SelectiveCS-FEM-T10: Selective cell-based smoothed finite element methods with 10-node tetrahedral elements

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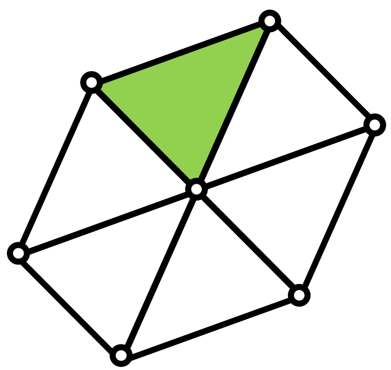
What is S-FEM?

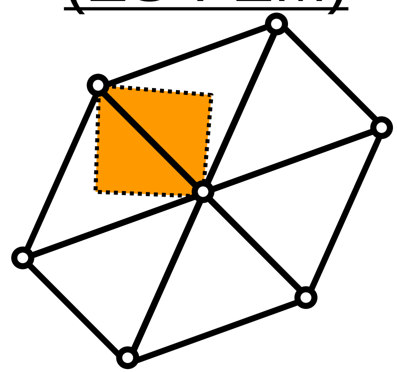
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example in 2D triangular mesh:

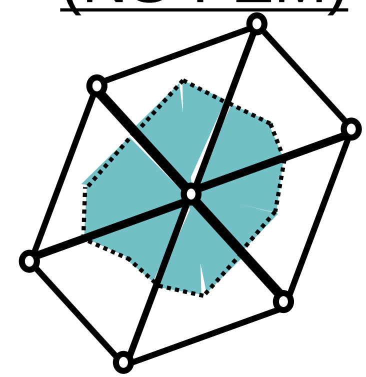
Standard FEM



Edge-based S-FEM (ES-FEM)

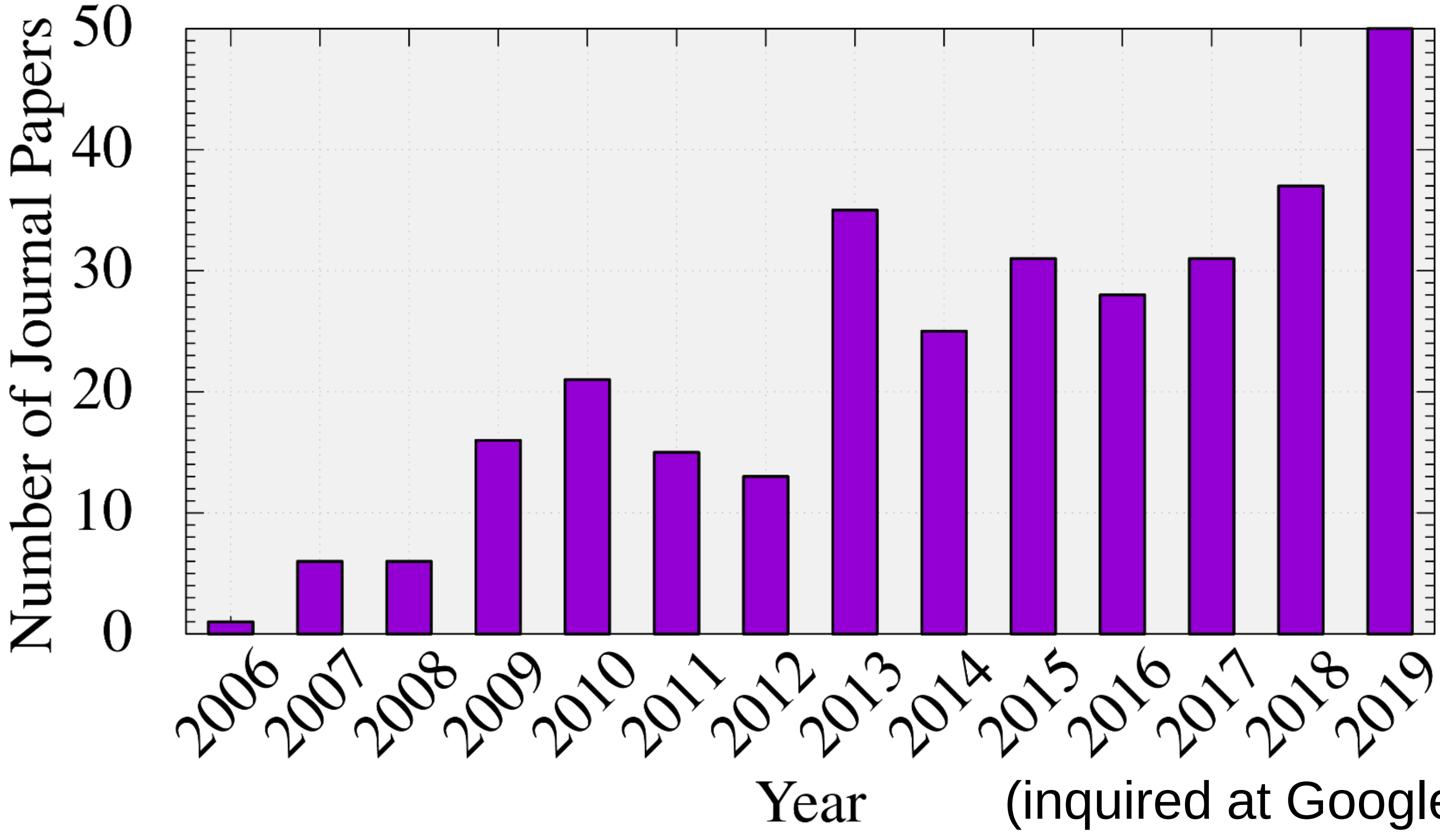


Node-based S-FEM (NS-FEM)



How popular is S-FEM?

Number of journal papers written **in English** whose **title** contains "**smoothed finite element**":



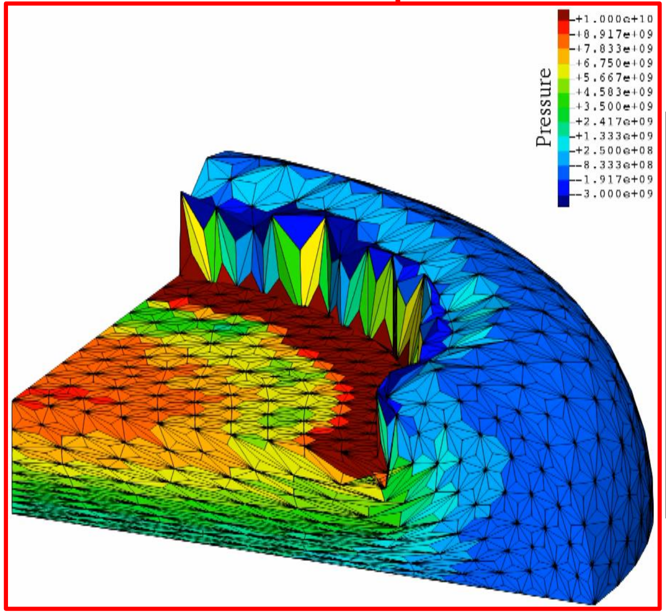
(inquired at Google Scholar)

The attraction of S-FEM is expanding continuously.

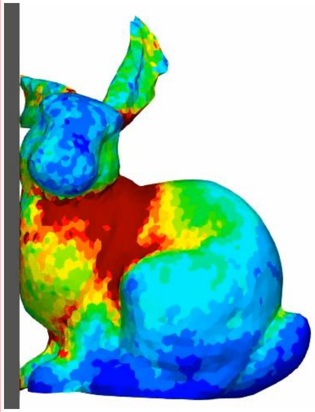
Applications of S-FEMs in Our Lab

- Solid mechanics

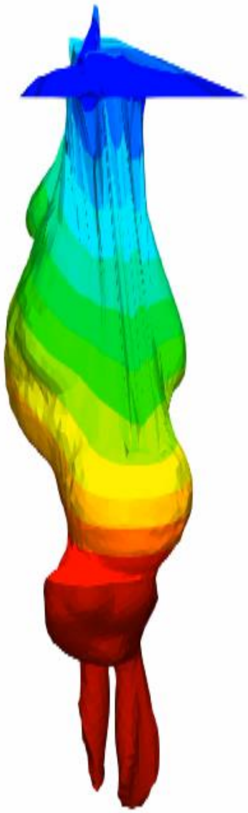
Static Implicit



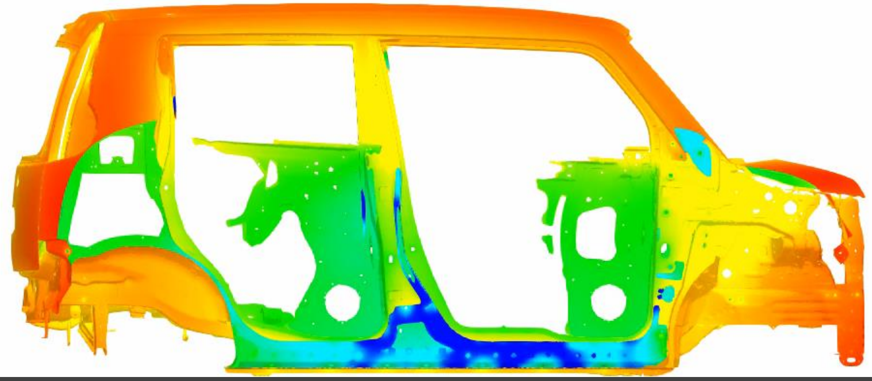
Dynamic Explicit



Viscous Implicit



- Electrostatic



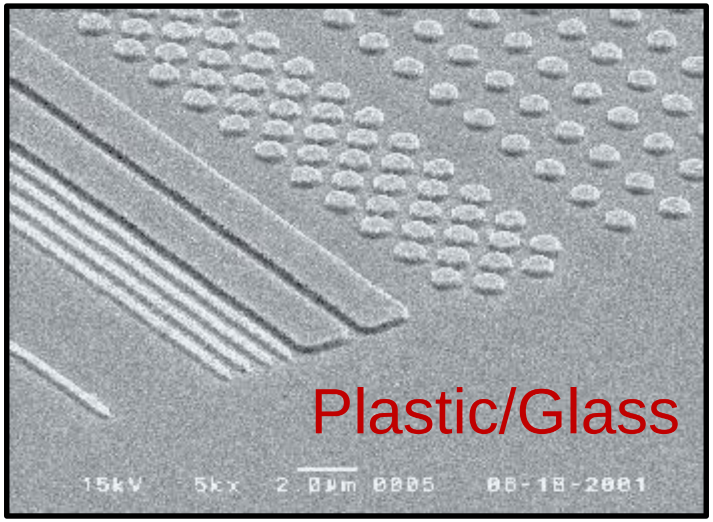
Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.


Rubber


Metal



Plastic/Glass

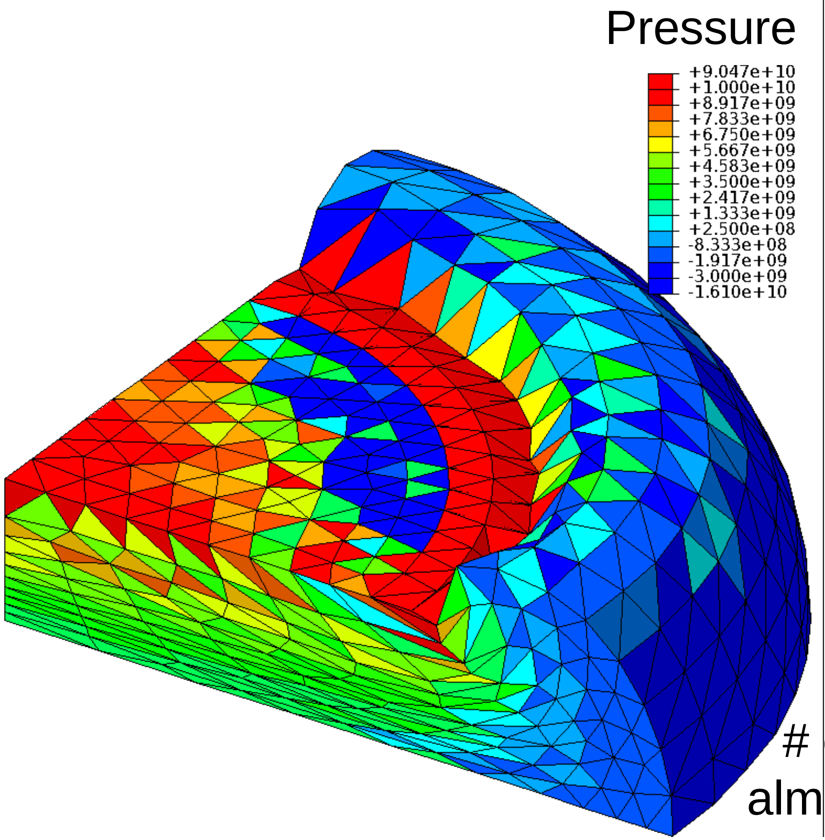
Issues

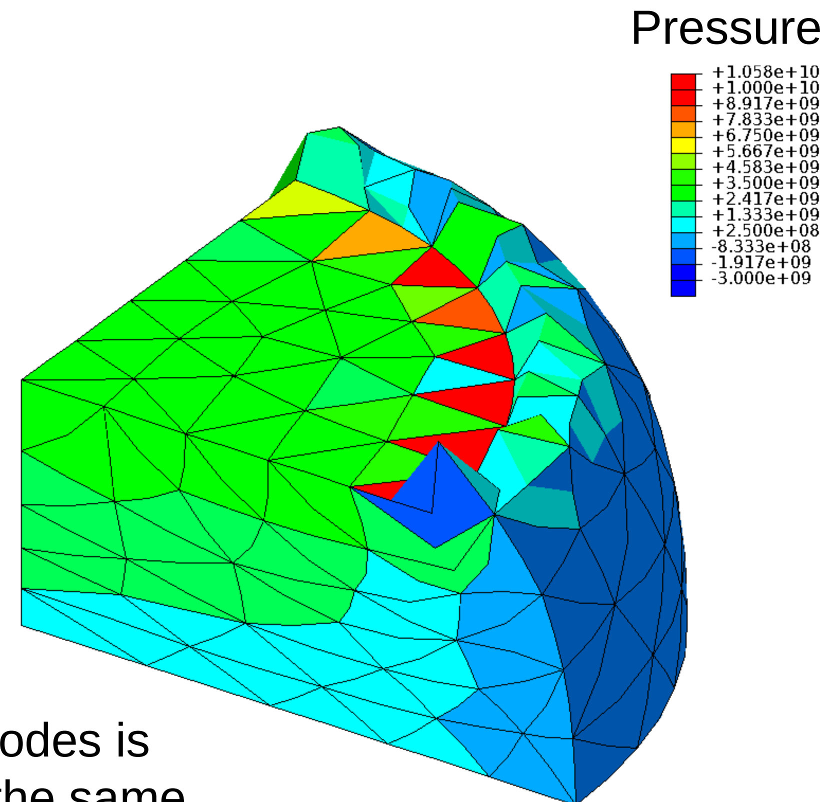
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
    Accuracy loss in large strain due to intermediate nodes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding, Early convergence failure etc..
- **F-bar type smoothed FEM (F-barES-FEM-T4):**
  - ✓ Accurate & stable ✗ Hard to implement in FEM codes.

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$





\# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

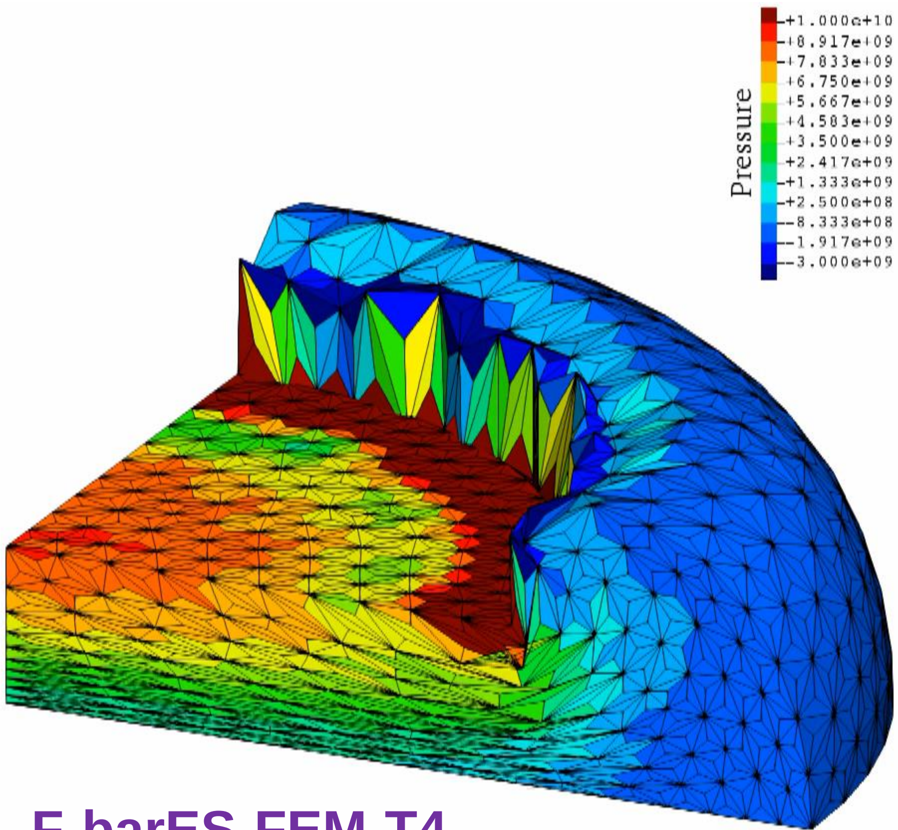
2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

Same mesh as C3D4H case.



F-barES-FEM-T4

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding
- ✓ No increase in DOF

Although F-barES-FEM-T4 is accurate and stable, ✗ it cannot be implemented in general-purpose FE software due to the adoption of ES-FEM. Also, it cosumes larger memory & CPU costs.

⇩

Another approach adopting CS-FEM with T10 element would be effective.

Objective

To develop an S-FEM formulation using T10 mesh (SelectiveCS-FEM-T10) for severe large deformation analyses.

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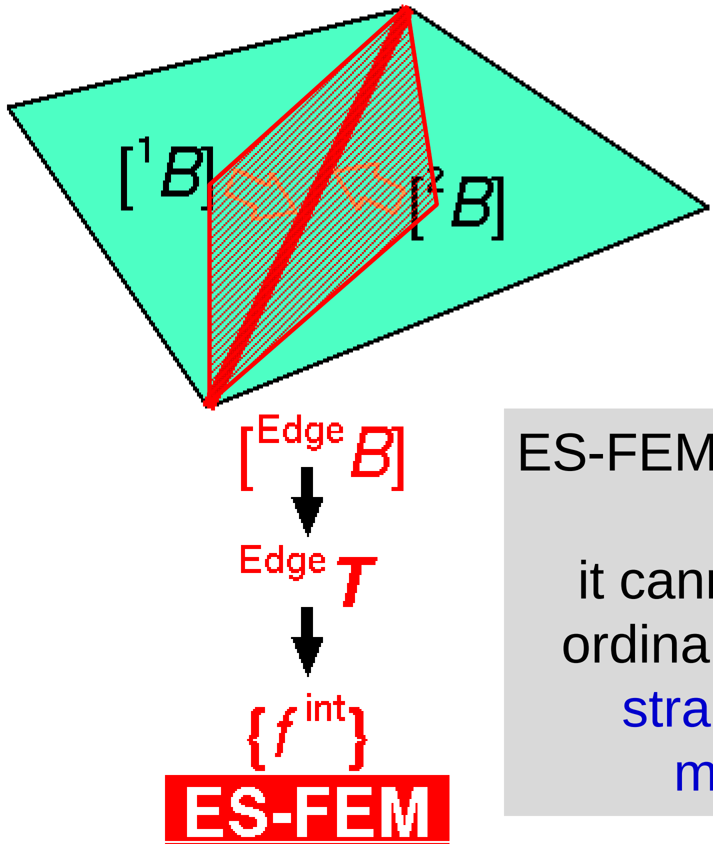
- Quick introduction of F-barES-FEM-T4
  – Why not T4 but T10? –
- Formulation of SelectiveCS-FEM-T10
- Demonstrations of SelectiveCS-FEM-T10
- Summary

Quick Introduction of F-barES-FEM-T4 – Why not T4 but T10? –

Brief of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each edge smoothing domain.

