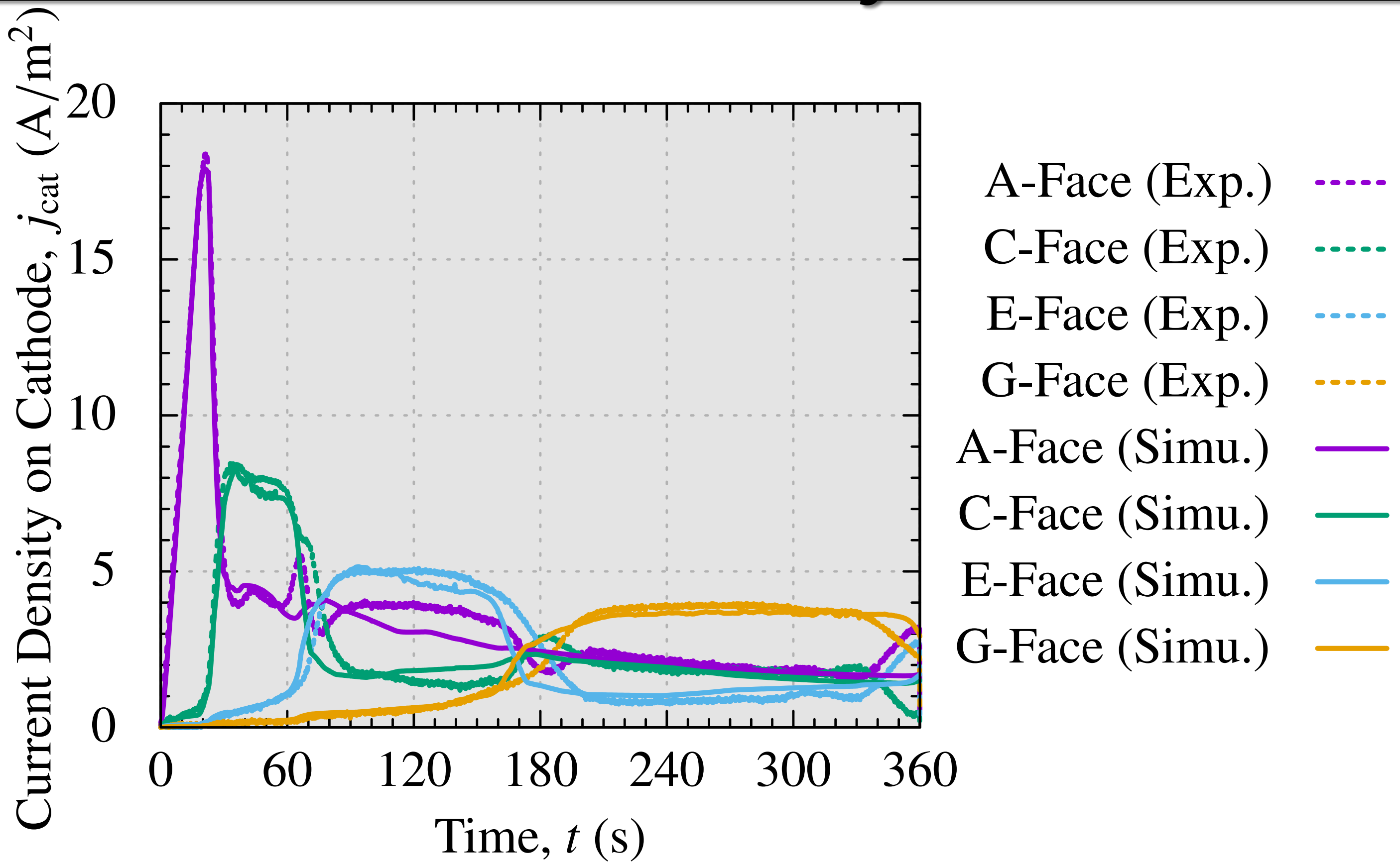
Comparison of Surface Potential Time Histories



The simulation curves agree well with the experimental ones.

