SelectiveCS-FEM-T10: Selective cell-based smoothed finite element methods with 10-node tetrahedral elements

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What is S-FEM?

- **Smoothed** finite element method (S-FEM) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example in 2D triangular mesh:

- Standard FEM
- Edge-based S-FEM (ES-FEM)
- Node-based S-FEM (NS-FEM)
How popular is S-FEM?

Number of journal papers written in English whose title contains “smoothed finite element”:

The attraction of S-FEM is expanding continuously.
Applications of S-FEMs in Our Lab

- Solid mechanics
  - Static Implicit
  - Dynamic Explicit
  - Viscous Implicit

- Electrostatic
Motivation

What we want to do:

- Solve **hyper large deformation analyses** accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** ($\nu \approx 0.5$).
- Support **contact problems**.
- Handle **auto re-meshing**.

[Images of rubber, plastic/glass, metal]
Issues

Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2\textsuperscript{nd} or higher order elements:
  - \texttimes\hspace{1em} Volumetric locking.
  - Accuracy loss in large strain due to intermediate nodes.

- B-bar method, F-bar method, Selective reduced integration:
  - \texttimes\hspace{1em} Not applicable to tetrahedral element directly.

- F-bar-Patch method:
  - \texttimes\hspace{1em} Difficulty in building good-quality patches.

- \textit{u/p mixed (hybrid) method}:
  - (e.g., ABAQUS/Standard C3D4H and C3D10MH)
  - \texttimes\hspace{1em} Pressure checkerboarding, Early convergence failure etc..

- F-bar type smoothed FEM (F-barES-FEM-T4):
  - ✓\hspace{1em} Accurate & stable \hspace{1em} \texttimes\hspace{1em} Hard to implement in FEM codes.
Issues (cont.)

E.g.) Compression of neo-Hookean hyperelastic body with $\nu_{ini} = 0.49$

1\textsuperscript{st} order hybrid T4 (C3D4H)
- ✓ No volumetric locking
- ✗ Pressure checkerboading
- ✗ Shear & corner locking

2\textsuperscript{nd} order modified hybrid T10 (C3D10MH)
- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

# of Nodes is almost the same.
E.g.) Compression of neo-Hookean hyperelastic body with $\nu_{ini} = 0.49$.

Although F-barES-FEM-T4 is accurate and stable, it cannot be implemented in general-purpose FE software due to the adoption of ES-FEM. Also, it consumes larger memory & CPU costs.

Another approach adopting CS-FEM with T10 element would be effective.

F-barES-FEM-T4
✓ No shear/volumetric locking
✓ No corner locking
✓ No pressure checkerboarding
✓ No increase in DOF

Same mesh as C3D4H case.
Objective

To develop an S-FEM formulation using T10 mesh (SelectiveCS-FEM-T10) for severe large deformation analyses.

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Quick Introduction of F-barES-FEM-T4
— Why not T4 but T10? —
Brief of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[\text{Edge } B]$. 
- Calculate $F, T, \{f^{\text{int}}\}$ etc. in each edge smoothing domain.

As if putting an integration point on each edge center.

ES-FEM can avoid shear locking. However, it cannot be implemented in ordinary FE codes due to the strain smoothing across multiple elements...
Brief of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[\text{Node } B]$.
- Calculate $F, T, \{f^{\text{int}}\}$ etc. in each node smoothing domain.

As if putting an integration point on each node

- Spurious low-energy mode (or hour-glass mode)
- Less pressure checkerboarding
- No shear locking
- No volumetric locking
Concept of F-barES-FEM

Concept: combining ES-FEM and NS-FEM using F-bar method

Outline

- **Edge** $\overline{F}_{iso}$ is given by ES-FEM.
- **Edge** $\overline{J}$ is given by cyclically applied NS-FEM.
- **Edge** $\overline{F}$ is calculated in the manner of F-bar method:

  $$\text{Edge } \overline{F} = \frac{1}{3} \text{ Edge } \overline{J} \overline{F}_{iso}.$$
Deformation gradient of each edge \((\mathbf{F})\) is derived as

\[
\mathbf{F} = \mathbf{F}^{\text{iso}} \cdot \mathbf{F}^{\text{vol}}
\]

in the manner of F-bar method.
Formulation of F-barES-FEM (2 of 2)