As if putting an integration point on each edge center

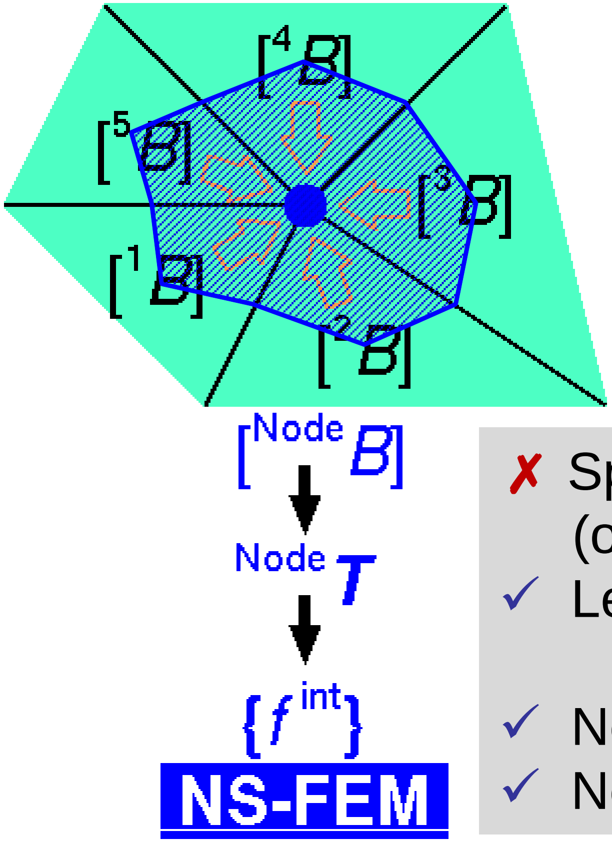


ES-FEM can avoid shear locking.
However,
it cannot be implemented in ordinary FE codes due to the strain smoothing across multiple elements...

Brief of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[{}^{\mathrm{Node}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each node smoothing domain.

As if putting an integration point on each node

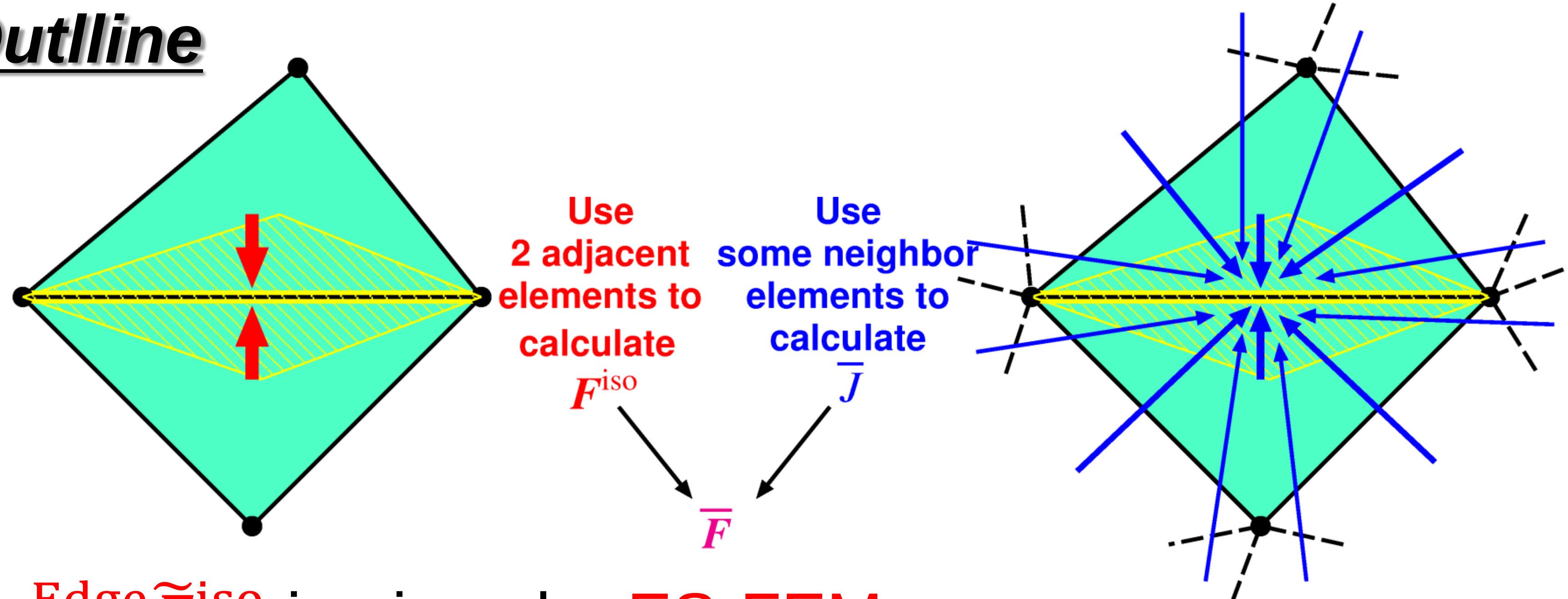


- ✗ Spurious low-energy mode (or hour-glass mode)
- ✓ Less pressure checkerboarding
- ✓ No shear locking
- ✓ No volumetric locking

Concept of F-barES-FEM

Concept: combining ES-FEM and NS-FEM using F-bar method

Outlline



- ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- ${}^{\mathrm{Edge}}\bar{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

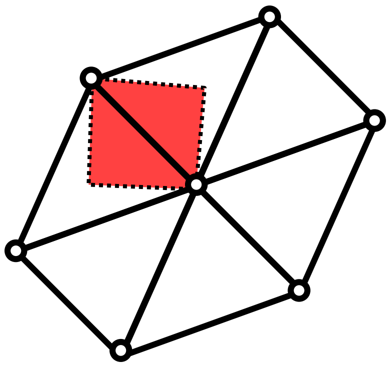
$${}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{\,1/3}\ {}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}.$$

Formulation of F-barES-FEM (1 of 2)

Deformation gradient of each edge ($\overline{\boldsymbol{F}}$) is derived as

$$\overline{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\text{iso}} \cdot \overline{\boldsymbol{F}}^{\text{vol}}$$

in the manner of F-bar method.

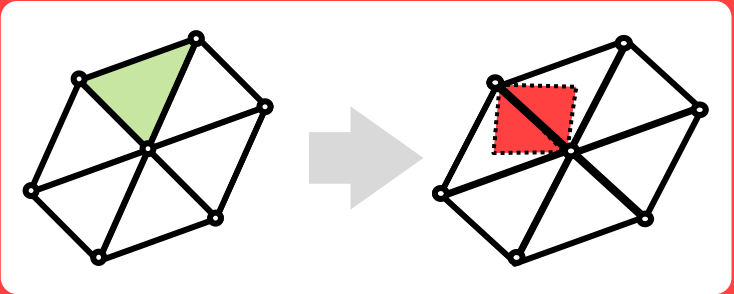

ES–FEM

Formulation of F-barES-FEM (2 of 2)

Each part of $\overline{\boldsymbol{F}}$ is calculated as

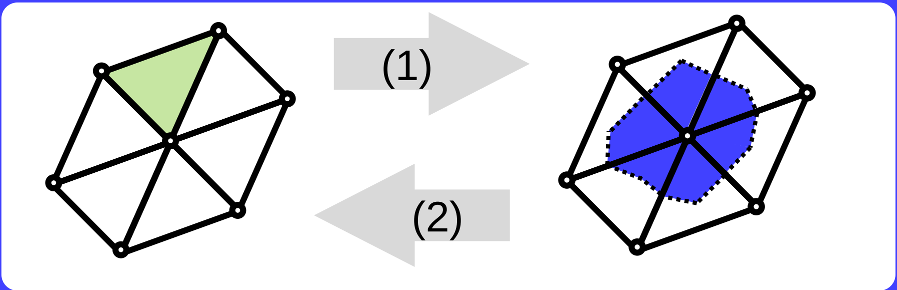
$$\overline{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \overline{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part



Smoothing the value of adjacent elements. (same manner as ES-FEM)

Volumetric part



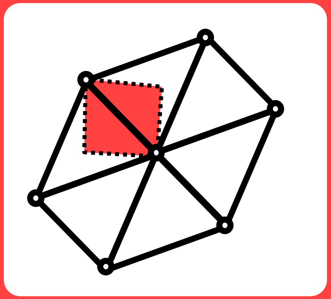
(1)Calculating node's value by smoothing the value of adjacent elements
(2)Calculating elements' value by smoothing the value of adjacent nodes
(3)Repeating (1) and (2) a few times

Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

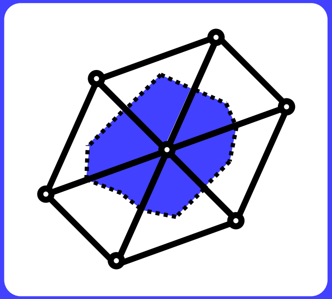
Isovolumetric part



Like a ES-FEM

1. Shear locking free

Volumetric part



Like a NS-FEM

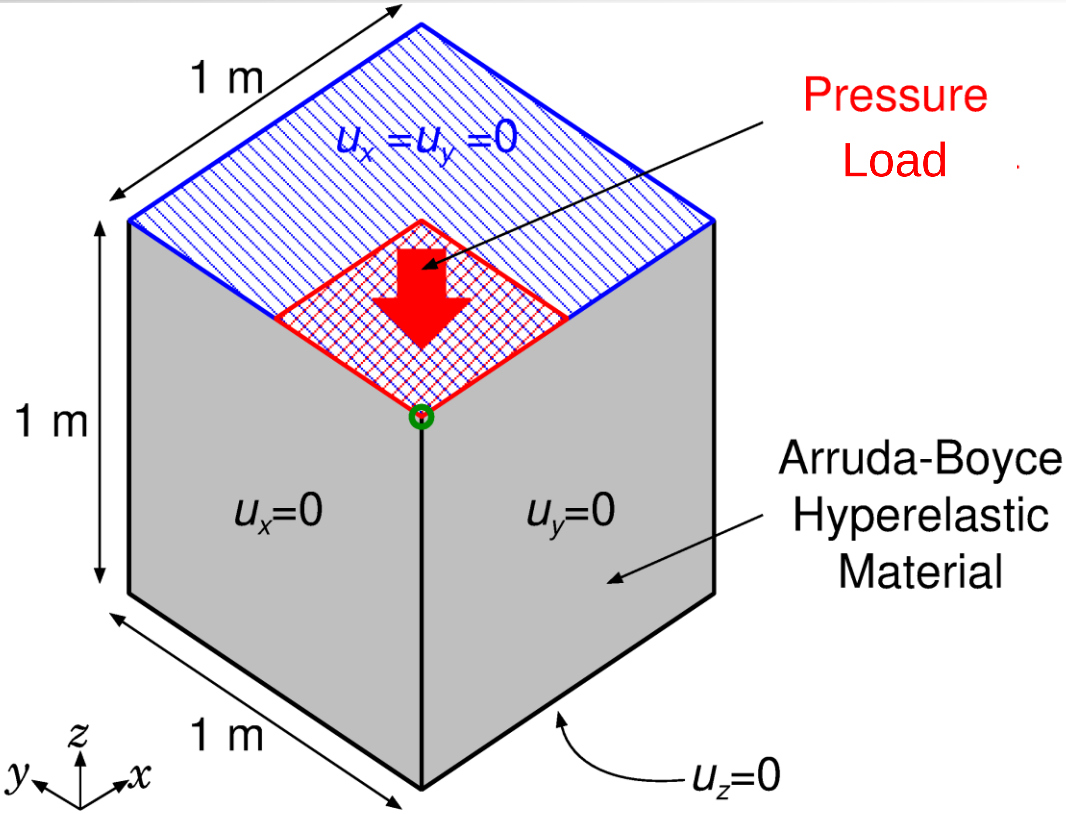
2. Little pressure oscillation

3. Volumetric locking free
with the aid of F-bar method

Static Implicit

Compression of Rubber Block

Outline



- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Result of F-barES-FEM-T4 is compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static Implicit

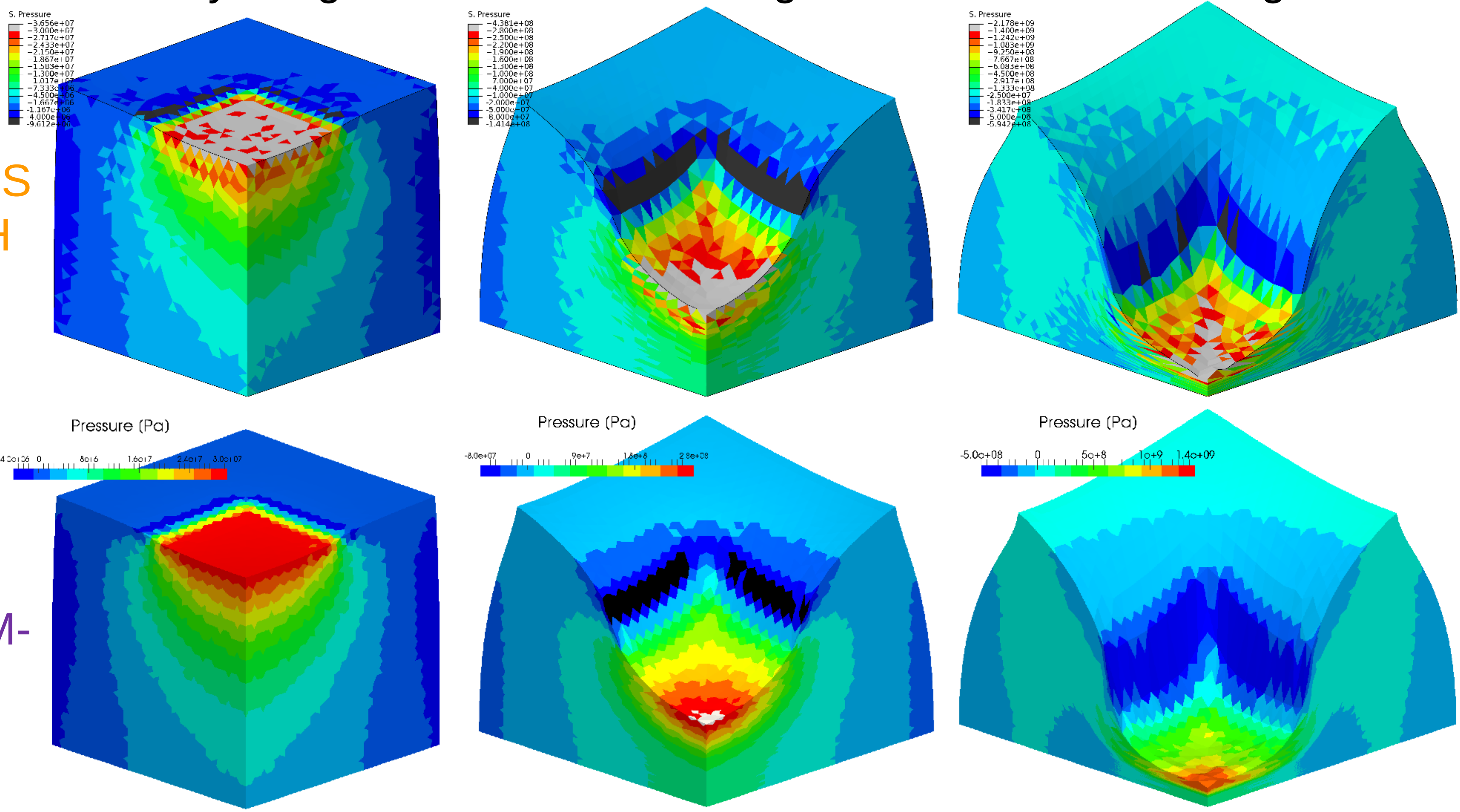
Compression of Rubber Block

Pressure dist.

Early stage | Middle stage | Later stage

ABAQUS C3D4H

F-bar ES-FEM-T4(3)

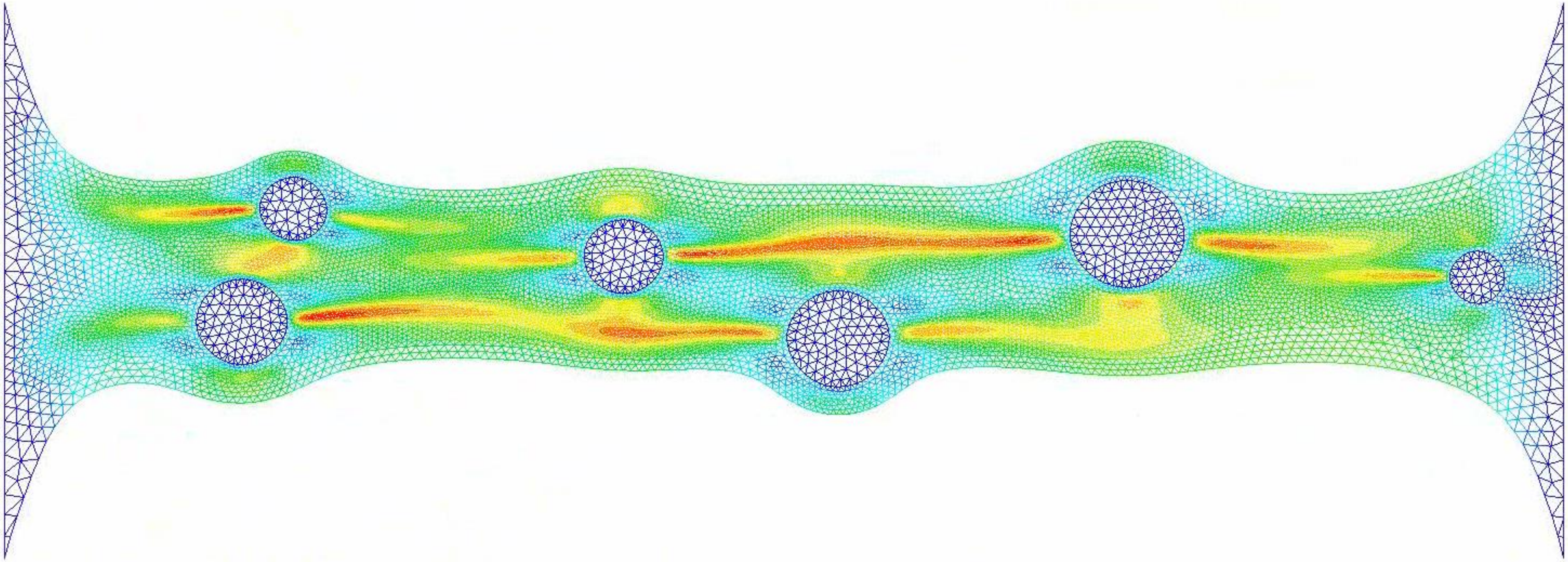


Smooth pressure distributions are obtained.

Static Implicit

Stretch of Filler-containing Rubber with 2D Remesing

- Several hard circular fillers are distributed in a square soft matrix rubber (neo-Hookean hyperelastic with $\nu_{\mathrm{ini}} = 0.49$).
- E_{ini} of the filler is 100 times larger than E_{ini} of the matrix.
- Left side is constrained and right side is displaced.

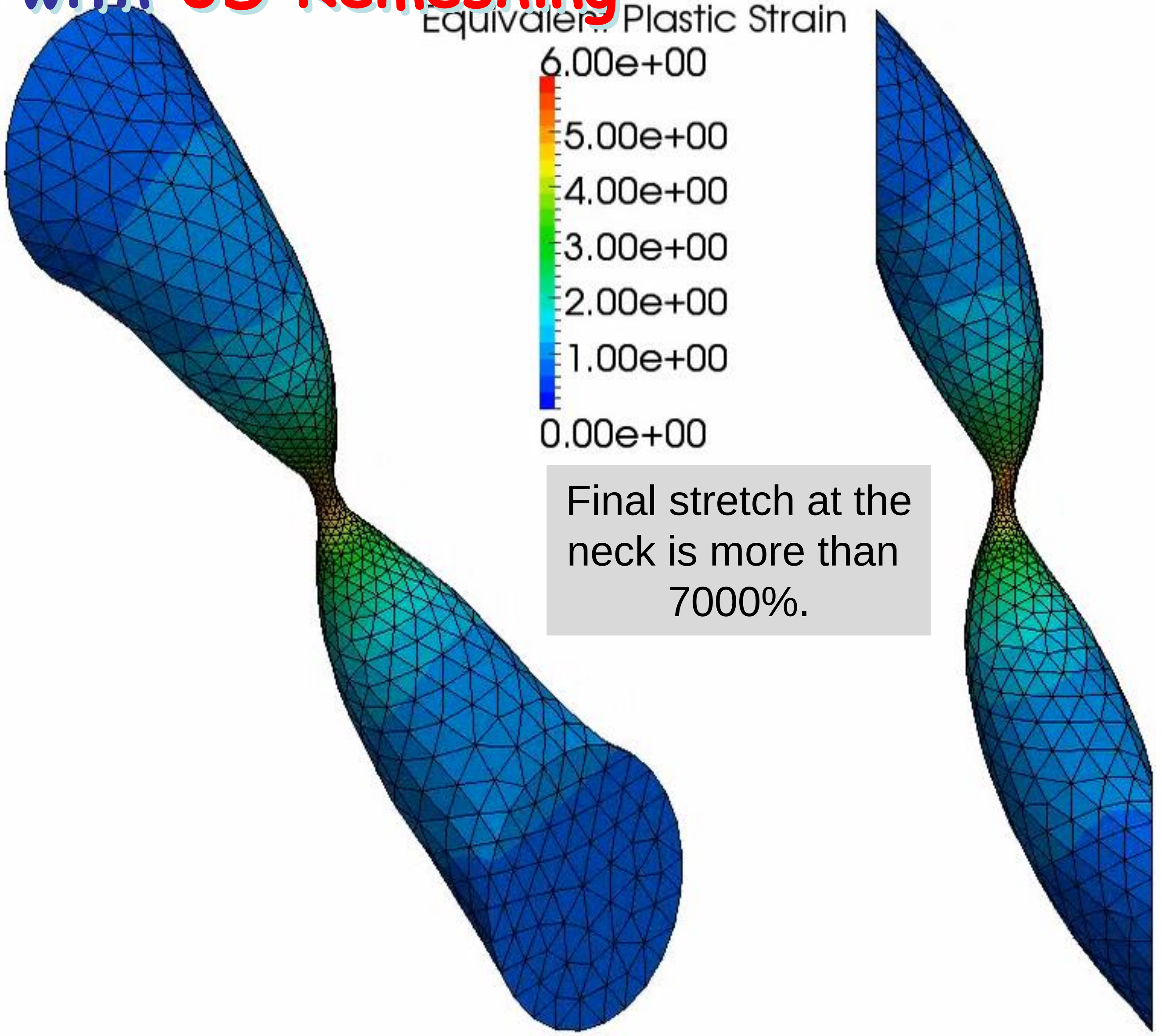


Valid Mises stress dist. is obtained after many time remeshings.