4-Plate BOX Test/Simulation

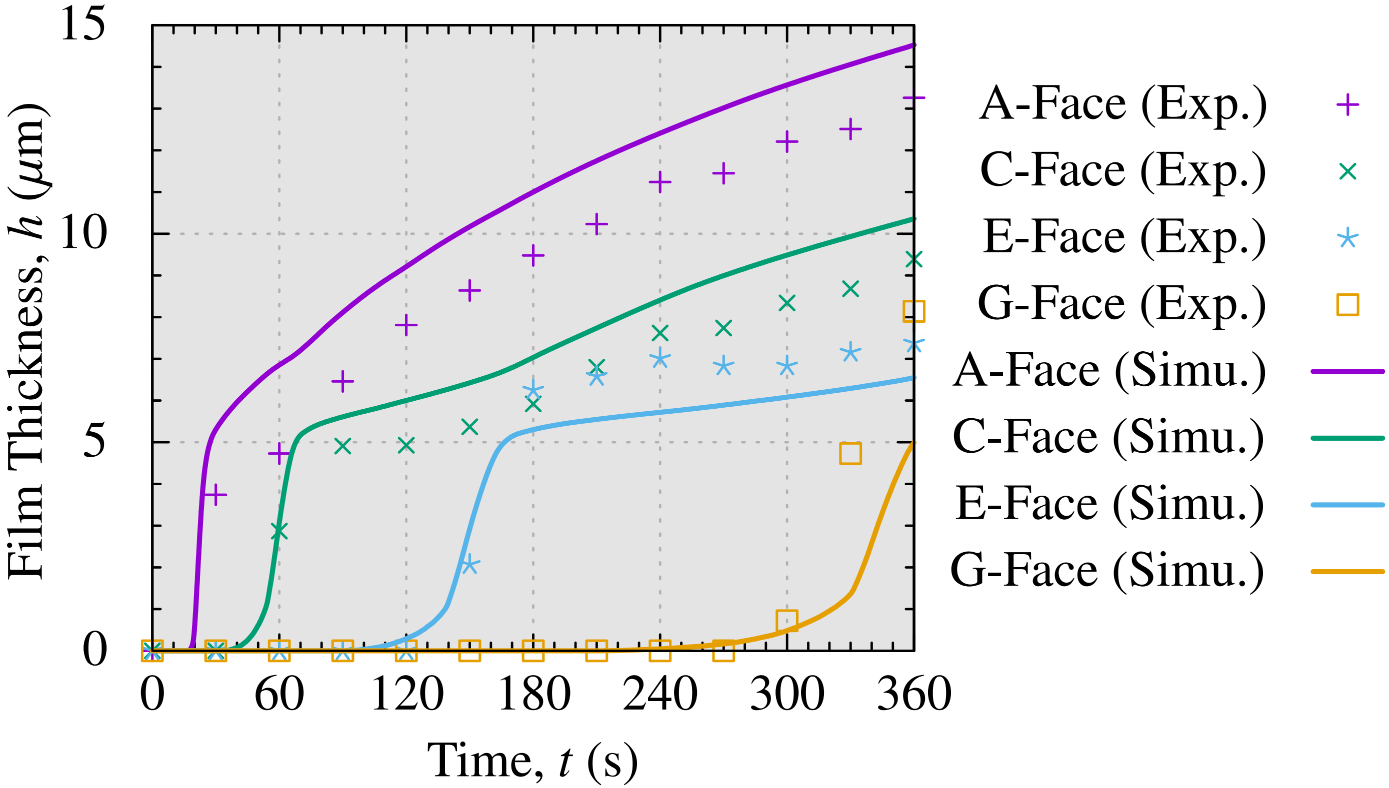
Comparison of Current Density Time Histories



The simulation curves agree well with the experimental ones.

4-Plate BOX Test/Simulation

Comparison of Film Thickness Time Histories

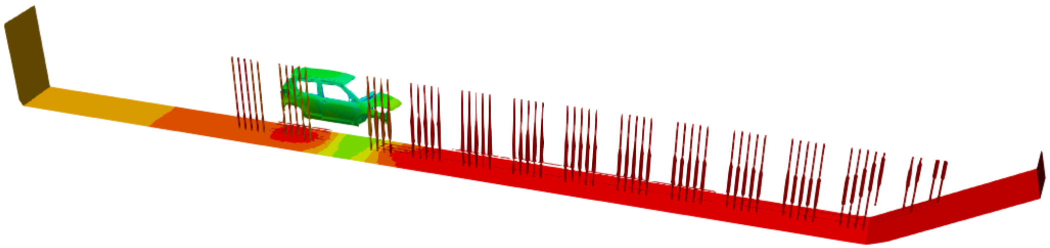


The simulation curves agree well with the experimental ones, except the curve on the innermost face (G-Face): 3 μm error.

