F-bar aided edge-based smoothed finite element method with 4-node tetrahedral elements (F-barES-FEM-T4) for viscoelastic large deformation problems

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Motivation

What we want to do:

- Solve hyper large deformation analyses accurately and stably.
- Treat complex geometries with tetrahedral meshes.
- Consider nearly incompressible materials ($\nu \approx 0.5$).
- Support contact problems.
- Handle auto re-meshing.
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- **2nd or higher order elements:**
  - ✗ Volumetric locking. Accuracy loss in large strain due to intermediate nodes.

- **Enhanced assumed strain method (EAS):**
  - ✗ Spurious low-energy modes.

- **B-bar method, F-bar method, Selective reduced integration:**
  - ✗ Not applicable to tetrahedral element directly.

- **F-bar-Patch method:**
  - ✗ Difficulty in building good-quality patches.

- **u/p mixed (hybrid) method:**
  - (e.g., ABAQUS/Standard C3D4H and C3D10MH)
  - ✗ Pressure checkerboarding, Early convergence failure etc..
**Issues (cont.)**

*E.g.*) Compression of neo-Hookean hyperelastic body with $\nu_{\text{ini}} = 0.49$.

1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

# of Nodes is almost the same.
A Recent Solution

A new idea of FE formulation called “Smoothed Finite Element Method (S-FEM)” was recently proposed and is in researching today widely.

Our group has proposed a latest S-FEM named “F-barES-FEM-T4” (detailed later):

- No intermediate node & No additional DOF, (i.e., Purely displacement-based 4-node tetrahedral (T4) element),
- Free from shear, volumetric and corner locking,
- No pressure checkerboarding,
- Long lasting in large deformation.
A Recent Solution (cont.)

E.g.1) Compression of **hyperelastic** body with $\nu_{ini} = 0.49$

F-barES-FEM-T4 (One of the latest S-FEM)
- No shear/volumetric locking
- No corner locking
- No pressure checkerboarding

Same mesh as C3D4H case.
E.g.2) Explicit dynamic twist of hyperelastic body with $\nu_{\text{ini}} = 0.49$
E.g.3) Shear of elastoplastic body with soft hardening coeff.

1\textsuperscript{st} order hybrid T4 (C3D4H)
- ✔ No volumetric locking
- ✗ Shear locking
- ✗ Pressure checkerboarding

F-barES-FEM-T4
- ✔ No volumetric locking
- ✔ No shear locking
- ✔ No pressure checkerboarding

We have evaluated F-barES-FEM-T4 in elastic and elastoplastic cases but NOT in viscoelastic cases yet.
To applying and demonstrate the latest S-FEM called F-barES-FEM-T4 to viscoelastic large deformation problems.

Table of Body Contents

- Introduction of F-barES-FEM-T4’s formulation
- Demonstration of F-barES-FEM-T4 in viscoelastic problems
- Summary
Introduction of F-barES-FEM-T4's formulation
1. Brief of **Edge-based S-FEM (ES-FEM)**

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[\text{Edge } B]$.
- Calculate $F, T, \{f^{\text{int}}\}$ etc. in each edge smoothing domain.

As if putting an integration point on each edge center

- ✗ Volumetric locking
- ✗ Pressure checkerboarding
- ✓ No shear locking
- ✓ No spurious modes
2. Brief of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[\text{Node } B]$.
- Calculate $F, T, \{f^{\text{int}}\}$ etc. in each node smoothing domain.

As if putting an integration point on each node

- [ ] Spurious low-energy mode
- [✓] Less pressure checkerboarding
- [✓] No shear locking
- [✓] No volumetric locking
3. Brief of F-bar Method

**Algorithm**

1. Calculate deformation gradient $F$ at the element center, and then make the relative volume change $\bar{J} (= \det(F))$.