Shear-tensioning of Elasto-plastic cylinder with 3D Remeshing

- Aluminium cylinder subjected to enforced disp..
- Pure shear at the initial stage, but stretch dominates at the later stage.
- Necking occurs in the end.

Valid plastic strain dist. is obtained after many time remeshings.



Final stretch at the neck is more than 7000%.

Characteristics of F-barES-FEM-T4

✓ No increase in DOF.
(No Lagrange multiplier. No static condensation.)

✓ Locking- & checkerboarding-free with T4 mesh.

✗ Higher costs in memory and CPU time due to wider bandwidth of $[K]$.

In case of standard unstructured T4 meshes:

Method	Approx. Bandwidth
Standard FEM-T4	40
F-barES-FEM-T4(1)	390

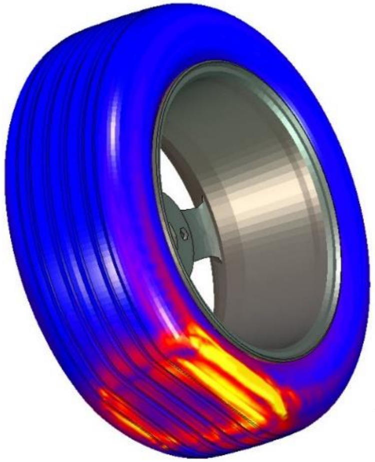
✗ Difficulty in implementation to existing FE codes due to the smoothing across elements. Critical Issue!!

Why Not T4 But T10?

If we cannot implement F-barES-FEM-T4 to existing FE codes, then we have to code ***everything*** in our in-house code for practical use.

For example in tire analyses:

- Material constitutive models,
- Structural elements,
- Cohesive elements,
- Contact functionality and so on.



MSC Software web page

Therefore, choosing S-FEM-T4 leads us to the long and winding road…

⇒ We gave up T4 and chose T10 for solid mechanics analyses.

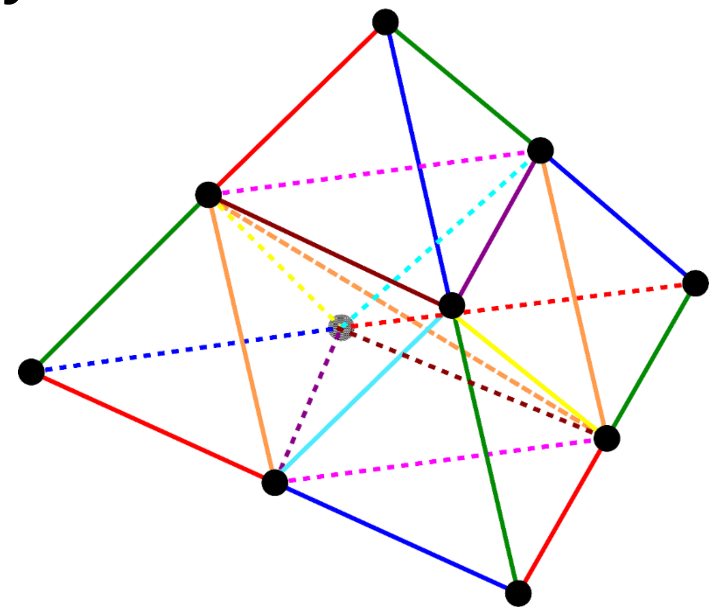
Formulation of SelectiveCS-FEM-T10

Concept of SelectiveCS-FEM-T10

- Our new approach using T10 mesh.
- Adopting CS-FEM having no smoothing across multiple elements, SelectiveCS-FEM-T10 becomes an independent finite element.

  ⇒ We can implement it as an element of existing FE code.

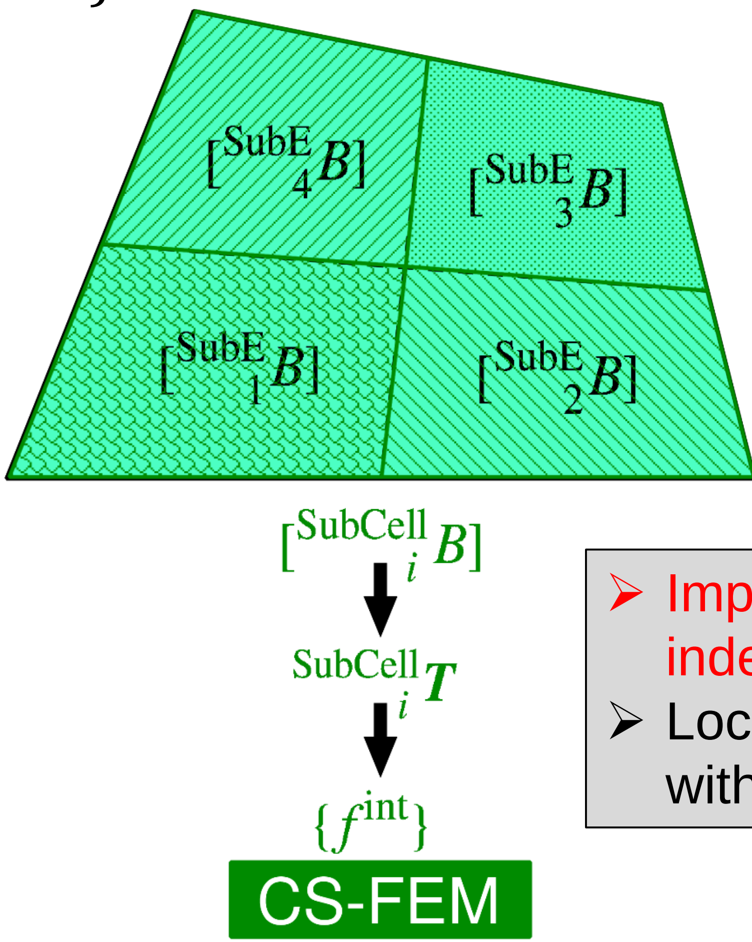
- Same memory & CPU costs as the T10 elements.



Brief of Cell-based S-FEM (CS-FEM)

- Subdivide each element into some sub-element.
- Calculate $[{}^{\mathrm{SubE}}B]$ at each sub-element.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each sub-element.

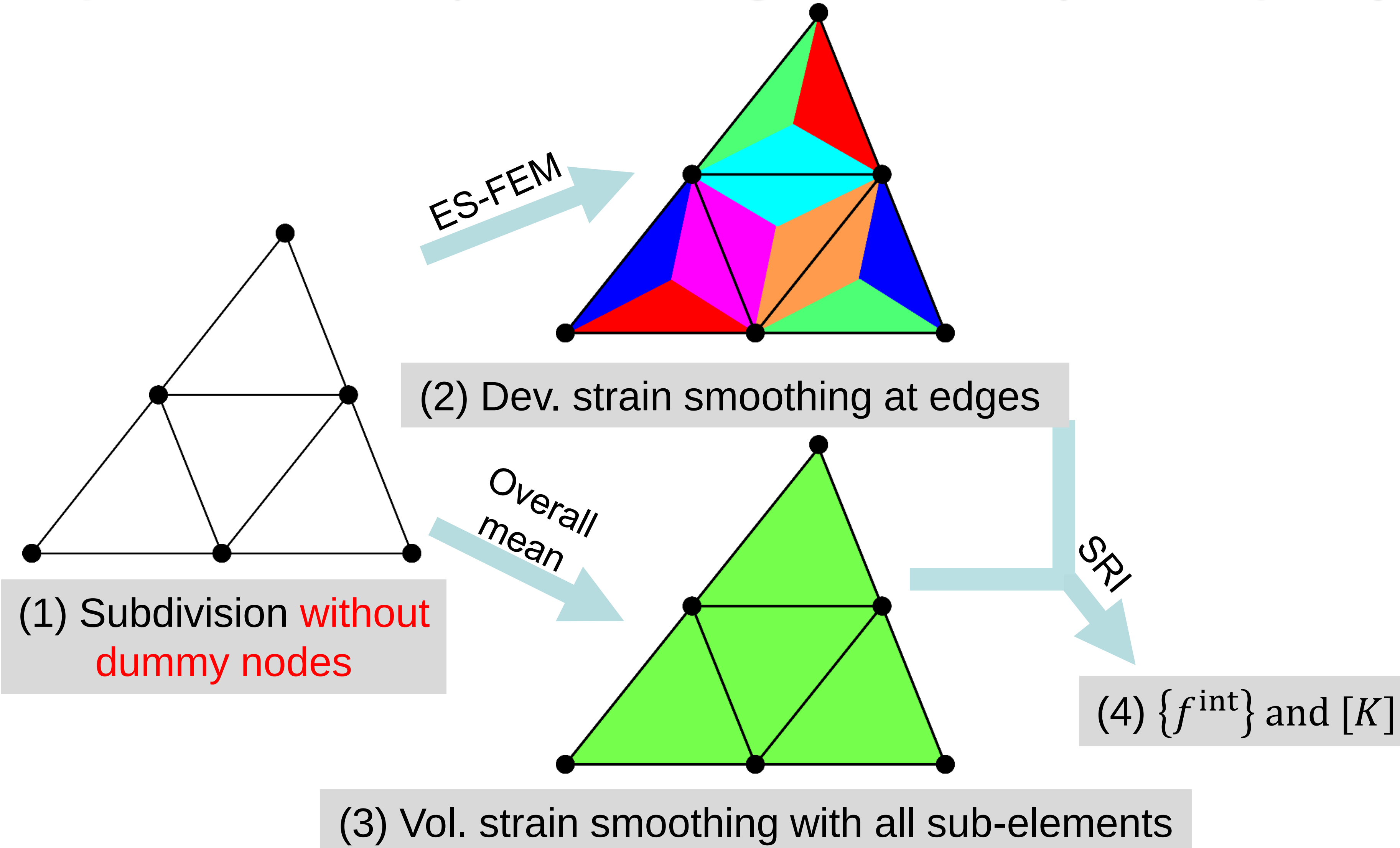
As if putting an integration point on each sub-element



- Implementable as an independent finite element.
- Locking can be avoided with SRI etc..

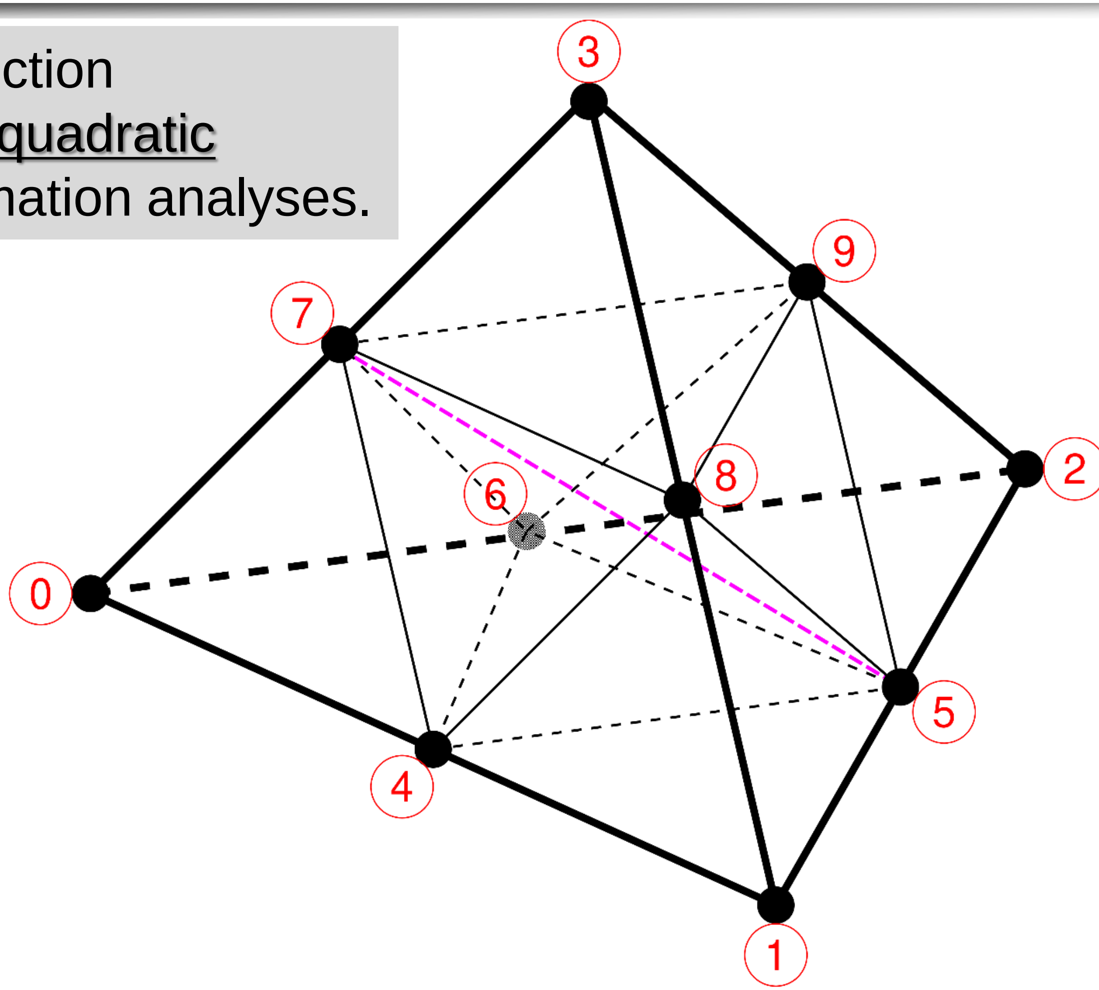
Flowchart of SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity



(1) Subdivision into T4 Sub-elements

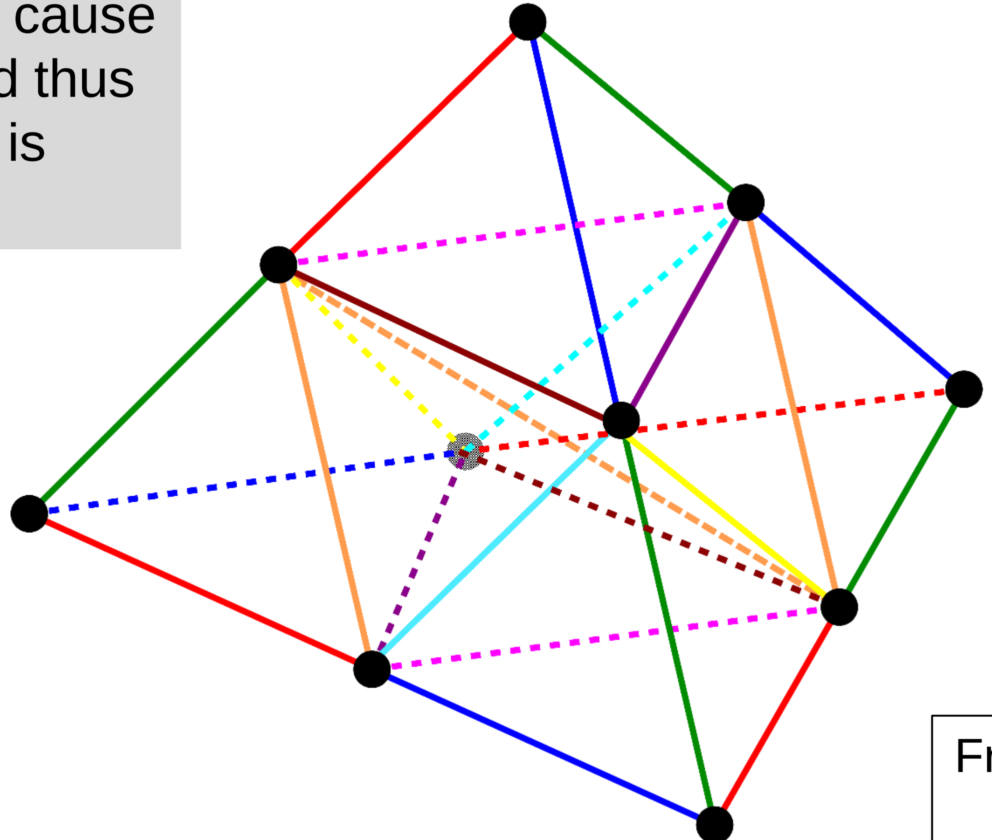
The shape function
<u>should not be quadratic</u>
in large deformation analyses.



- Introduce no dummy node (i.e., asymmetric element).
- Subdivide a T10 element into <u>eight T4 sub-elements</u> and calculate their *B*-matrices and strains.

(2) Deviatoric Strain Smoothing

T4 sub-elements cause shear locking and thus <u>strain smoothing</u> is necessary.

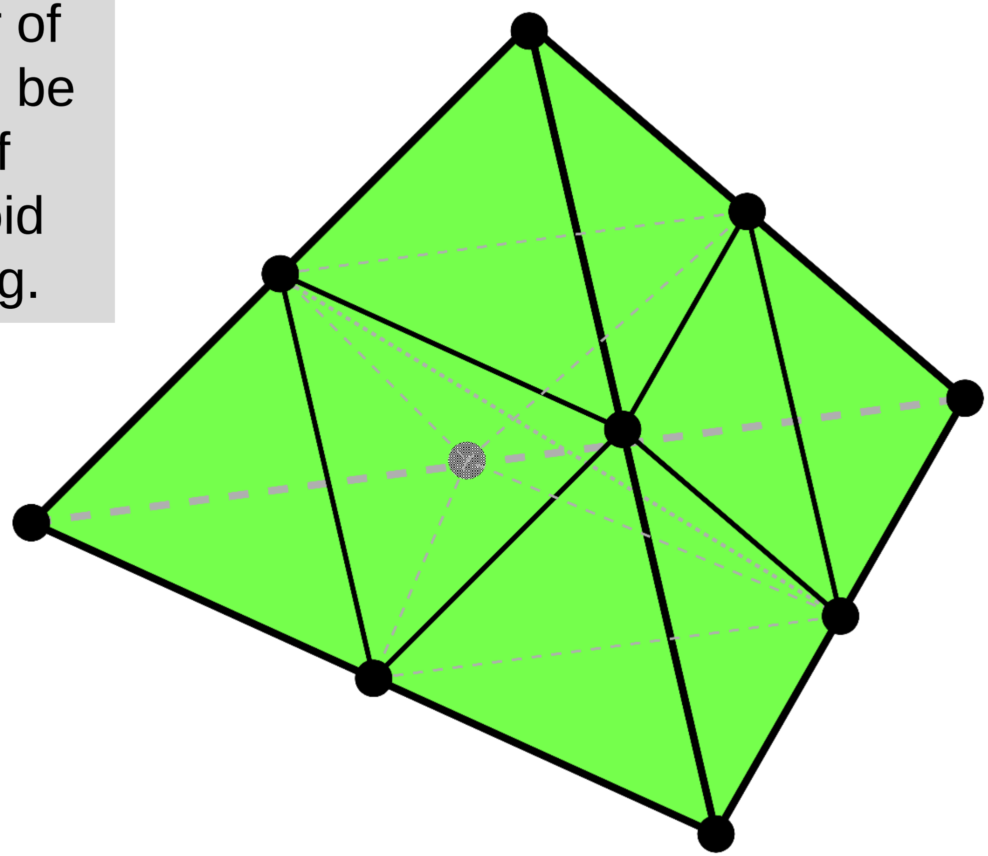


From 8 sub-elements to 25 edges

- Perform strain smoothing in the manner of ES-FEM (i.e., average dev. strains of sub-elements at edges).
- Evaluate deviatoric strain and stress at edges.