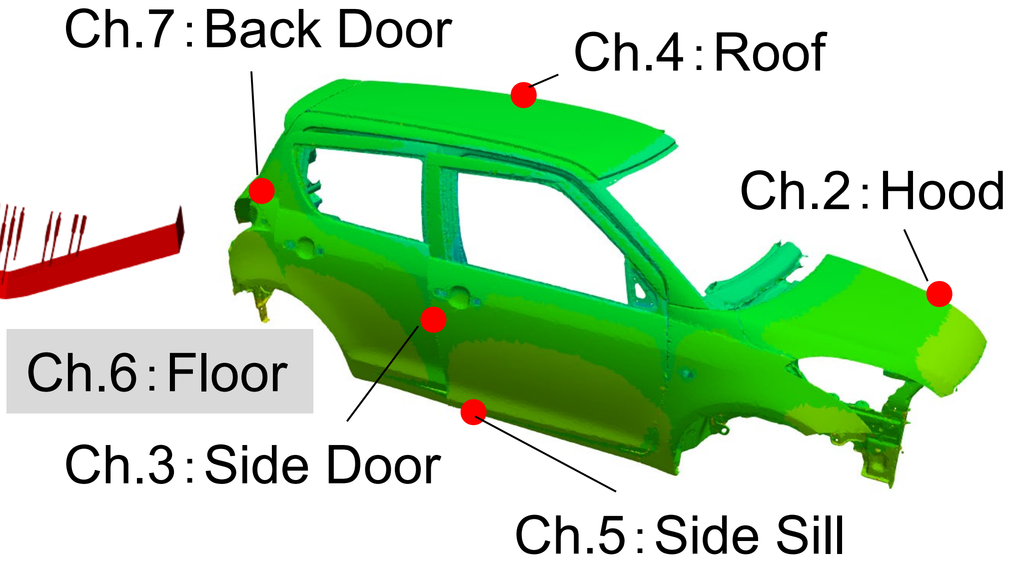
Further improvement of the ED constitutive model is required.

Actual Line Test/Simulation

Overview



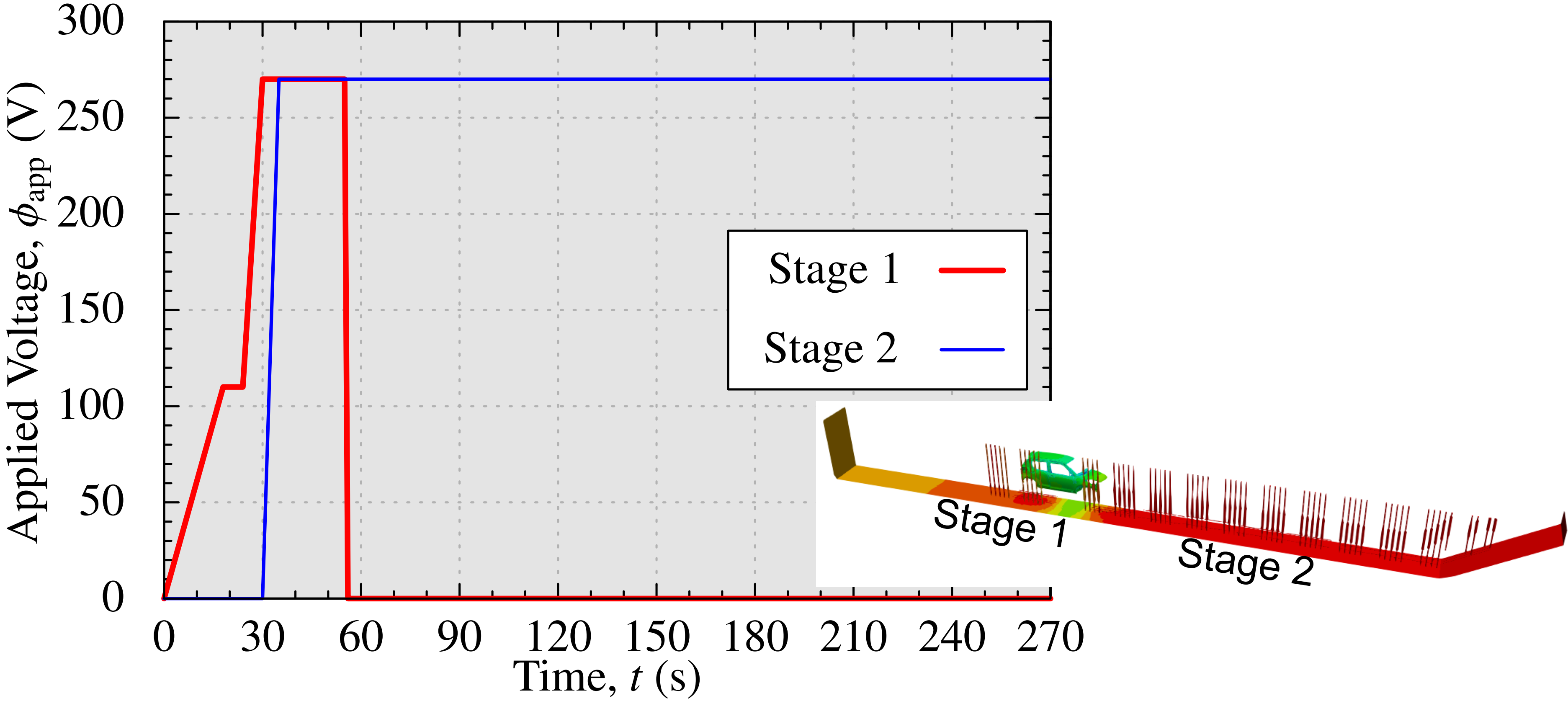
Six Measurement Points



- Half-body analysis (only right-hand side).
- Entire line shape, carbody motion, and electrode conditions are faithfully reproduced.
- Model for stirred BC is assigned to the outer surface, while that for unstirred BC is assigned to the inner surface.
- **Accuracy of surface potential and final film thickness at the measurement points are evaluated** using ES-FEM-T4.