Each part of $\overline{F}$ is calculated as

$$\overline{F} = \overline{F}^{\text{iso}} \cdot \overline{F}^{\text{vol}}$$

Isovolumetric part

- Smoothing the value of adjacent elements.
  - (same manner as ES-FEM)

Volumetric part

1. Calculating node’s value by smoothing the value of adjacent elements
2. Calculating elements’ value by smoothing the value of adjacent nodes
3. Repeating (1) and (2) a few times
Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

\[ \bar{F} = \bar{F}_{\text{iso}} \cdot \bar{F}_{\text{vol}} \]

- **Isovolumetric part**
  - Like a ES-FEM
  - 1. Shear locking free

- **Volumetric part**
  - Like a NS-FEM
  - 2. Little pressure oscillation
  - 3. Volumetric locking free with the aid of F-bar method
Arruda-Boyce hyperelastic material \((\nu_{\text{ini}} = 0.499)\).

- Applying pressure on \(\frac{1}{4}\) of the top face.
- Result of F-barES-FEM-T4 is compared to ABAQUS C3D4H with the same unstructured T4 mesh.
Compression of Rubber Block

**Pressure dist.**

Early stage  Middle stage  Later stage

ABAQUS C3D4H

F-bar ES-FEM-T4(3)

Smooth pressure distributions are obtained.
Several hard circular fillers are distributed in a square soft matrix rubber (neo-Hookean hyperelastic with $\nu_{ini} = 0.49$).

$E_{ini}$ of the filler is \textbf{100 times larger} than $E_{ini}$ of the matrix.

Left side is constrained and right side is displaced.

Valid Mises stress dist. is obtained after many time remeshings.
Shear-tensioning of Elasto-plastic cylinder with 3D Remeshing

- Aluminium cylinder subjected to enforced disp..
- Pure shear at the initial stage, but stretch dominates at the later stage.
- Necking occurs in the end.

Valid plastic strain dist. is obtained after many time remeshings.

Final stretch at the neck is more than 7000%.
Characteristics of F-barES-FEM-T4

✓ No increase in DOF.
  (No Lagrange multiplier. No static condensation.)
✓ Locking- & checkerboarding-free with T4 mesh.

✗ Higher costs in memory and CPU time due to wider bandwidth of $[K]$.

In case of standard unstructured T4 meshes:

<table>
<thead>
<tr>
<th>Method</th>
<th>Approx. Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard FEM-T4</td>
<td>40</td>
</tr>
<tr>
<td>F-barES-FEM-T4(1)</td>
<td>390</td>
</tr>
</tbody>
</table>

✗ Difficulty in implementation to existing FE codes due to the smoothing across elements. Critical Issue!!
Why Not T4 But T10?

If we cannot implement F-barES-FEM-T4 to existing FE codes, then we have to code everything in our in-house code for practical use.

For example in tire analyses:
- Material constitutive models,
- Structural elements,
- Cohesive elements,
- Contact functionality and so on.

Therefore, choosing S-FEM-T4 leads us to the long and winding road...

We gave up T4 and chose T10 for solid mechanics analyses.
Formulation of SelectiveCS-FEM-T10
Our new approach using T10 mesh.

Adopting CS-FEM having no smoothing across multiple elements, SelectiveCS-FEM-T10 becomes an independent finite element.

⇒ We can implement it as an element of existing FE code.

Same memory & CPU costs as the T10 elements.
Brief of Cell-based S-FEM (CS-FEM)

- Subdivide each element into some sub-element.
- Calculate $[\text{Sub}E \, B]$ at each sub-element.
- Calculate $F, T, \{f^{\text{int}}\}$ etc. in each sub-element.

As if putting an integration point on each sub-element

- Implementable as an independent finite element.
- Locking can be avoided with SRI etc.
Flowchart of SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity

(1) Subdivision without dummy nodes

(2) Dev. strain smoothing at edges

(3) Vol. strain smoothing with all sub-elements

(4) \( \{ f^{\text{int}} \} \) and \([K]\)
(1) Subdivision into T4 Sub-elements

- **Introduce no dummy node** (i.e., asymmetric element).
- **Subdivide a T10 element into eight T4 sub-elements** and calculate their $B$-matrices and strains.

The shape function **should not be quadratic** in large deformation analyses.
(2) Deviatoric Strain Smoothing

- Perform strain smoothing in the manner of ES-FEM (i.e., average dev. strains of sub-elements at edges).
- Evaluate deviatoric strain and stress at edges.