2. Calculate deformation gradient $F$ at each gauss point as usual, and then make $F^{iso} (= F / J^{1/3})$.

3. Modify $F$ at each gauss point to obtain $\bar{F}$ as
   \[
   \bar{F} = \bar{J}^{1/3} F^{iso}.
   \]

4. Use $\bar{F}$ to calculate the stress $T$, nodal force $\{f^{int}\}$ etc..

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.
Outline of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

Outline

- Edge $\vec{F}_{iso}$ is given by ES-FEM.
- Edge $\vec{J}$ is given by cyclically applied NS-FEM.
- Edge $\vec{F}$ is calculated in the manner of F-bar method:
  \[ \text{Edge } \vec{F} = \text{Edge } \vec{J} \cdot \frac{1}{3} \text{ Edge } \vec{F}_{iso}. \]
Outline of F-barES-FEM (cont.)

Brief Formulation

1. Make $\text{Elem } F$ as usual and calculate $\text{Elem } J$.

2. Smooth $\text{Elem } J$ at nodes and get $\text{Node } \tilde{J}$.

3. Smooth $\text{Node } \tilde{J}$ at elements and get $\text{Elem } \tilde{J}$.

4. Repeat 2. and 3. as necessary ($c$ times).

5. Smooth $\text{Elem } \tilde{J}$ at edges and get $\text{Edge } \overline{J}$.

6. Combine $\text{Edge } \overline{J}$ and $\text{Edge } F_{\text{iso}}$ of ES-FEM as

   \[
   \text{Edge } \overline{F} = \text{Edge } \overline{J} \frac{1}{3} \text{ Edge } F_{\text{iso}}.
   \]

Hereafter, F-barES-FEM-T4 with $c$ cycles of smoothing is called “F-barES-FEM-T4($c$)”. 

Cyclic Smoothing of $J$

A kind of low-pass filter

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How to Treat Viscoelastic Model

The target constitutive model to treat is the Hencky’s viscoelastic model based on the generalized Maxwell model.

### Stress

\[
\begin{align*}
\mathbf{T}^\text{hyd} &= K \operatorname{tr}(\mathbf{H}) \mathbf{I}, \\
\mathbf{T}^\text{dev} &= 2G_0 \left( \mathbf{H}^\text{dev} - \sum g_i \mathbf{H}_i^v \right).
\end{align*}
\]

- Bulk modulus
- Hencky (Logarithmic) strain
- Instantaneous shear modulus
- Prony coeff
- Viscous strain
- Viscosity only in deviatoric stress.

### Time advance of viscous strain

\[
\mathbf{H}_i^{v+} = \mathbf{R} \cdot \mathbf{H}_i^v \cdot \mathbf{R}^T + \Delta \mathbf{H}_i^v.
\]

- Rigid rotation in an increment
- Viscous strain increment

### Equation to solve

\[
[K]\{u\} = \{f\}
\]

Same as static problems due to the absence of inertia.
Demonstration of F-barES-FEM-T4 in viscoelastic problems
Tensile Suspension of Viscoelastic Block

Outline

- 1 m × 2 m × 3 m block subjected to 100% stretch in 10 s, and hold the enforced displacement for 1000 s under the gravity.
- Hencky’s viscoelastic body based on the generalized Maxwell model with 1 maxwell element & 1 long-term spring.
  - Poisson’s ratio: \( \nu_0 = 0.3 \), and \( \nu_\infty = 0.49 \).
  - Relaxation time: \( \tau = 10 \text{ s} \).
- Compare the results of F-barES-FEM-T4(2), ABAQUS C3D4, C3D4H, and C3D8.
Tensile Suspension of Viscoelastic Block

Animation of Mises Stress (F-barES-FEM-T4(2))

Mises Stress

Stress relaxation progresses after stretch.
Tensile Suspension of Viscoelastic Block

Pressure at the end of stretch (common contour range)

ABAQUS C3D4

ABAQUS C3D4H

Checkerboarding

ABAQUS C3D8

F-barES-FEM-T4(2)

No Checkerboarding!!
Tensile Suspension of Viscoelastic Block

**Pressure at the final state** (common contour range)

- **ABAQUS C3D4**
  - Small Deflection
- **ABAQUS C3D4H**
  - Severe Checkerboarding
- **ABAQUS C3D8**
  - No Checkerboarding!!
- **F-barES-FEM-T4(2)**
  - Pressure