Actual Line Test/Simulation

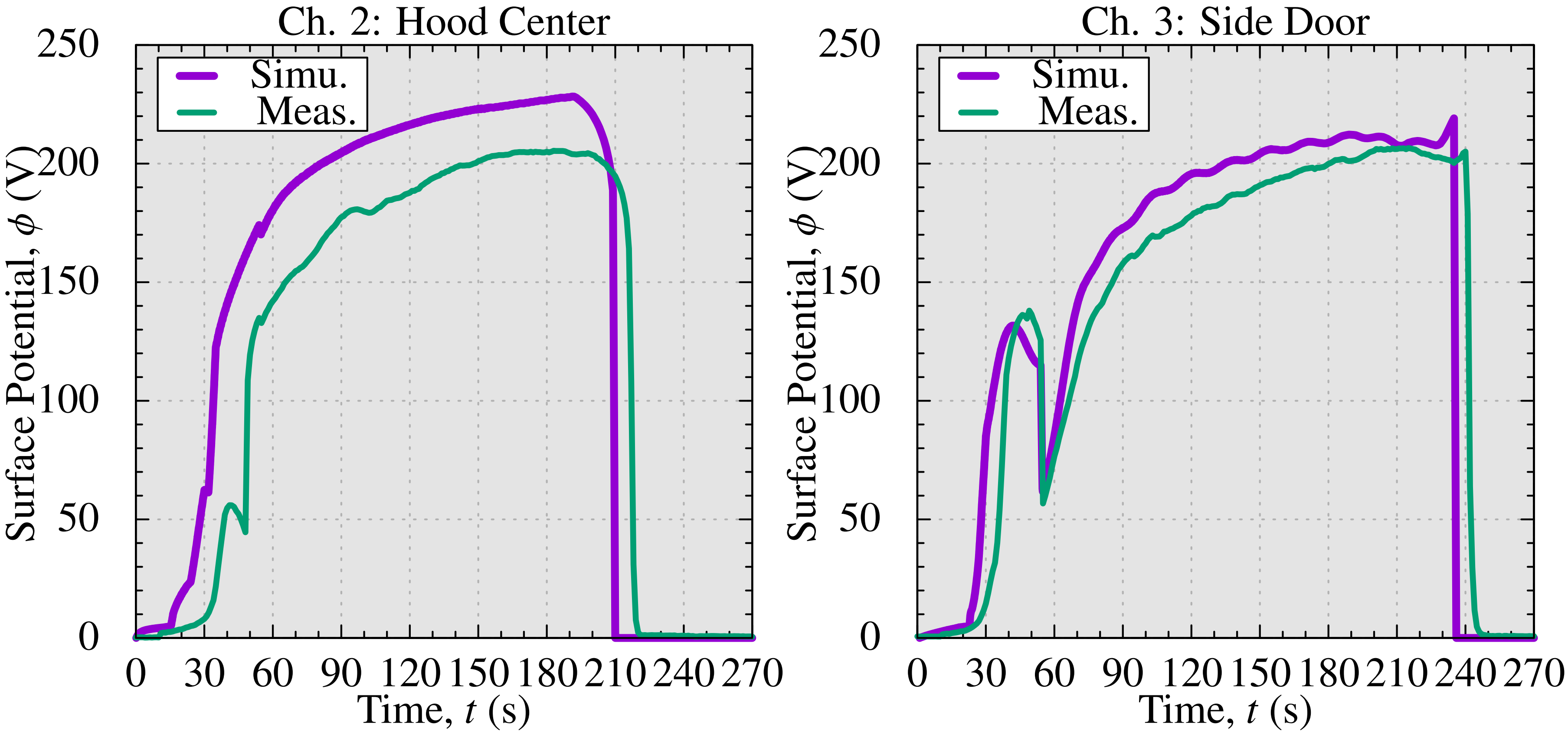
Time History of Applied Voltage to Anodes



- “Stage 1” denotes anodes on the entry side, whereas “Stage 2” denotes anodes on the exit side.
- Note that **there is a sudden turn on/off of power**.