T4 sub-elements cause shear locking and thus **strain smoothing** is necessary.

From 8 sub-elements to 25 edges.
Treat the overall mean vol. strain of all sub-elements as the uniform element vol. strain (i.e., same approach as SRI elements).

The spatial order of vol. strain should be lower than that of dev. strain to avoid volumetric locking.
Apply SRI method to combine the Dev. & Vol. parts and obtain \( \{ f^{\text{int}} \} \) and \( [K] \).
Demonstration of SelectiveCS-FEM-T10
Bending of Hyperelastic Cantilever

Outline

- Neo-Hookean hyperelastic material
- Initial Poisson’s ratio: $\nu_0 = 0.49$
- Compared to ABAQUS C3D10MH (modified hybrid T10 element) with the same mesh.
Bending of Hyperelastic Cantilever

Comparison of the deflection disp. at the final state

No volumetric locking is observed.

ABAQUS C3D10MH

Selective CS-FEM-T10
Bending of Hyperelastic Cantilever

Comparison of the pressure dist. at the final state

Almost the same pressure distributions with no checkerboarding.

ABAQUS C3D10MH

Selective CS-FEM-T10
Comparison of the Mises stress dist. at the final state

Almost the same Mises stress distributions.
Barreling of Hyperelastic Cylinder

Outline

- Enforce axial displacement on the top face.
- Neo-Hookean body with \( \nu_{\text{ini}} = 0.49 \).
- Compare results with ABAQUS T10 hybrid elements (C3D10H, C3D10MH, C3D10HS) using the same mesh.
Barreling of Hyperelastic Cylinder

**Animation of Mises stress**

(ABAQUS C3D10MH)

Convergence failure at **24%** compression

Unnaturally oscillating distributions are obtained around the rim.
Barreloning of Hyperelastic Cylinder

Animation of Mises stress

(Selective CS-FEM-T10)

Smooth distributions are obtained except around the rim.

Convergence failure at 43% compression

The present element is more robust than ABAQUS C3D10MH
Barreling of Hyperelastic Cylinder

Comparison of Mises stress at 24% comp.

Selective CS-FEM-T10
ABAQUS C3D10H
ABAQUS C3D10MH
ABAQUS C3D10HS

All results are similar to each other except around the rim having stress singularity.
Barreling of Hyperelastic Cylinder

Comparison of pressure at 24% comp.

Selective CS-FEM-T10

ABAQUS C3D10H

ABAQUS C3D10MH

ABAQUS C3D10HS

All results are similar to each other except around the rim having stress singularity.
Barreling of Hyperelastic Cylinder

Comparison of nodal reaction force at 24% comp.

Selective CS-FEM-T10

ABAQUS C3D10H

ABAQUS C3D10MH

ABAQUS C3D10HS

ABAQUS C3D10H and C3D10HS suffer from nodal force oscillation.
Compression of Hyperelastic Block

Outline

- Arruda-Boyce hyperelastic material ($\nu_{ini} = 0.499$).
- Applying pressure on $\frac{1}{4}$ of the top face.
- Compared to ABAQUS C3D10MH with the same unstructured T10 mesh.
Compression of Hyperelastic Block

Animation of pressure dist. (ABAQUS C3D10MH)

Convergence failure at 0.7 GPa pressure
Compression of Hyperelastic Block

Animation of Mises stress dist. (ABAQUS C3D10MH)

Convergence failure at 0.7 GPa pressure
Compression of Hyperelastic Block

Animation of pressure dist. (Selective CS-FEM-T10)

Convergence failure at 1.3 GPa pressure

The present element is more robust than ABAQUS C3D10MH
Compression of Hyperelastic Block

**Animation of Mises stress dist.**

*(Selective CS-FEM-T10)*

The present element presents Mises stress oscillation.
Compression of Hyperelastic Block

**Mises stress dist. at 0.7 GPa pressure**

ABAQUS C3D10MH

SelectiveCS-FEM-T10

Less smoothed Mises stress is observed in SelectiveCS-FEM-T10. Further improvement is still required.
Characteristics of SelectiveCS-FEM-T10

**Benefits**

- ✓ Accurate
  (no locking, no checkerboarding, no force oscillation).
- ✓ Robust (long-lasting in large deformation).
- ✓ No increase in DOF (No static condensation).
- ✓ Same memory & CPU costs as the other T10 elements.
- ✓ Implementable to commercial FE codes (e.g., ABAQUS UEL).