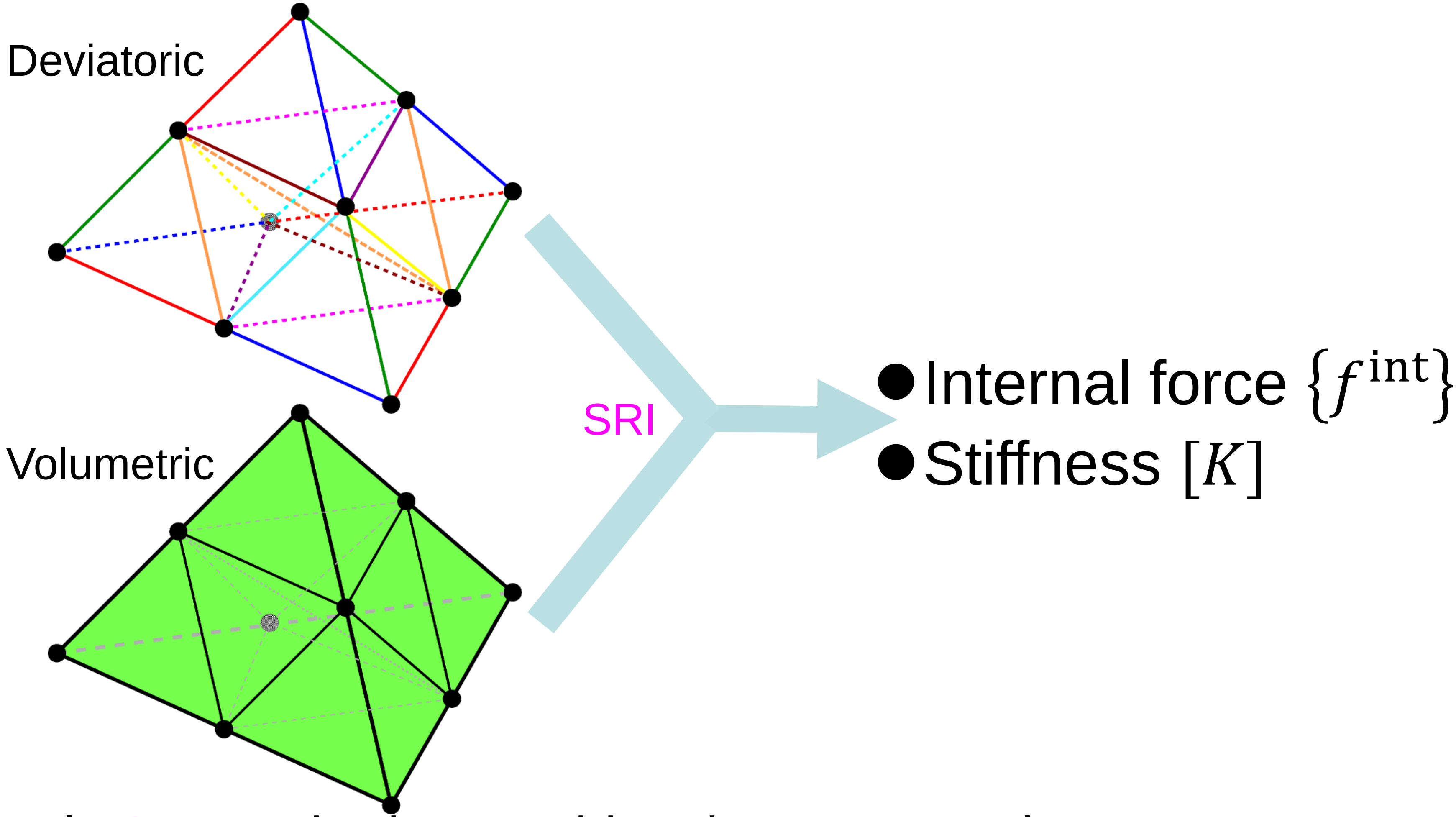
(3) Volumetric Strain Smoothing

The spatial order of vol. strain should be lower than that of dev. strain to avoid volumetric locking.



- Treat the overall mean vol. strain of all sub-elements as the uniform element vol. strain (i.e., same approach as SRI elements).

(4) Combining with SRI Method

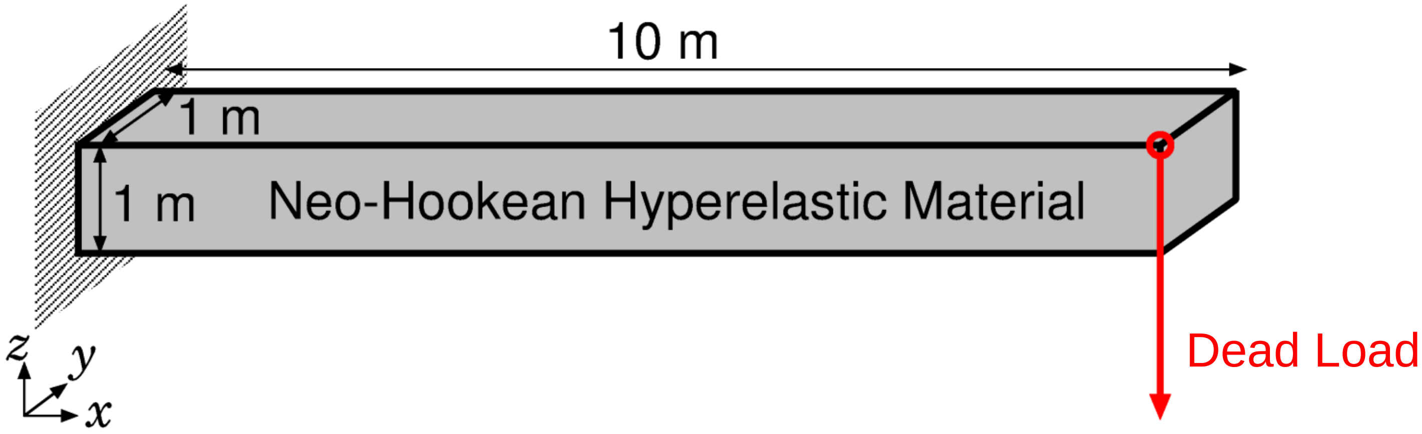


■ Apply SRI method to combine the Dev. & Vol. parts and obtain $\{f^{\mathrm{int}}\}$ and $[K]$.

Demonstration of
SelectiveCS-FEM-T10

Bending of Hyperelastic Cantilever

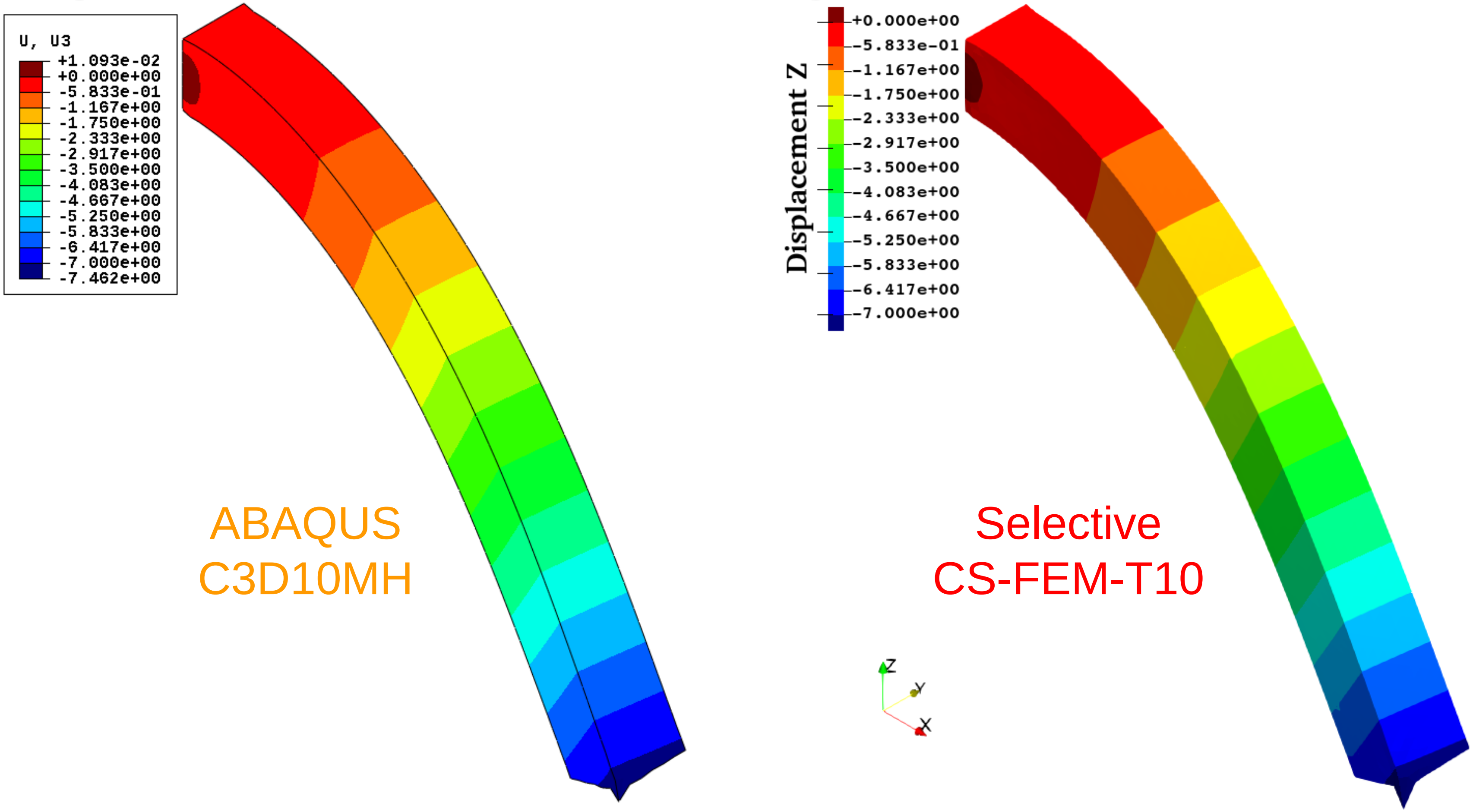
Outline



- Neo-Hookean hyperelastic material
- Initial Poisson's ratio: $\nu_0 = 0.49$
- Compared to ABAQUS C3D10MH (modified hybrid T10 element) with the same mesh.

Bending of Hyperelastic Cantilever

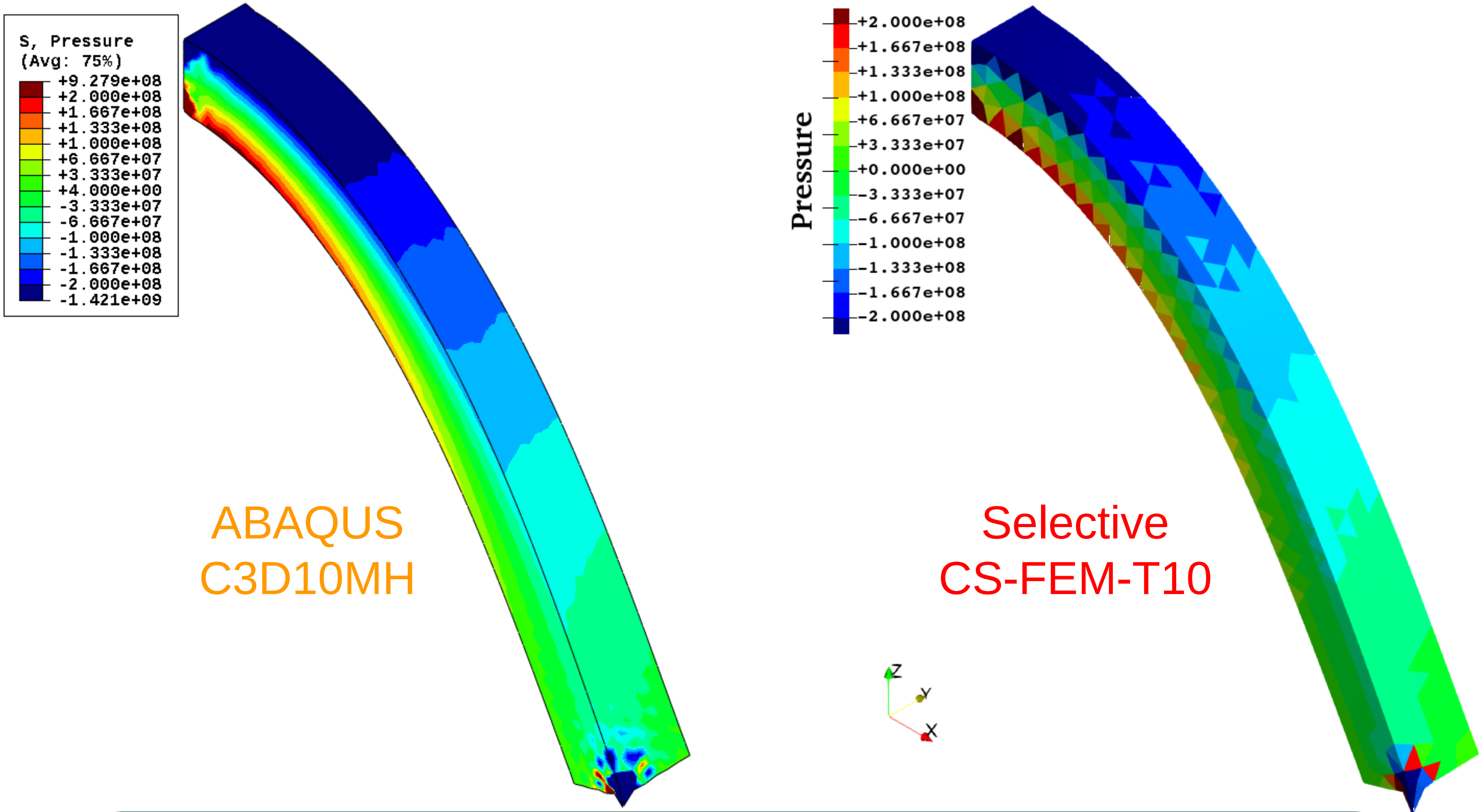
Comparison of the deflection disp. at the final state



No volumetric locking is observed.

Bending of Hyperelastic Cantilever

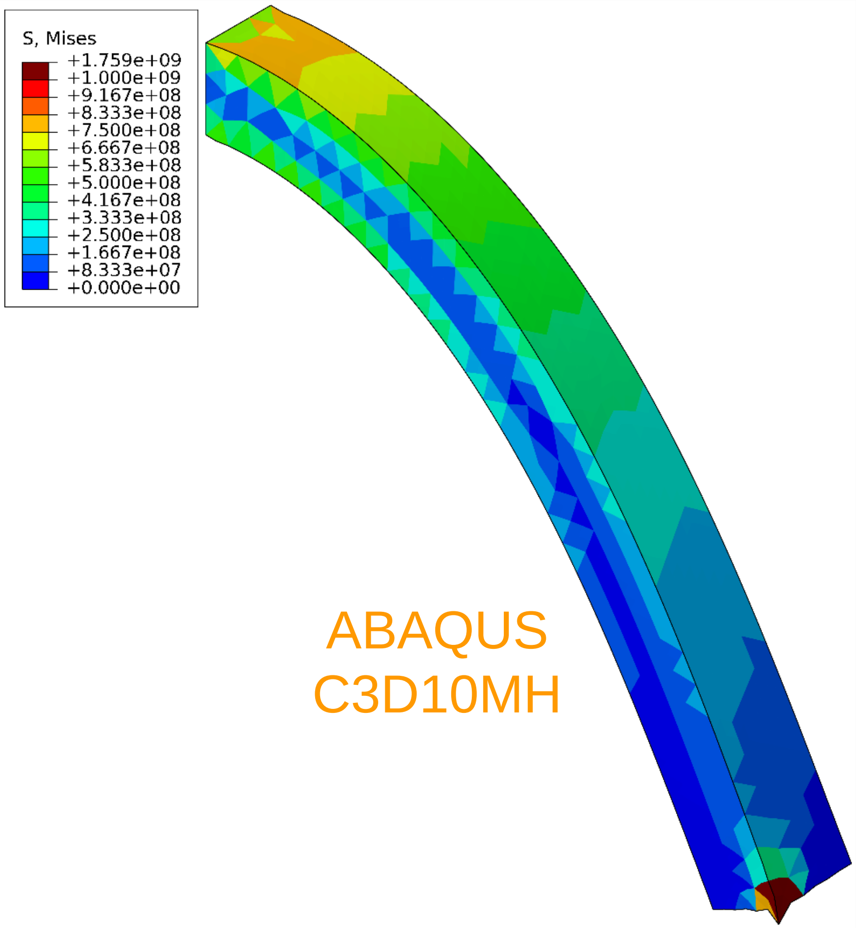
Comparison of the pressure dist. at the final state

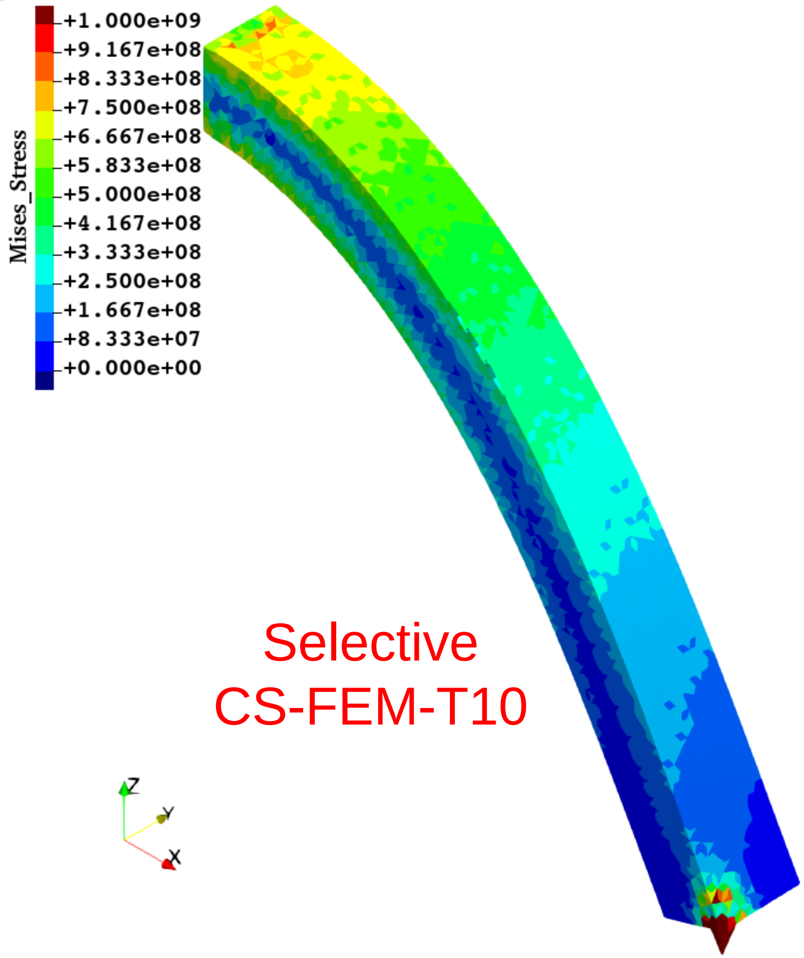


Almost the same pressure distributions with no checkerboarding.

Bending of Hyperelastic Cantilever

Comparison of the Mises stress dist. at the final state

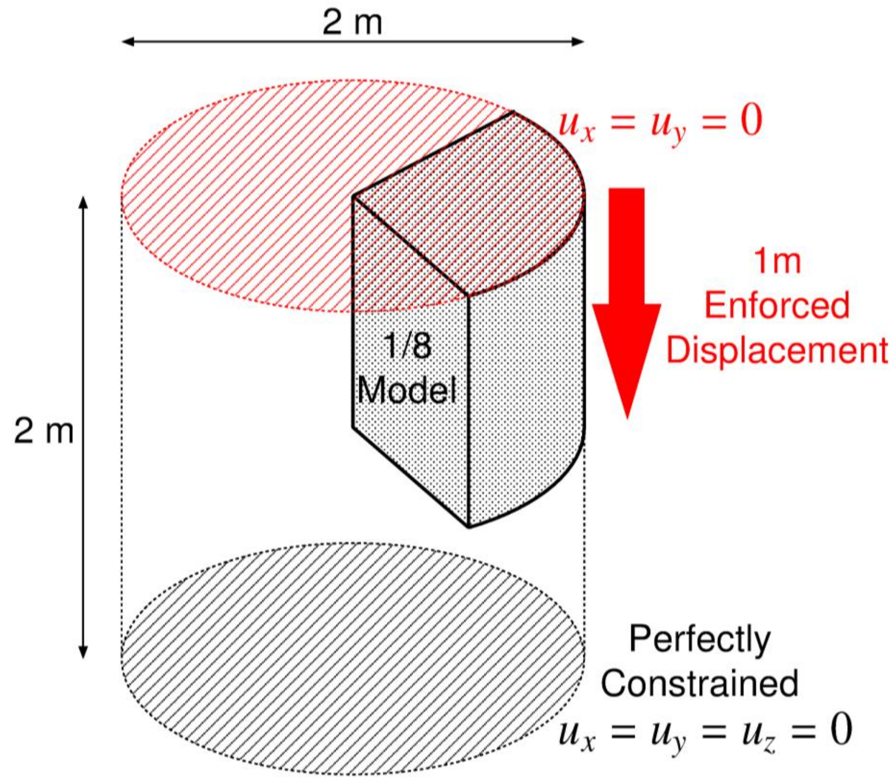




Almost the same Mises stress distributions.