Actual Line Test/Simulation

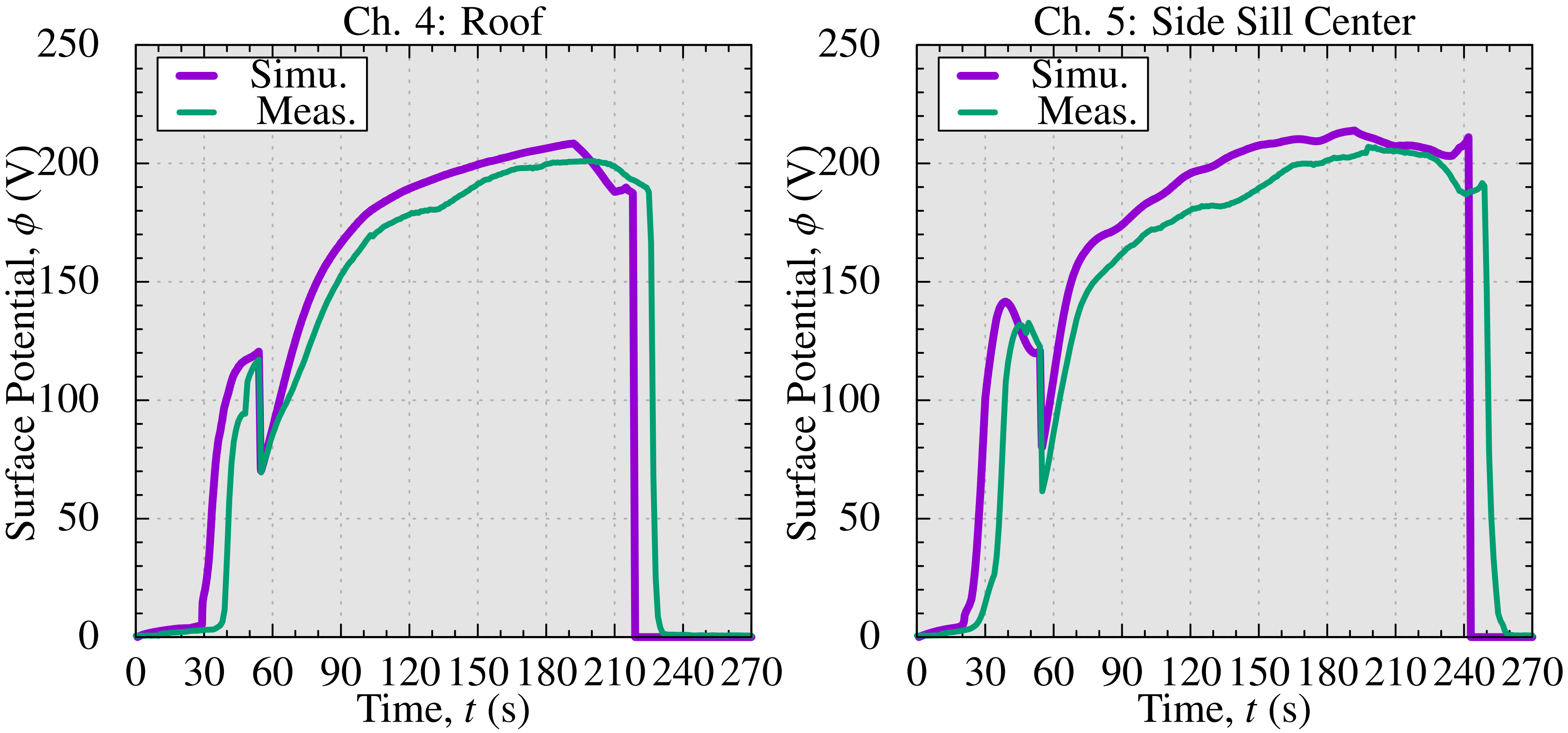
Comparison of Surface Potential Time Histories



Although the rise in surface potential tends to be faster in the simulation, the measurement curves are nearly reproduced.