**Drawbacks**

- ✗ Mises stress oscillation in same extreme analyses.
- ✗ No longer a T4 formulation.

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SelectiveCS-FEM-T10 is competitive with the best ABAQUS T10 element, C3D10MH.
Summary
Summary

One-sentence summary
SelectiveCS-FEM-T10 is already good enough for practical use as compared to ABAQUS Tet elements.

Take-home message
Please consider implementing SelectiveCS-FEM-T10 to your in-house code. It’s supremely useful & easy to code!!

Thank you for your kind attention!
Appendix
Differences between Old and New

1. The new formulation has **NO dummy node** at the center of an element.
   - Fewer sub-elements and edges.
   - Asymmetric element.

2. The new formulation has **No ES-FEM\(^{-1}\)** after ES-FEM.
   - Strain & stress evaluation at edges.
   - No strain smoothing at frame edges.

Intuitively, the lack of **element symmetry** and **frame edge smoothing** is not good for accuracy and stability; however, the new formulation is better in fact.

Its reason has not revealed yet.
Outline

- Applying gravity to the Stanford Bunny and let it collapsed by its self-weight.
- Soft viscoelastic material \( (\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10 \text{ s}) \).
- Contact is NOT considered.
- Comparing F-barES-FEM-T4(2) and ABAQUS C3D4H.
Because contact is not considered, the body penetrates the feet and finally becomes upside downside. The analysis lasts till the necking.
Collapse Analysis of Viscoelastic Bunny

Mises stress dist. when C3D4H get a convergence failure

- ABAQUS C3D4H shows a stiffer result due to shear locking.
- The result of F-barES-FEM-T4 would be better.
A bunny made of rubber (Neo-Hookean) is crushed to a rigid wall.

Compared with ABAQUS/Explicit C3D4 using a same T4 mesh.

Note that neither Hex mesh nor hybrid elements is not available in this problem.

Rubber body

\[
\begin{align*}
E &= 6.0 \text{ MPa} \\
\nu &= 0.49 \\
\rho &= 920 \text{ kg/m}^3
\end{align*}
\]
Impact of Rubber Bunny

Animation of Pressure Dist.

ABAQUS/Explicit
C3D4
× Pressure Checkerboarding
× Shear Locking

SymF-barES-FEM-T4(1)
✓ Smooth pressure
✓ No Locking
Impact of Rubber Bunny

Sign of Pressure at Initial Phase

ABAQUS/Explicit  C3D4 (Standard T4 element)  SymF-barES-FEM-T4(1)

The proposed S-FEM captures the pressure wave in a complex body successfully!!


**E.g.)** Compression of neo-Hookean *hyperelastic* body with $\nu_{ini} = 0.49$

As other S-FEMs, SelectiveCS-FEM-T10 has many varieties in the formulation.

The proposed method last year was *not an optimal formulation yet.*

**SelectiveCS-FEM-T10 (Old Ver.)**
- No shear/volumetric locking
- Little corner locking
- Little pressure checkerboarding
- Same cost & usability as T10 elements.
Shear-tensioning of **Elasto-plastic** Bar

**Outline**
- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

**Elasto-plastic** material:
- Hencky elasticity with $E = 1 \text{ GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_Y = 1 \text{ MPa}$ and $H = 0.1 \text{ GPa}$ (constant).

1.2 k nodes & 4.8 k elems.
Shear-tensioning of *Elasto-plastic* Bar

**Result of**

*F-bar ES-FEM(2)*

*(Equiv. plastic strain)*

Extreme large deformation with smooth strain dist. is successfully achieved.
Shear-tensioning of Elasto-plastic Bar

Equivalent plastic strain dist. in middle states

\[ u_z = 0.5 \text{ m} \]

Smooth distribution.

\[ u_z = 1.0 \text{ m} \]

There is minor oscillation.

F-barES-FEM-T4(2)  ABAQUS C3D4H
Shear-tensioning of Elasto-plastic Bar

Pressure dist. in middle states

$u_z = 0.5 \text{ m}$

Smooth distribution.

$u_z = 1.0 \text{ m}$

There is major oscillation.

F-barES-FEM-T4(2)

ABAQUS C3D4H