Barreling of Hyperelastic Cylinder

Outline

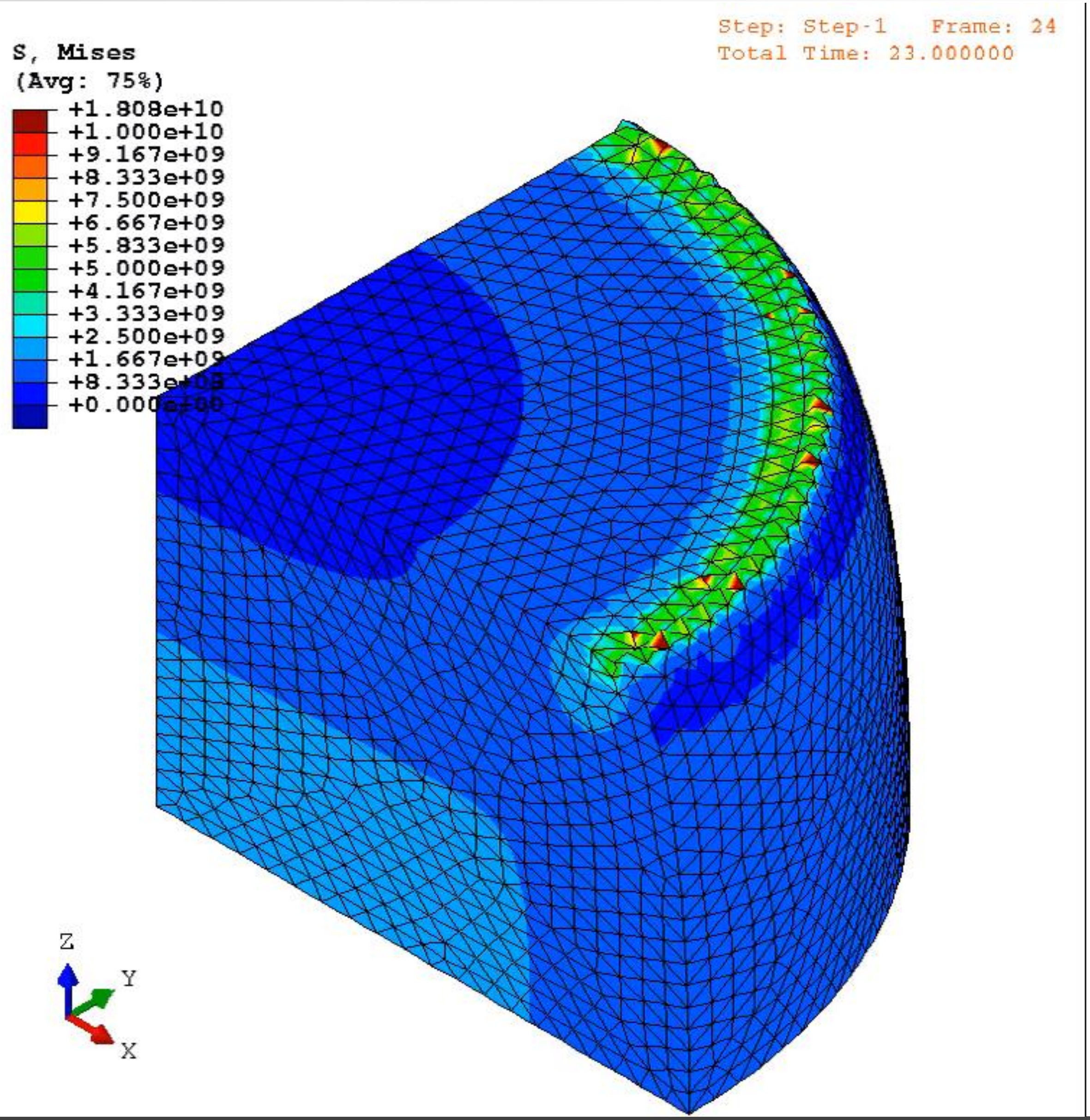


- Enforce axial displacement on the top face.
- Neo-Hookean body with $\nu_{\mathrm{ini}} = 0.49$.
- Compare results with ABAQUS T10 hybrid elements (C3D10H, C3D10MH, C3D10HS) using the same mesh.

Barreling of Hyperelastic Cylinder

Animation of Mises stress (ABAQUS C3D10MH)

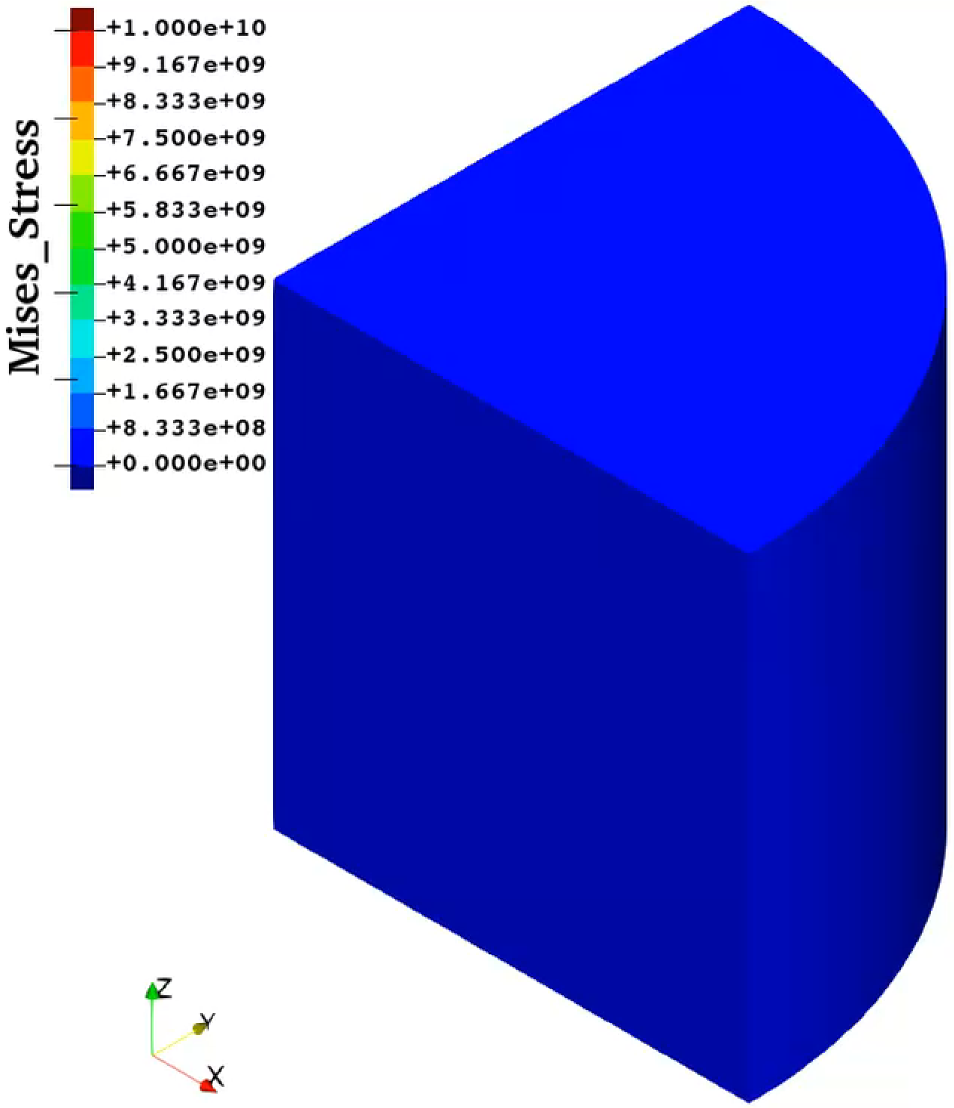
Convergence failure at **24%** compression



Unnaturally oscillating distributions are obtained around the rim.

Barreling of Hyperelastic Cylinder

Animation of Mises stress (Selective CS-FEM-T10)



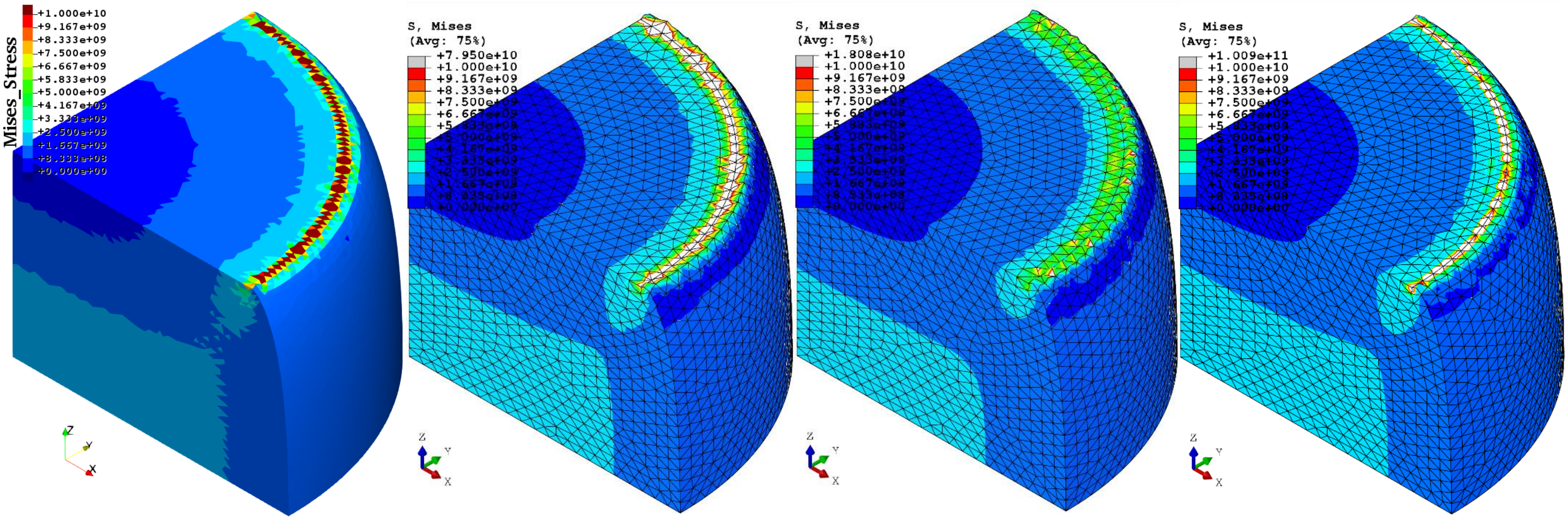
Smooth distributions are obtained except around the rim.

Convergence failure at **43%** compression

The present element is more **robust** than ABAQUS C3D10MH

Barreling of Hyperelastic Cylinder

Comparison of Mises stress at 24% comp.

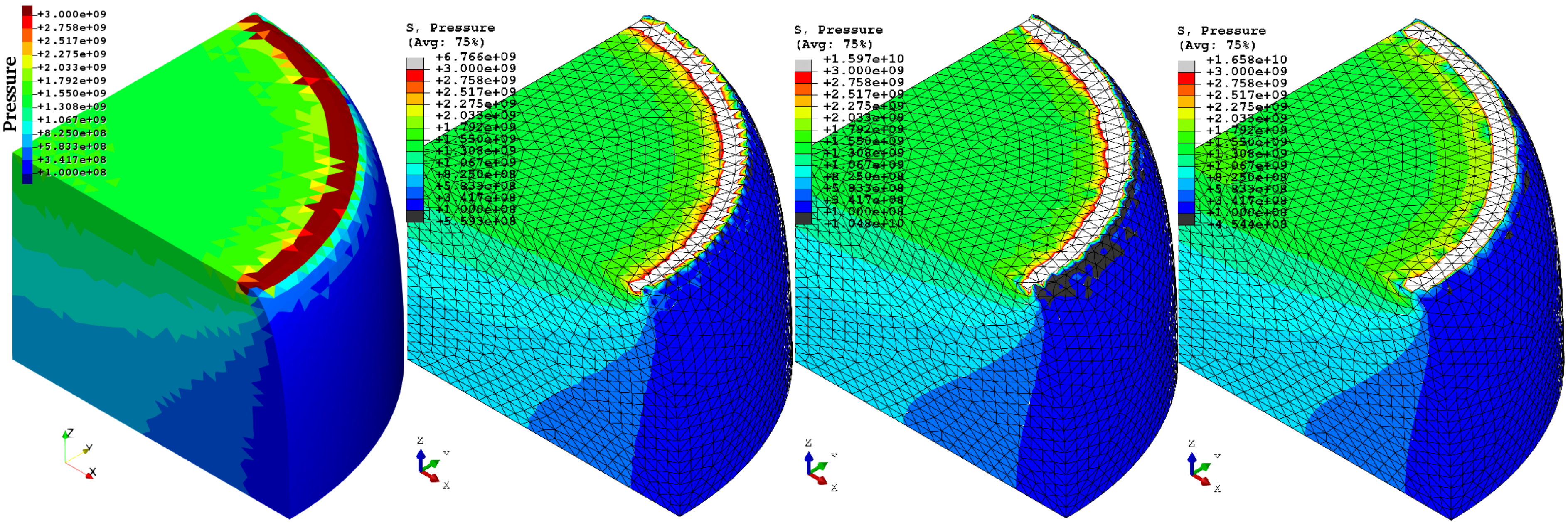


Selective CS-FEM-T10 | ABAQUS C3D10H | ABAQUS C3D10MH | ABAQUS C3D10HS

All results are similar to each other
except around the rim having stress singularity.

Barreling of Hyperelastic Cylinder

Comparison of pressure at 24% comp.



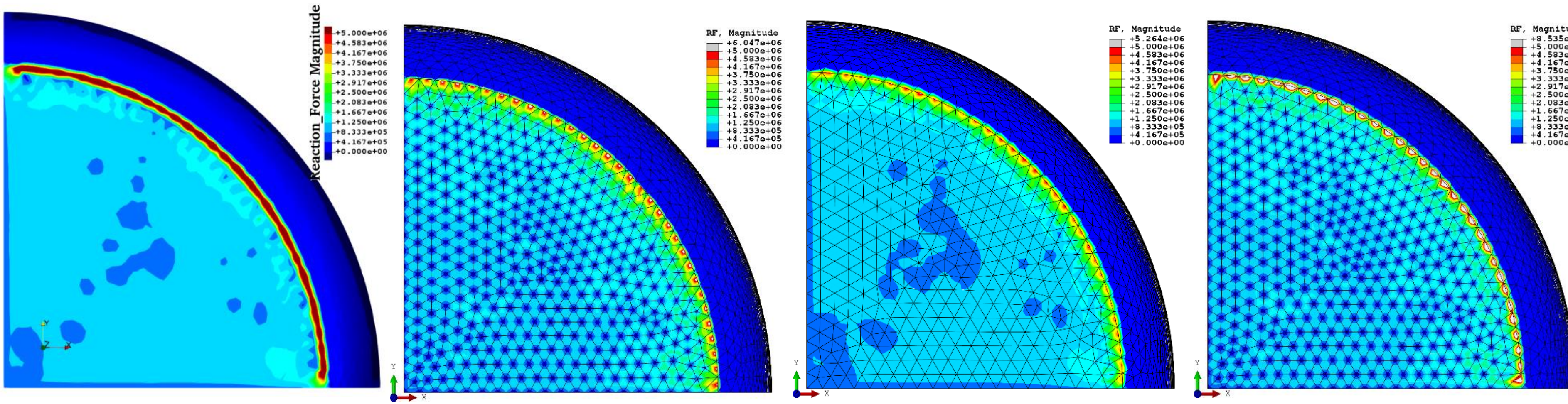
Selective CS-FEM-T10 | ABAQUS C3D10H | ABAQUS C3D10MH | ABAQUS C3D10HS

All results are similar to each other except around the rim having stress singularity.

Static Implicit

Barreling of Hyperelastic Cylinder

Comparison of nodal reaction force at 24% comp.

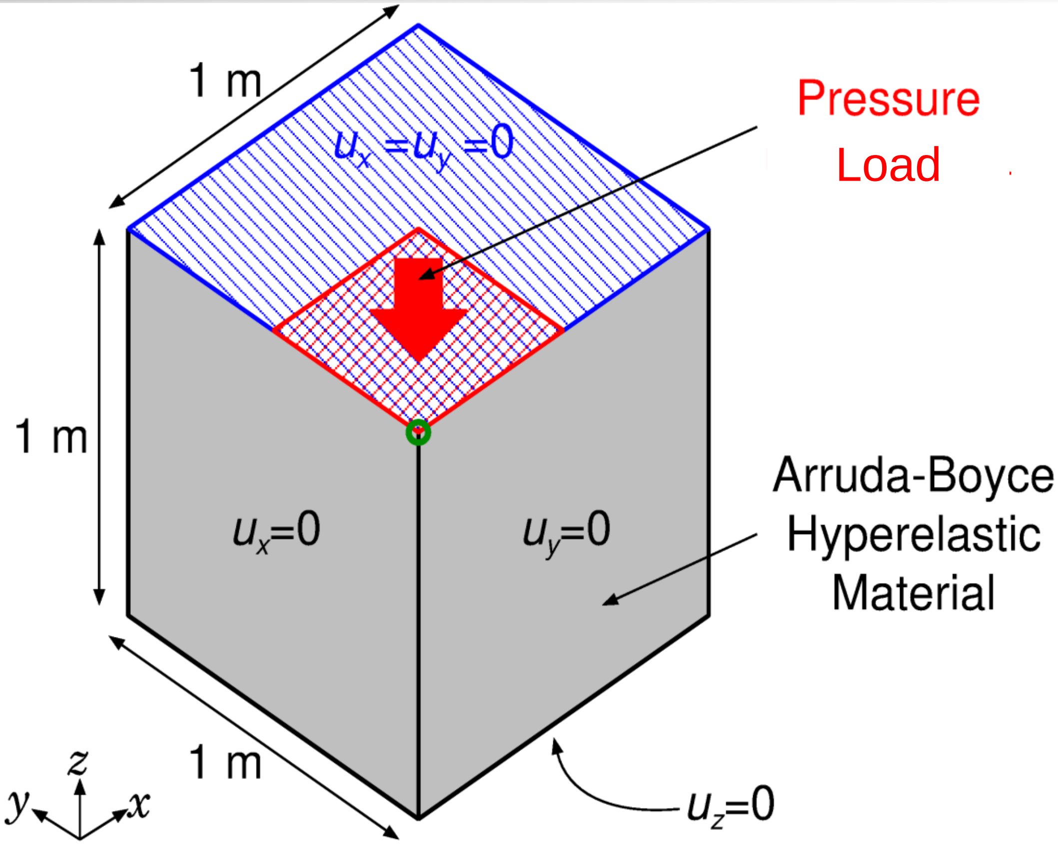


Selective CS-FEM-T10 | ABAQUS C3D10H | ABAQUS C3D10MH | ABAQUS C3D10HS

ABAQUS C3D10H and C3D10HS suffer from nodal force oscillation.