Actual Line Test/Simulation

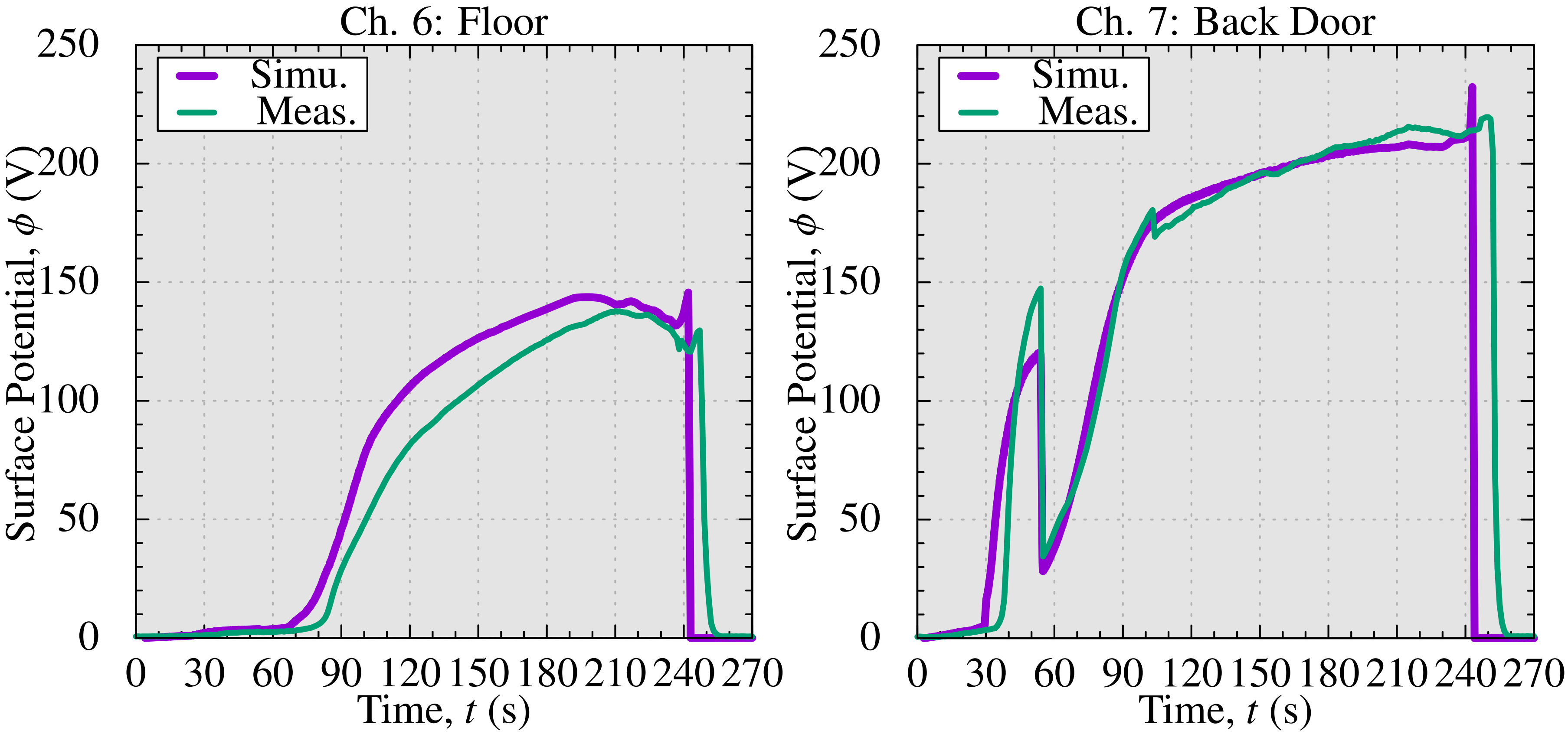
Comparison of Surface Potential Time Histories



Although the rise in surface potential tends to be faster in the simulation, the measurement curves are nearly reproduced.

Actual Line Test/Simulation

Comparison of Surface Potential Time Histories



The measurement results are reproduced with good accuracy even for inner plates (Ch.6: Floor), where lack of accuracy has been an issue.

Actual Line Test/Simulation

Comparison of Final Film Thickness

Point	Measued (μm)	Simulated (μm)	Error (μm)
Ch.2: Hood	20.1	21.4	+1.3 (+6.5%)
Ch.3: Side Door	19.0	21.0	+2.0 (+10.5%)
Ch.4: Roof	17.0	19.3	+2.3 (+13.5%)
Ch.5: Side Sill	20.0	21.6	+1.6 (+8.0%)
Ch.6: Floor	–	14.5	–
Ch.7: Back Door	23.0	20.3	−2.7 (−11.7%)

The maximum error is less than 3 μm, and thus our ED constitutive model has practical accuracy in the actual line simulation.

Summary

Summary

Conclusion

- ES-FEM-T4 was applied to the actual ED line simulations.
- The high accuracy of ES-FEM-T4, owing to its superlinear (almost quadratic) mesh convergence rate in ED simulation, was confirmed compared to the poor accuracy of FEM-T4.
- Our parallelized ES-FEM-T4 code enabled us to obtain mesh-converged accurate solutions of actual line simulations in reasonable time with relatively coarse meshes.