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(Images of pressure contour plots with varying elements and defect visualizations.)
ABAQUS’s T4 elements cannot avoid shear locking, whereas F-barES-FEM-T4 has good accuracy as H8-SRI element.
Tensile Drooping of Viscoelastic Twisted Prism

Outline

■ 180º twisted prism having a cross-section of a right triangle with 3, 4 and 5 m edges.
■ 50% vertical stretch in 10 s and drooping under gravity.
■ Same viscoelastic properties: ($\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10 \text{ s}$)
■ Solved by ABAQUS C3D4H and F-barES-FEM-T4(2).

Quick Enforced Displacement up to 5 m and Hold

Gravity

10 m

y

z

x

4 m

3 m

Fixed

Quick Enforced Displacement up to 5 m and Hold
Tensile Drooping of Viscoelastic Twisted Prism

Mises Stress Dist.
(F-barES-FEM-T4(2))

- Continuous stretch in the upper part after the enforced stretch.
- Stress relaxation in the lower part.
Tensile Drooping of Viscoelastic Twisted Prism

Pressure dist. at the final state (common contour range)

Seems unnaturally hard.

Seems naturally soft.
**Pressure sign dist. at the final state**

- **ABAQUS C3D4H**
- **F-barES-FEM-T4(2)**

No Checkerboarding!!

Pressures:
- S, Pressure
  - $+2.730e+05$
  - $+1.000e+00$
  - $+0.000e+00$
  - $-1.000e+00$
  - $-1.521e+05$

Pressure checkerboarding.

Title: Tensile Drooping of Viscoelastic Twisted Prism
Summary
Benefits and Drawbacks of F-barES-FEM-T4

Benefits

✓ Locking-free with 1\textsuperscript{st} order tetra meshes.
   No difficulty in severe strain or contact analysis.
✓ No increase in DOF.
   Purely displacement-based formulation.
✓ Long lasting.
✓ Less pressure checkerboarding.

Drawbacks

✗ The more cyclic smoothing necessitates
   the more CPU time due to the \textit{wider bandwidth}.

More stable & accurate than other T4 elements!!!

Slower than other T4 elements…
F-barES-FEM-T4 is the current best T4 FE formulation for the large deformation with near incompressibility:

- Rubber-like materials,
- Elastoplastic materials, and
- Viscoelastic materials.

Thank you for your kind attention!
Appendix
Characteristics of $[K]$ in F-barES-FEM-T4

✓ No increase in DOF.
  (No Lagrange multiplier. No static condensation.)

✓ Positive definite.

✗ Wider bandwidth.
  In case of standard unstructured T4 meshes,

<table>
<thead>
<tr>
<th>Method</th>
<th>Approx. Bandwidth</th>
<th>Approx. Ratio</th>
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</thead>
<tbody>
<tr>
<td>Standard FEM-T4</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>F-barES-FEM-T4(1)</td>
<td>390</td>
<td>x10</td>
</tr>
<tr>
<td>F-barES-FEM-T4(2)</td>
<td>860</td>
<td>x20</td>
</tr>
<tr>
<td>F-barES-FEM-T4(3)</td>
<td>1580</td>
<td>x40</td>
</tr>
<tr>
<td>F-barES-FEM-T4(4)</td>
<td>2600</td>
<td>x65</td>
</tr>
</tbody>
</table>

✗ Ill-posedness in nearly incompressible cases.
  (No improvement in condition number.)
Tensile Suspension of Viscoelastic Block

Mises stress at the end of stretch (common contour range)

ABAQUS C3D4

ABAQUS C3D4H

ABAQUS C3D8

F-barES-FEM-T4(2)

Checkerboarding

Mises Stress
-1.5996e+05
-4.4294e+05
-3.2884e+05
-2.2444e+05
-1.2004e+05
-1.1996e+05
Mises stress at the final state (common contour range)

ABAQUS C3D4

ABAQUS C3D4H

ABAQUS C3D8

F-barES-FEM-T4(2)

Checkerboarding

Mises Stress

F-barES-FEM-T4(2)