Compression of Hyperelastic Block

Outline

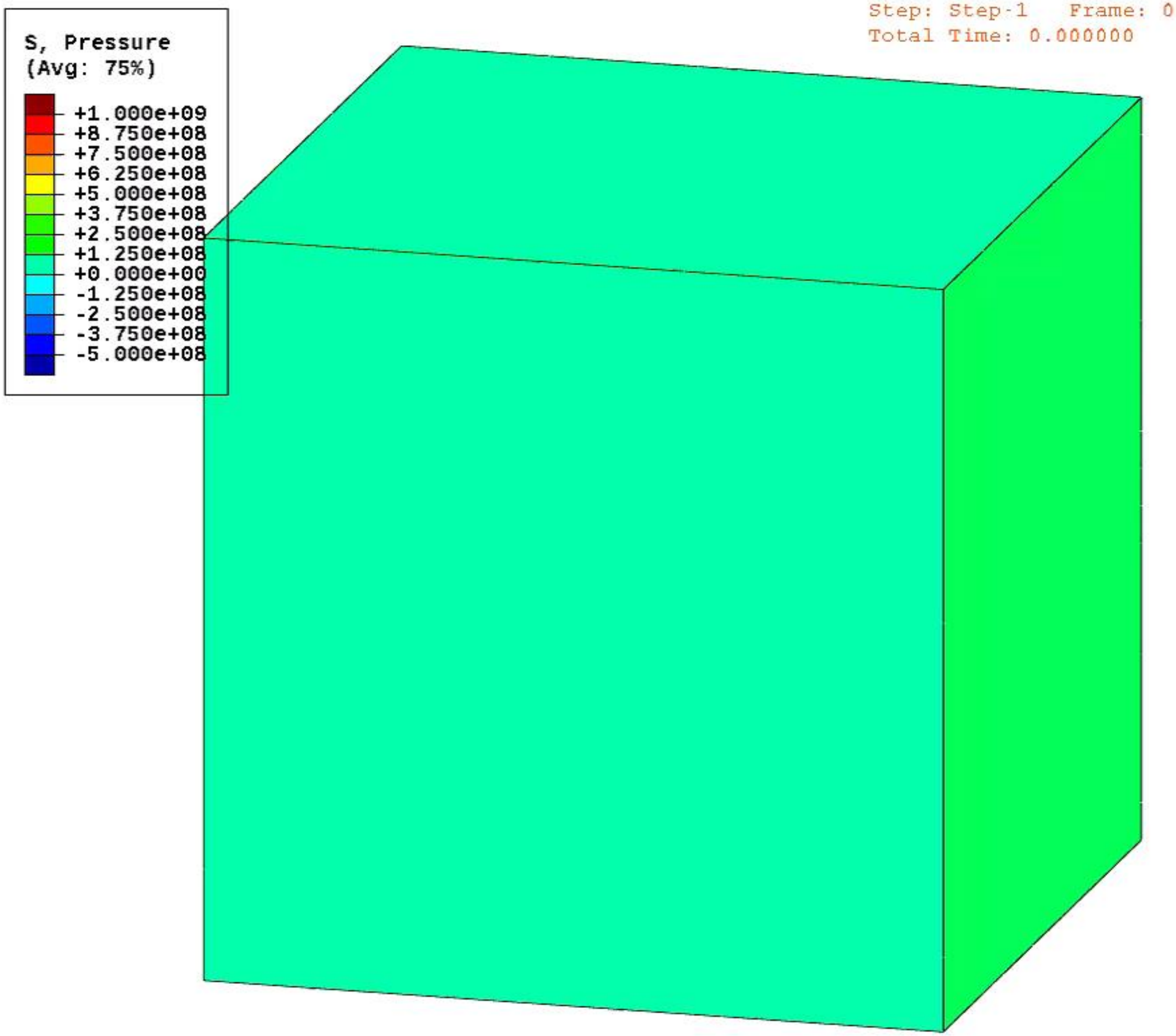


- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D10MH with the same unstructured T10 mesh.

Static Implicit

Compression of Hyperelastic Block

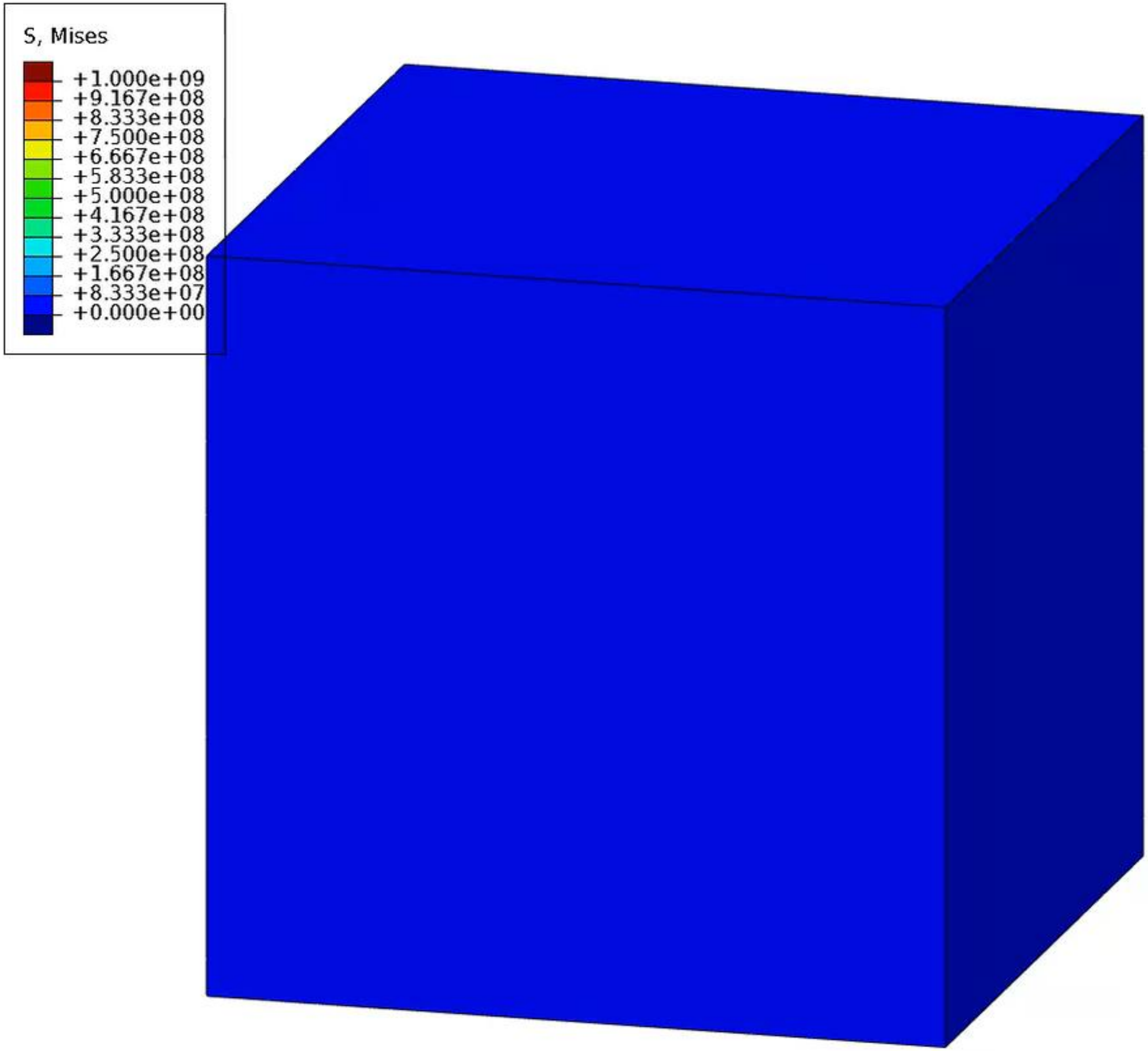
Animation of pressure dist. (ABAQUS C3D10MH)



Convergence failure at **0.7 GPa** pressure

Compression of Hyperelastic Block

<u>Animation of Mises stress dist. (ABAQUS C3D10MH)</u>



Convergence failure at **<u>0.7 GPa</u>** pressure

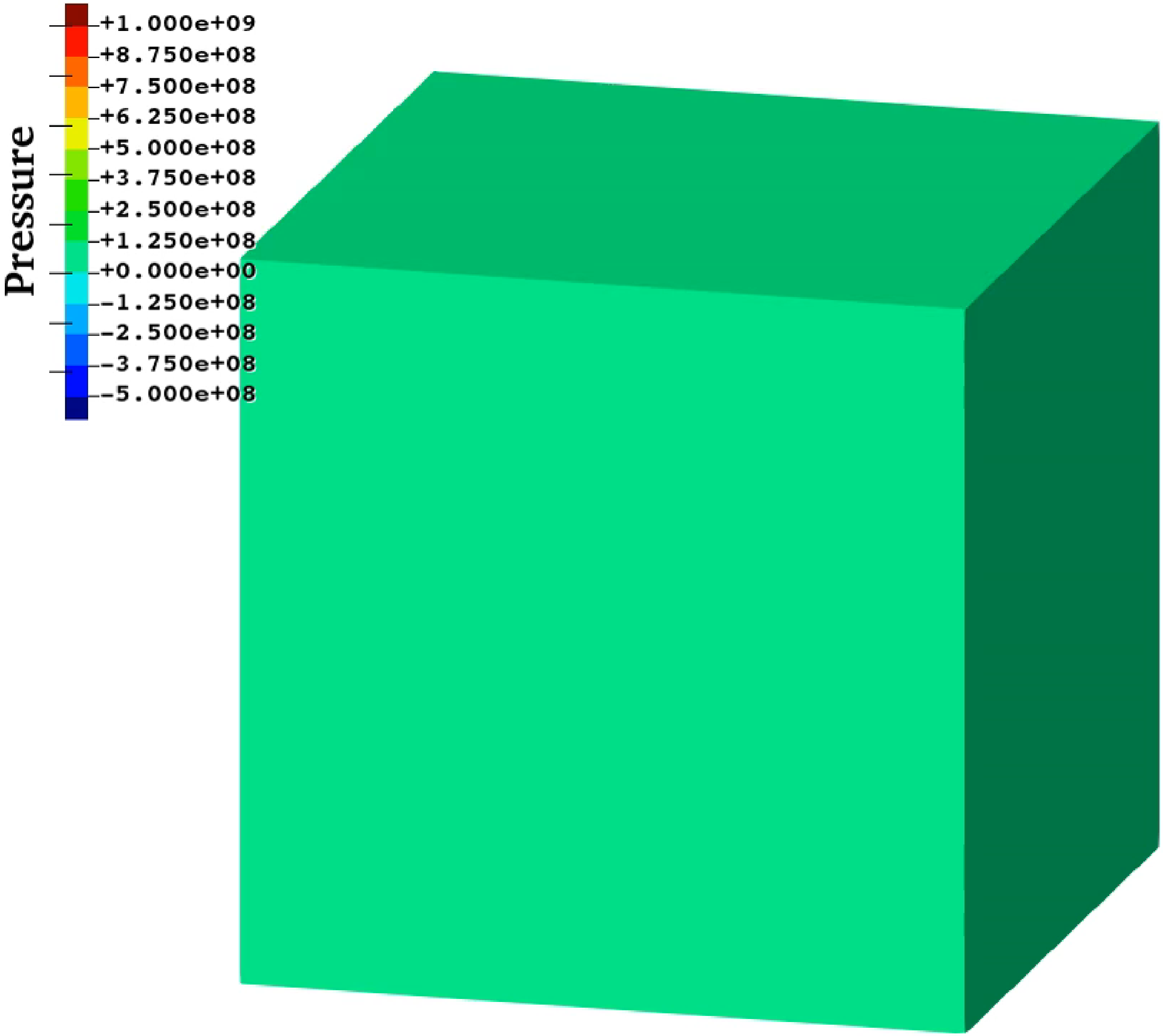
Static Implicit

Compression of Hyperelastic Block

Animation of pressure dist. (Selective CS-FEM-T10)

Convergence failure at **1.3 GPa** pressure

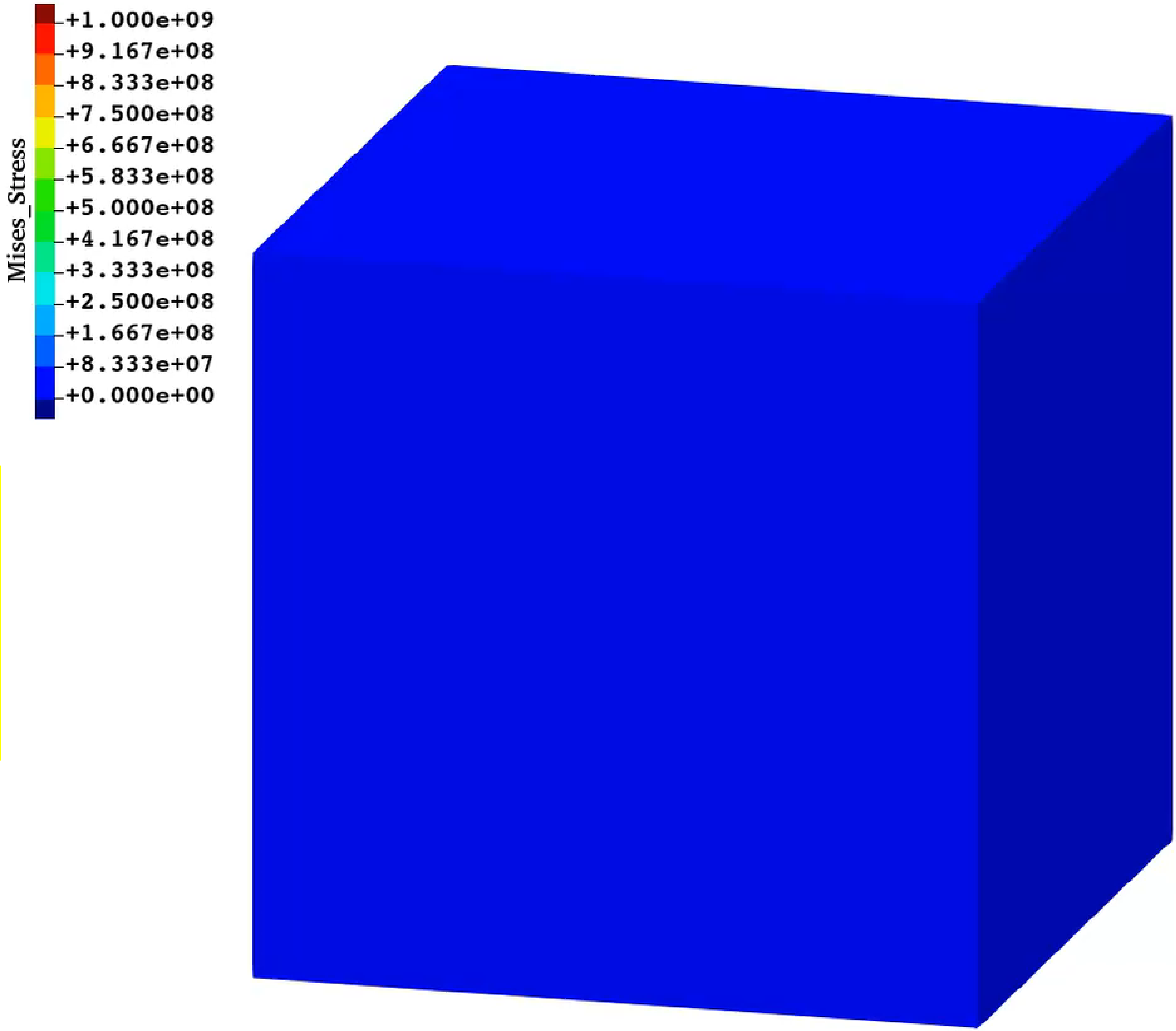
The present element is more **robust** than ABAQUS C3D10MH



Static Implicit

Compression of Hyperelastic Block

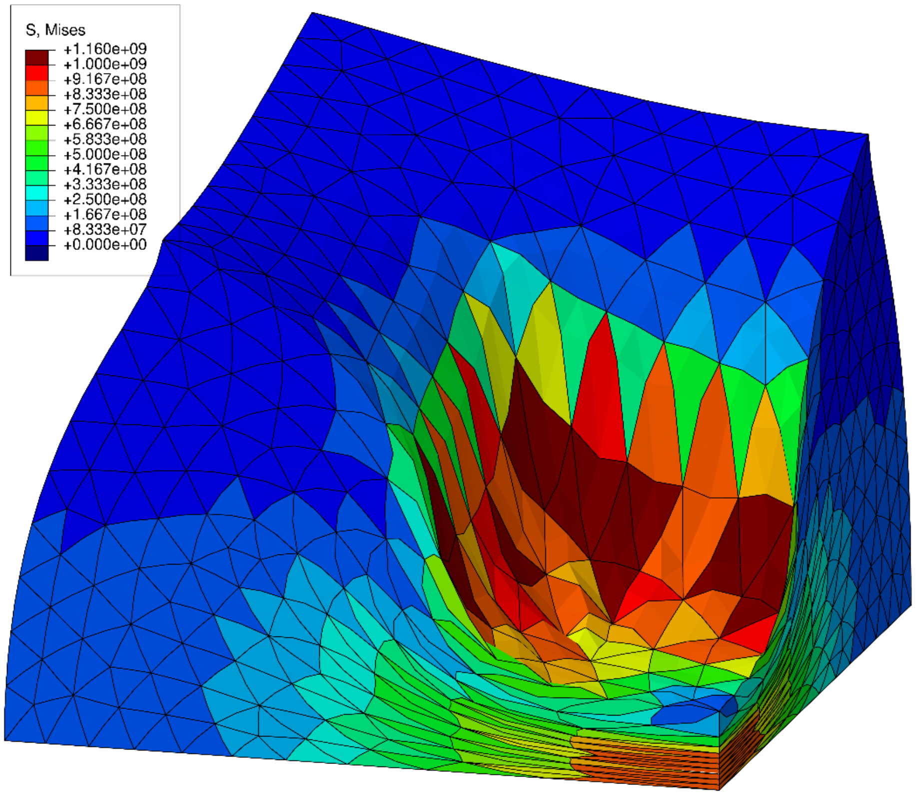
Animation of Mises stress dist. (Selective CS-FEM-T10)



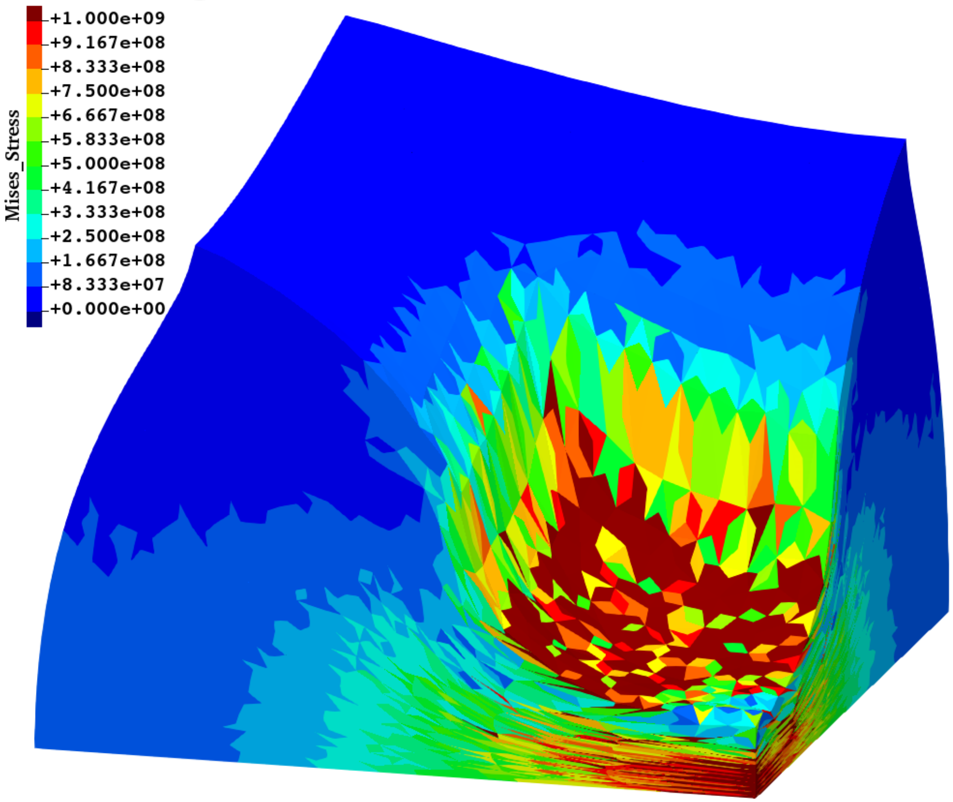
The present element presents Mises stress oscillation.

Compression of Hyperelastic Block

Misess stress dist. at 0.7 GPa pressre



ABAQUS C3D10MH



SelectiveCS-FEM-T10

Less smoothed Mises stress is observed in SelectiveCS-FEM-T10.
Further improvement is still required.

Characteristics of SelectiveCS-FEM-T10

Benefits

- ✓ Accurate
  (no locking, no checkerboarding, no force oscillation).
- ✓ Robust (long-lasting in large deformation).
- ✓ No increase in DOF (No static condensation).
- ✓ Same memory & CPU costs as the other T10 elements.
- ✓ Implementable to commercial FE codes
  (e.g., ABAQUS UEL).

Drawbacks

- ✗ Mises stress oscillation in same extreme analyses.
- ✗ No longer a T4 formulation.

SelectiveCS-FEM-T10 is competitive
with the best ABAQUS T10 element, C3D10MH.

Summary

Summary

One-sentence summary

SelectiveCS-FEM-T10 is already good enough for practical use as compared to ABAQUS Tet elements.

Take-home message

Please consider implementing SelectiveCS-FEM-T10 to your in-house code.
It’s supremely useful & easy to code!!

Thank you for your kind attention!

Appendix

Differences between Old and New

1. The new formulation has NO dummy node at the center of an element.
   - Fewer sub-elements and edges.
   - Asymmetric element.

2. The new formulation has No ES-FEM^{-1} after ES-FEM.
   - Strain & stress evaluation at edges.
   - No strain smoothing at frame edges.