Future Works

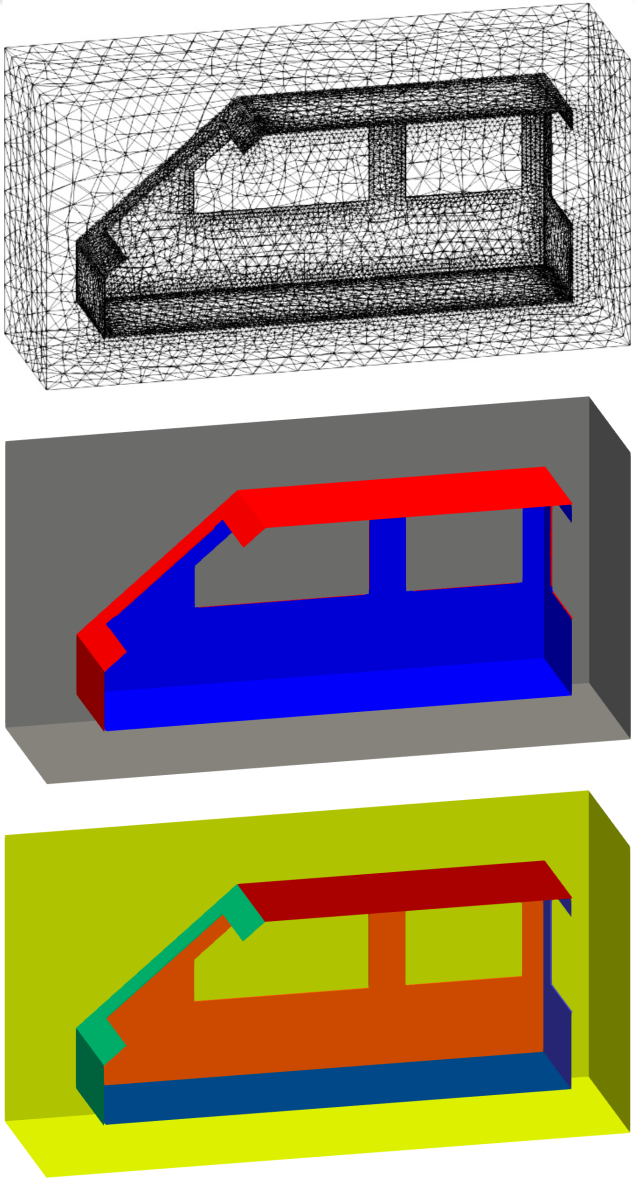
- Improvement of the ED resistance/growth models.
- Further validation of the ED models on the actual lines.

Take-home Message: **Why don’t you us ES-FEM-T4?**

Thank you for your kind attention.

Appendix

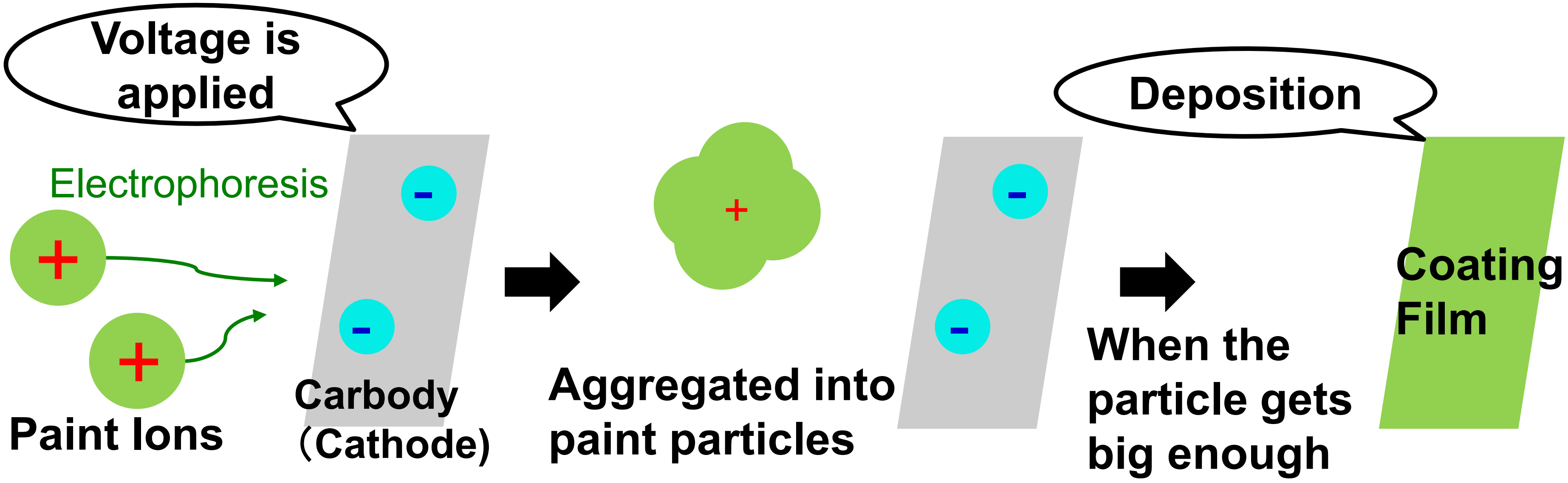
Pre-processes for ED Simulation



1. Mesh generation for body & pool.
2. Classification of body surfaces into inner & outer parts to assign different BCs (with/without stirring BC).
3. Mesh partitioning & reordering for MPI parallelization.
4. Preparation of input file including body motion definition.