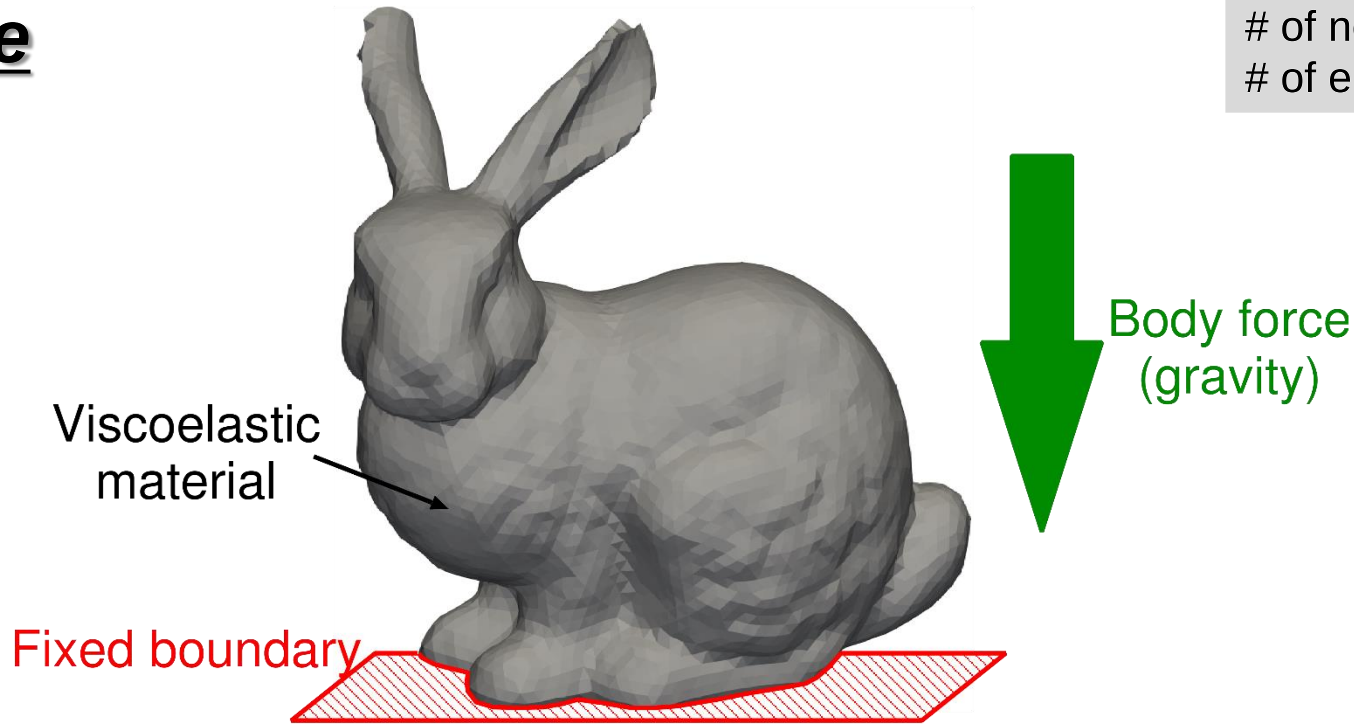
Intuitively, the lack of element symmetry and frame edge smoothing is not good for accuracy and stability; however, the new formulation is better in fact.

Its reason has not revealed yet.

Collapse Analysis of Viscoelastic Bunny

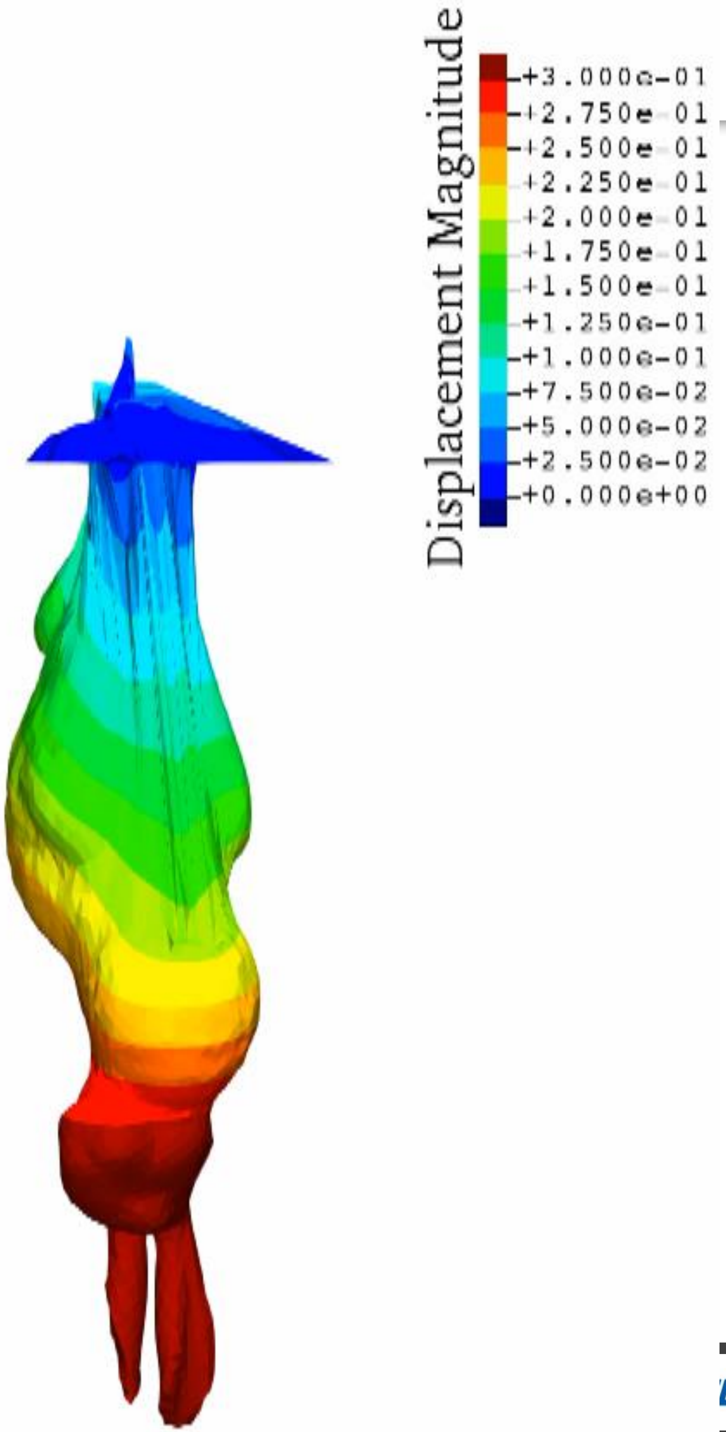
Outline

of nodes: 24136
# of elems: 126231



- Applying gravity to the Stanford Bunny and let it collapsed by its self-weight.
- Soft viscoelastic material ($\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10\ s$).
- Contact is NOT considered.
- Comparing F-barES-FEM-T4(2) and ABAQUS C3D4H.

Animation of Deformation

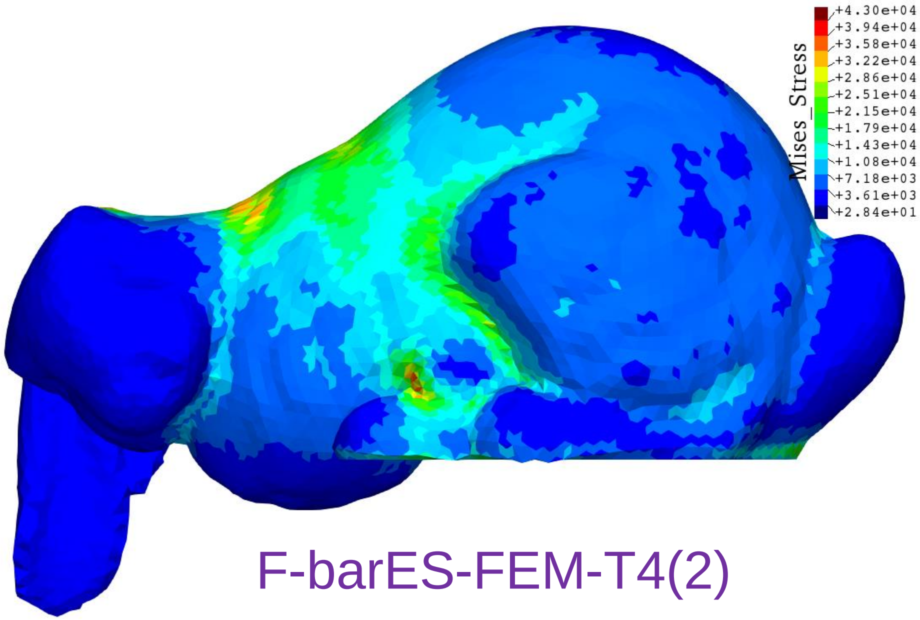


Because contact is not considered, the body penetrates the feet and finally becomes upside downside. The analysis lasts till the necking.

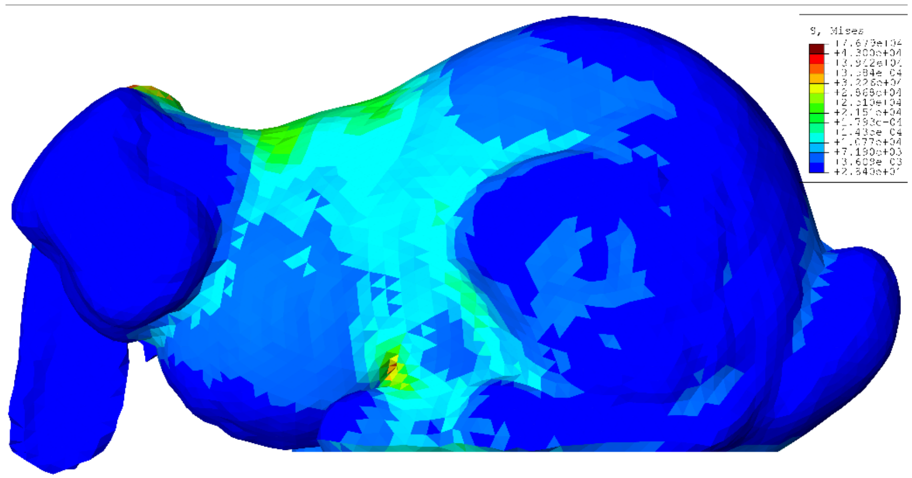
Viscous Implicit

Collapse Analysis of Viscoelastic Bunny

Mises stress dist. when C3D4H get a convergence failure



F-barES-FEM-T4(2)

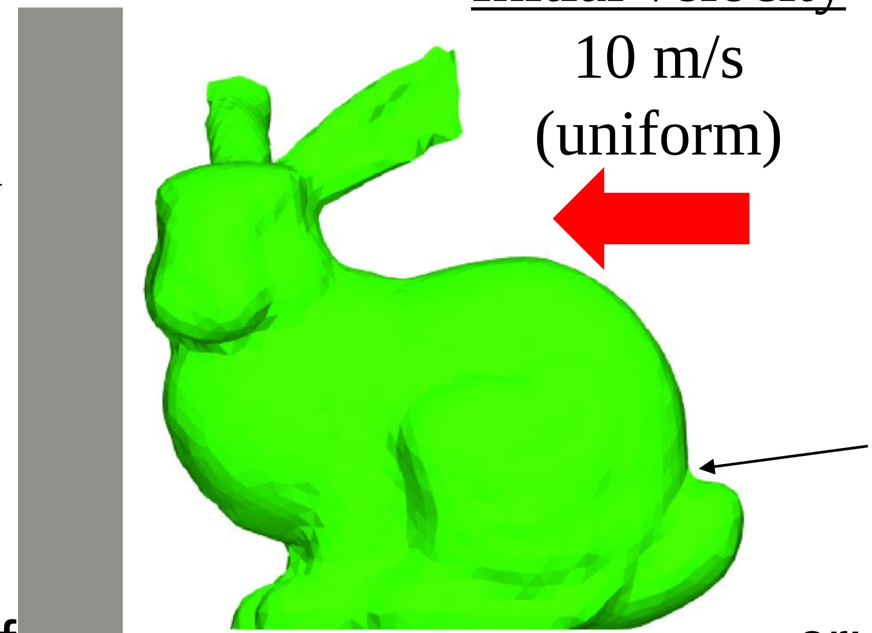


ABAQUS C3D4H

- ABAQUS C3D4H shows a stiffer result due to shear locking.
- The result of F-barES-FEM-T4 would be better.

Impact of Rubber Bunny

Outline



- A bunny made of rubber (Neo-Hookean) is crushed to a rigid wall.
- Compared with ABAQUS/Explicit C3D4 using a same T4 mesh.
- Note that neither Hex mesh nor hybrid elements is not available in this problem.

Impact of Rubber Bunny

Animation of Pressure Dist.

ABAQUS/Explicit
C3D4

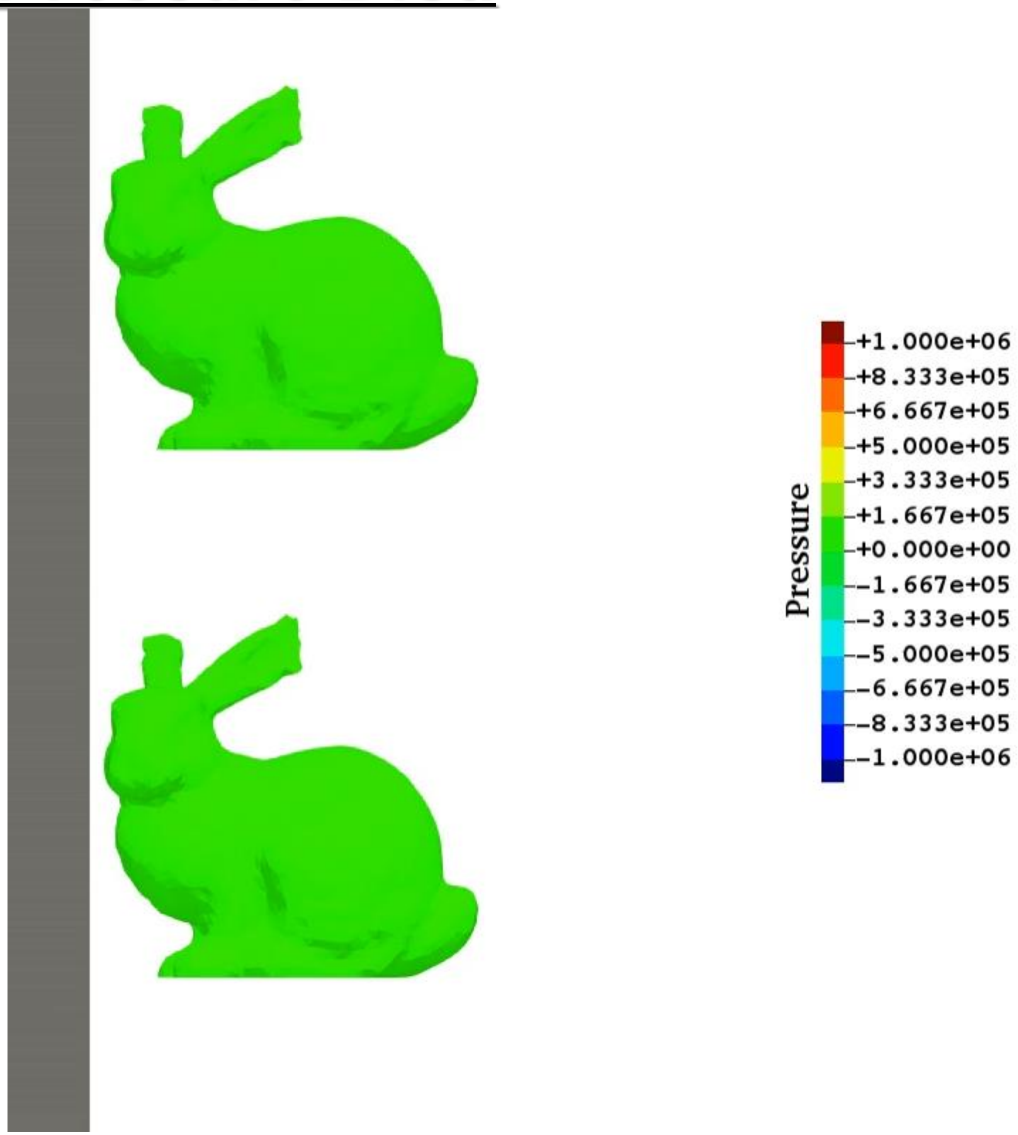
✗ Pressure Checkerboarding

✗ Shear Locking

SymF-barES-FEM-T4(1)

✓ Smooth pressure

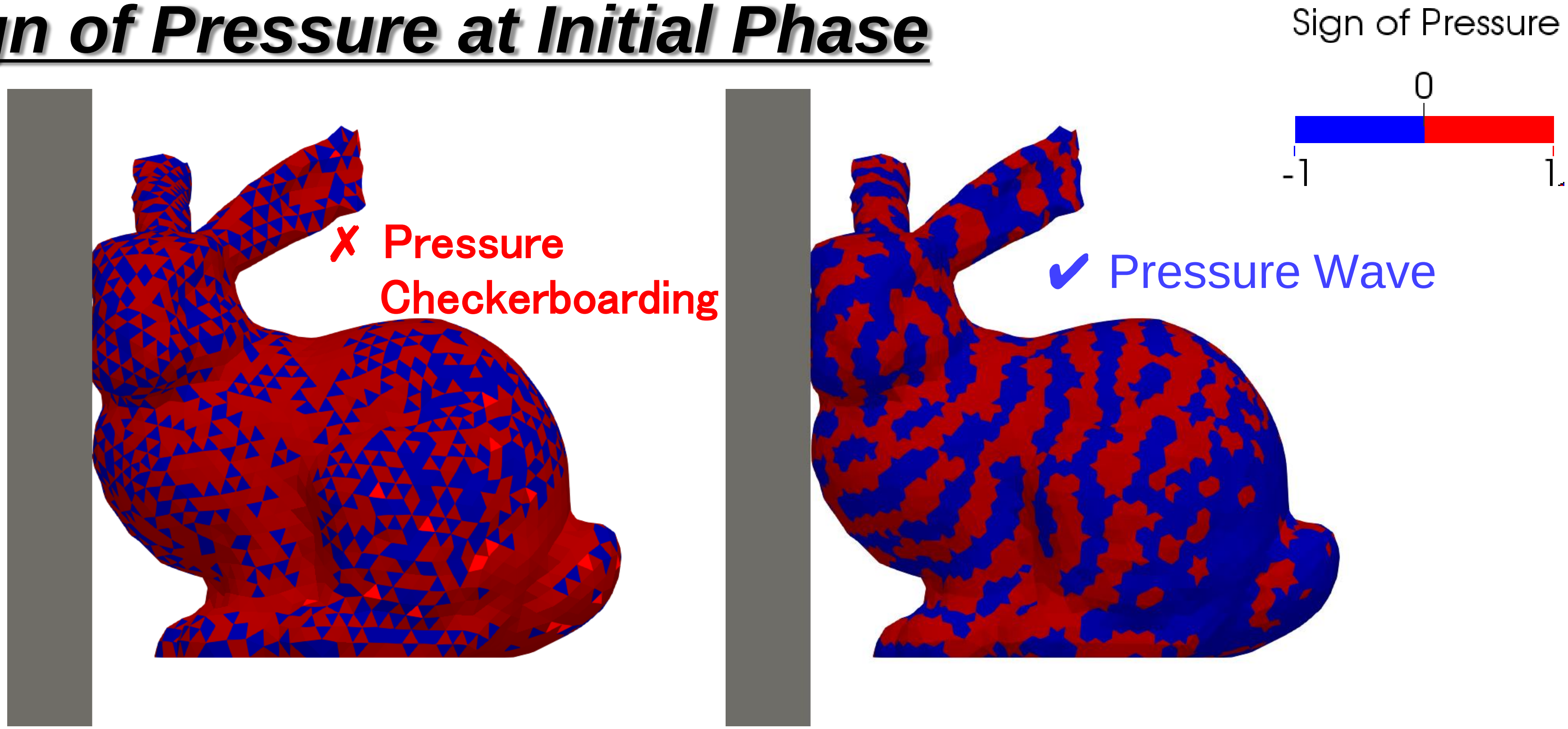
✓ No Locking



Dynamic Explicit

Impact of Rubber Bunny

Sign of Pressure at Initial Phase



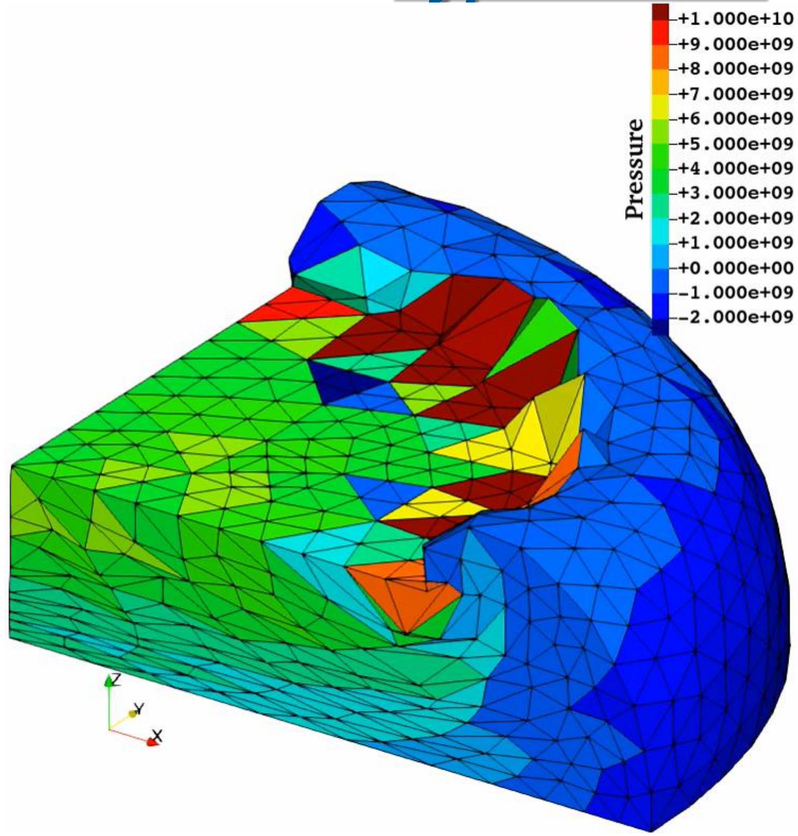
ABAQUS/Explicit C3D4
(Standard T4 element)

SymF-barES-FEM-T4(1)

The proposed S-FEM captures the pressure wave in a complex body successfully!!