⟹ Send to ED Solver

Mechanism of Electrodeposition



Voltage is applied

Electrophoresis

Paint Ions

Carbody (Cathode)

Aggregated into paint particles

Deposition

When the particle gets big enough

Coating Film

- Paint cations ("+" ions) are attracted to the cathode.
- Paint ions gradually lose their electrical charge and are aggregated into paint particles.
- Some of the paint particles are **deposited as coating film**. Meanwhile, the rests are diffused and re-dissolved.
- In contrast to a simple electroplating, accurate numerical simulation of ED film thickness is quite difficult.

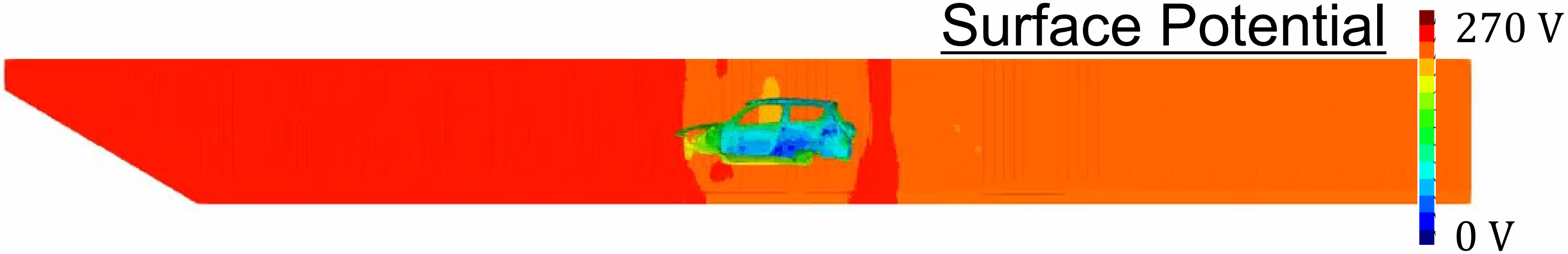
What is ED Simulation ?

ED simulation provides film thickness, surface potential, surface current density and so on.

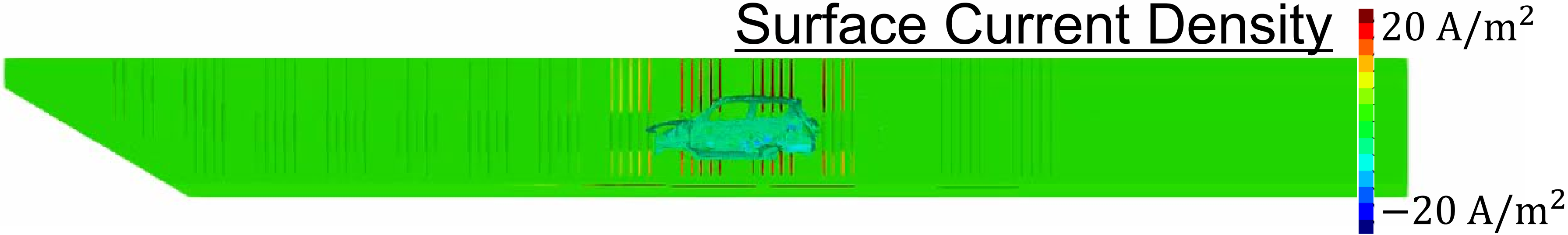
<u>Film Thickness</u>

You can see that the paint film starts to be deposited from the outside surface.

<u>Surface Potential</u>



<u>Surface Current Density</u>



ED Boundary Models

Film Resistance Model

- It represents the relation between $h, \Delta\phi_{\mathrm{cat}}$ and j_{cat}.
- Used to decide film resistance.
- Flow rate dependency is considered.

$$j_{\mathrm{cat}}(\Delta\phi_{\mathrm{cat}}, h) = \begin{cases} c_1(h)\Delta\phi_{\mathrm{cat}} & : \text{With stirring} \\ c_1(h)\left(e^{c_2(h)\Delta\phi_{\mathrm{cat}}} - e^{-c_2(h)\Delta\phi_{\mathrm{cat}}}\right) & : \text{Without stirring} \end{cases}$$

Film Growth Model

- It represents the relation between $\mathrm{h}, j_{\mathrm{cat}}$ and j_{dif}.
- Used to decide film growth rate.

After deposition : $j_{\mathrm{difA}}(j_{\mathrm{cat}}, h) = \dfrac{(j_{\mathrm{cat}}+d_1(h))^{d_2(h)}}{d_1^{d_2(h)-1} d_2(h)} - \dfrac{d_1(h)}{d_2(h)}$