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$



Same mesh as C3D10MH case.

As other S-FEMs, SelectiveCS-FEM-T10 has many varieties in the formulation.

The proposed method last year was **not an optimal formulation yet.**

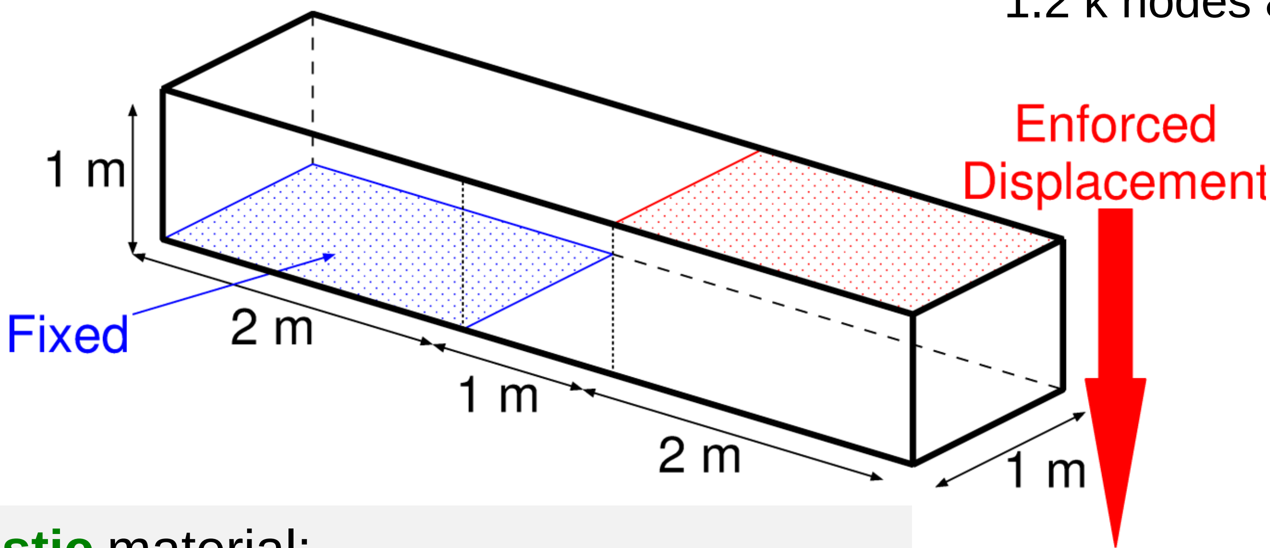
SelectiveCS-FEM-T10 (Old Ver.)

- ✓ No shar/voluemetric locking
- ✓ Little corner locking
- ✓ Little pressure checkerboarding
- ✓ Same cost & userbility as T10 elements.

Shear-tensioning of Elasto-plastic Bar

Outline

1.2 k nodes & 4.8 k elems.



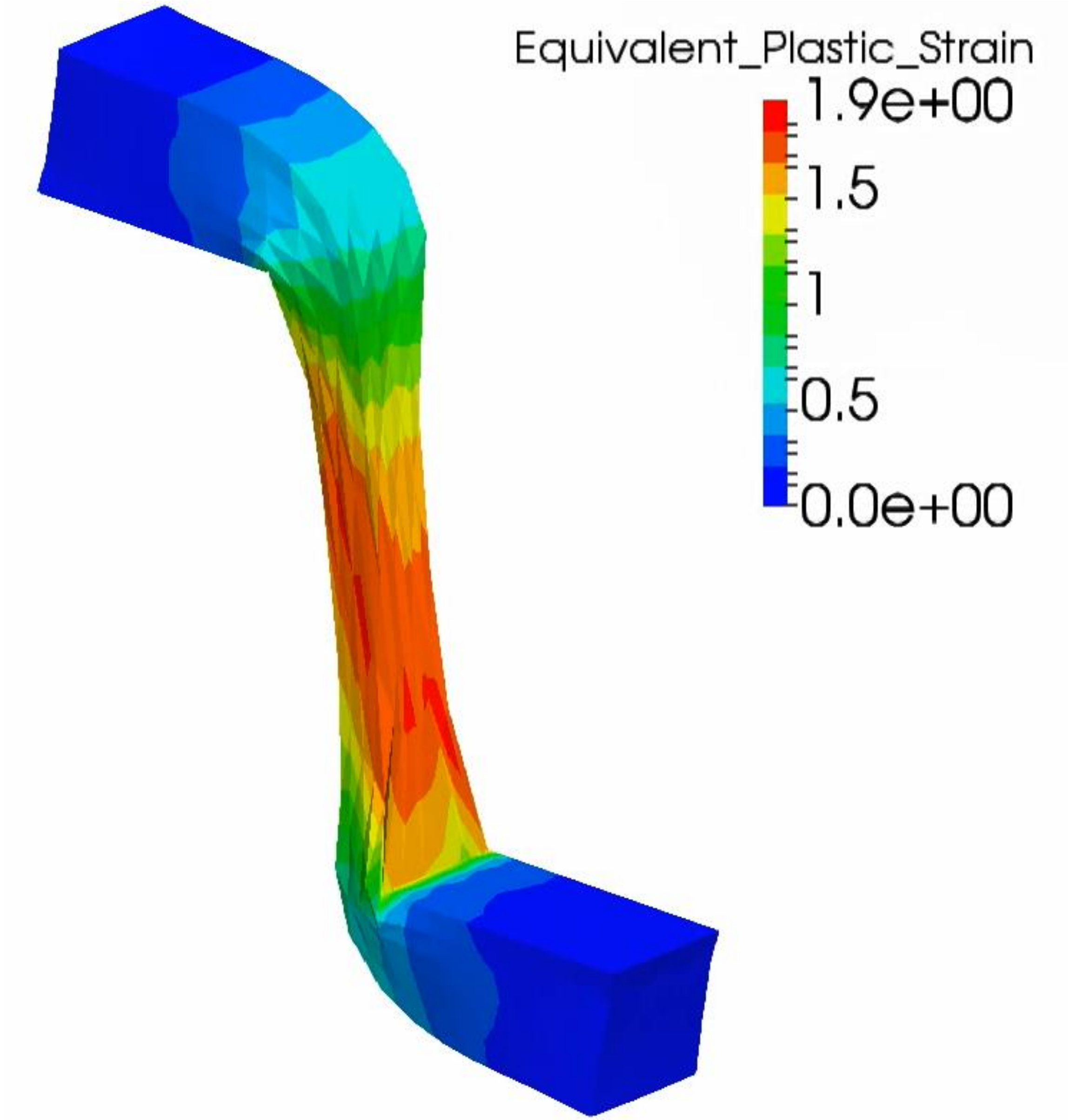
Elasto-plastic material:

- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Shear-tensioning of Elasto-plastic Bar

Result of
F-bar
ES-FEM(2)
(Equiv. plastic strain)

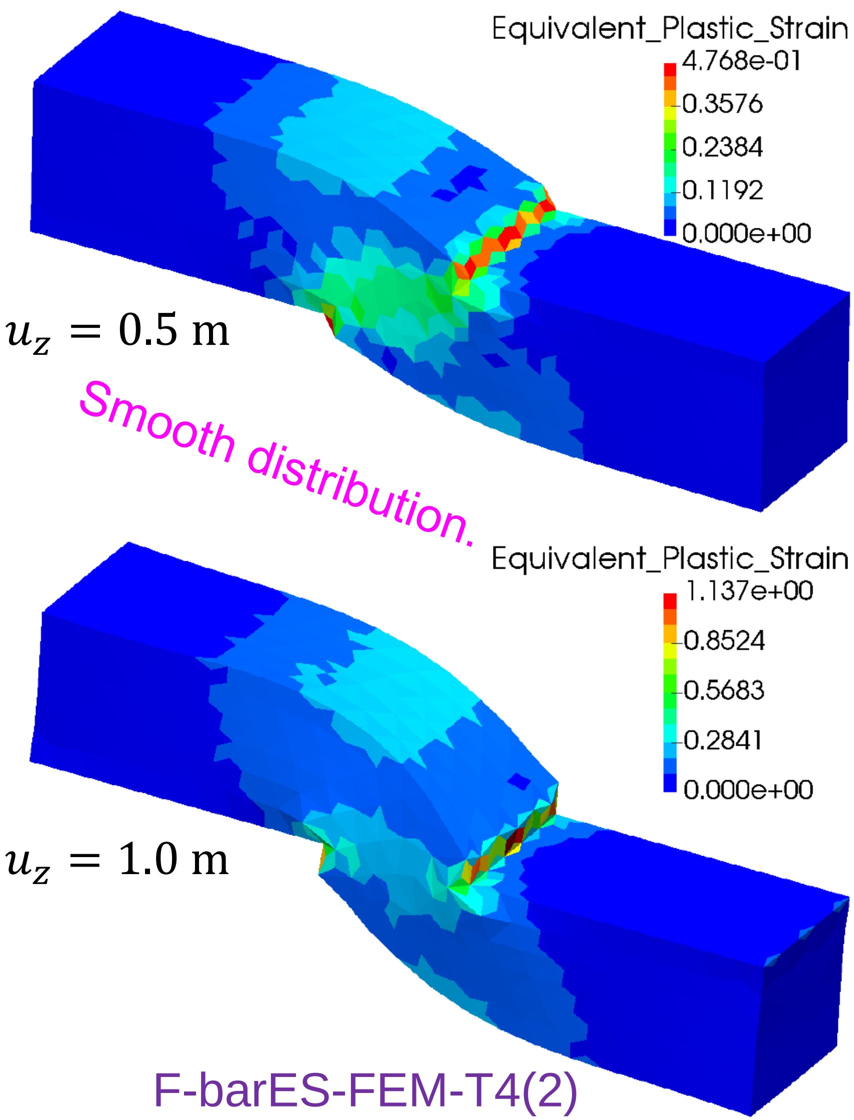


Extreme large deformation with smooth strain dist. is successfully achieved.

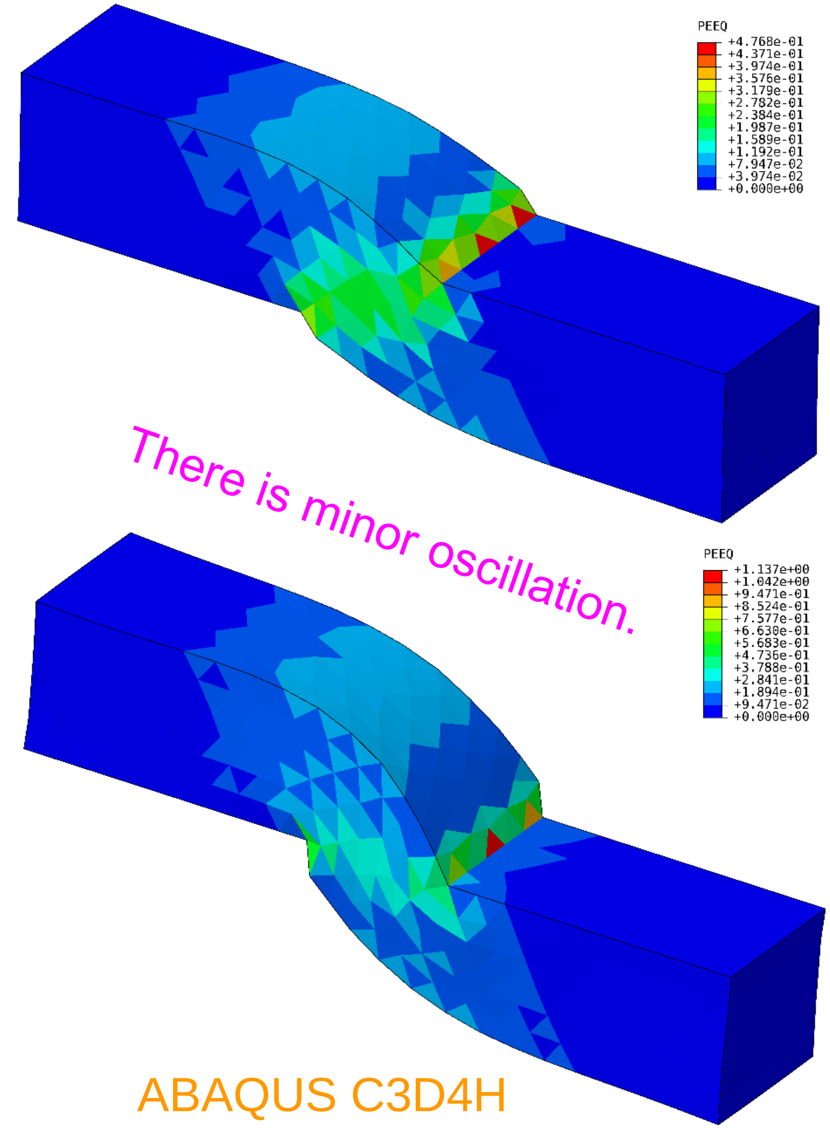
Static Implicit

Shear-tensioning of Elasto-plastic Bar

Equivalent plastic strain dist. in middle states



F-barES-FEM-T4(2)

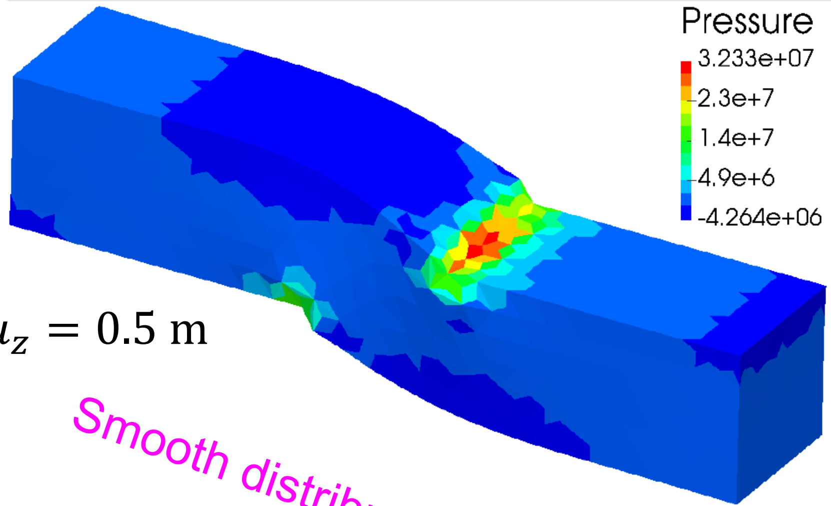


ABAQUS C3D4H

Static Implicit

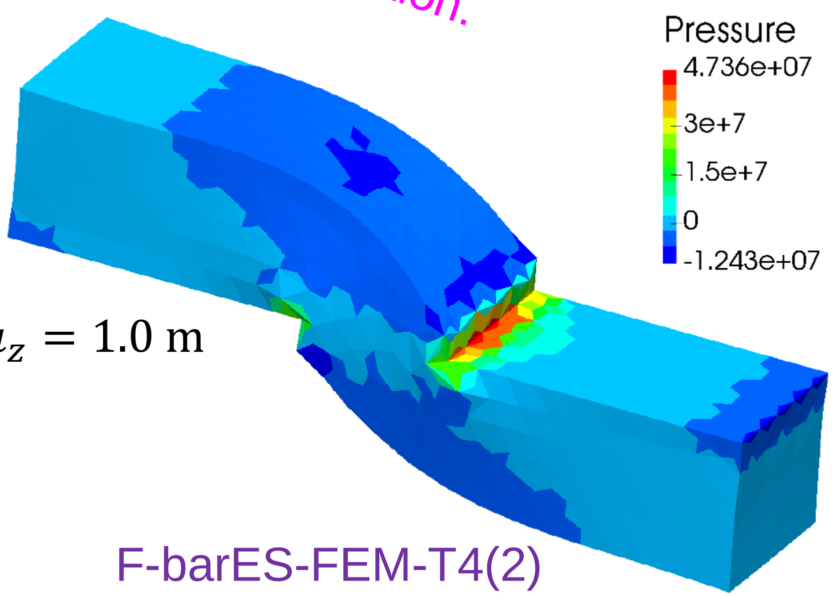
Shear-tensioning of Elasto-plastic Bar

Pressure dist. in middle states



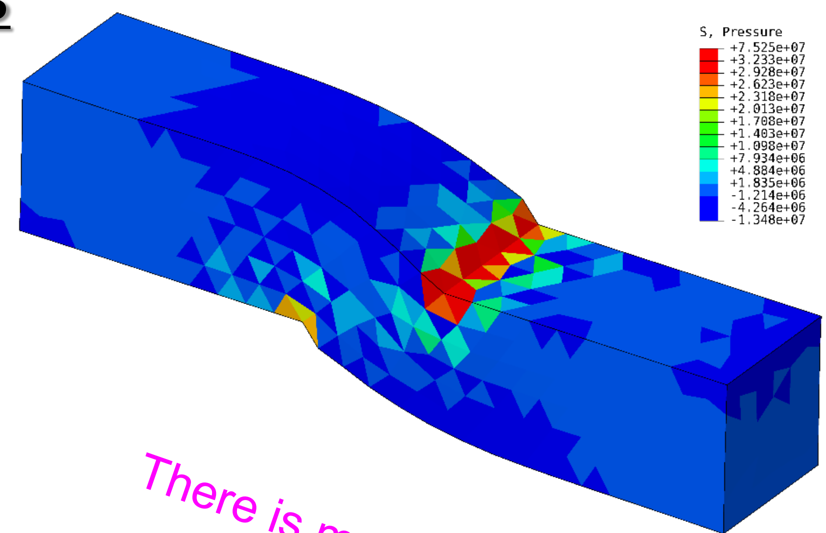
$u_z = 0.5\ \mathrm{m}$

Smooth distribution.

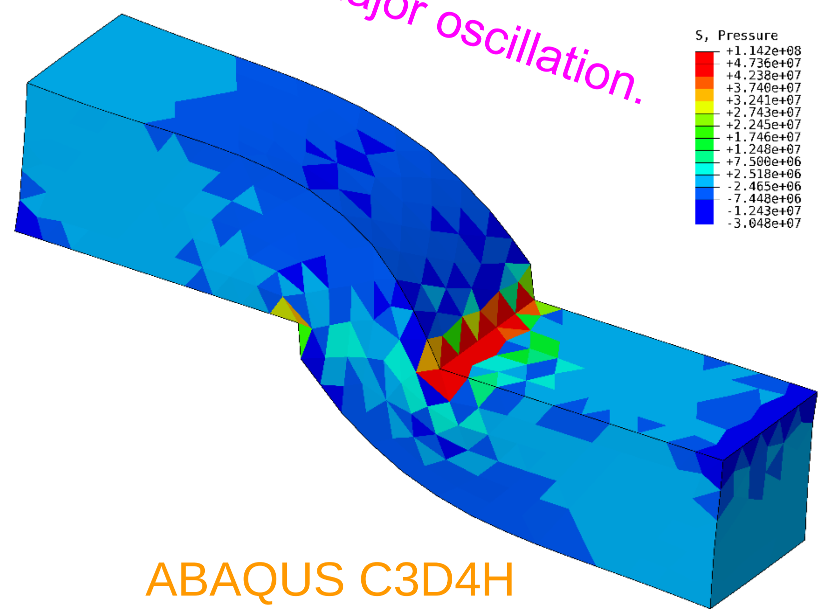


$u_z = 1.0\ \mathrm{m}$

F-barES-FEM-T4(2)



There is major oscillation.



ABAQUS C3D4H